X-ray tomography: history, principles, and algorithms

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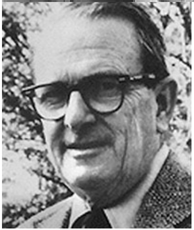
Outline

History

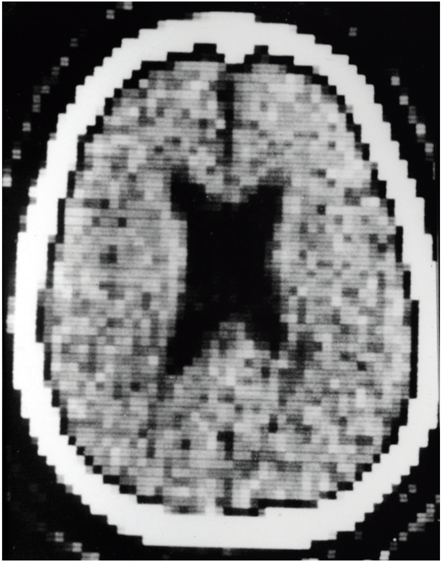
Modelling X-ray attenuation

Matrix model approach to tomography

Godfrey Hounsfield and Allan McLeod Cormack were the first to develop X-ray tomography



Cormack (left) and Hounsfield (top) received Nobel prizes in 1979. Right: an early tomographic image.



Reconstruction of a function from its line integrals was first invented by Johann Radon in 1917

$$f(P) = -\frac{1}{\pi}\int_0^\infty \frac{d\overline{F_p}(q)}{q}$$

This is an illustration of the Radon transform

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Tomographic reconstruction using the classical Filtered Back-Projection (FBP) algorithm

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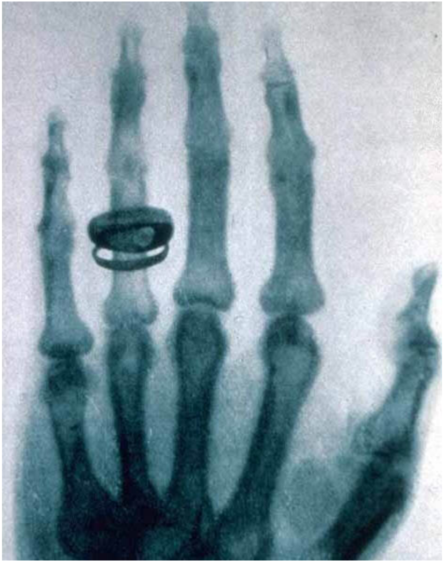
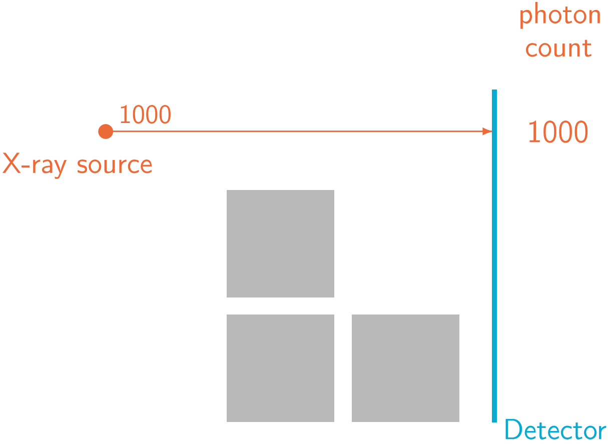
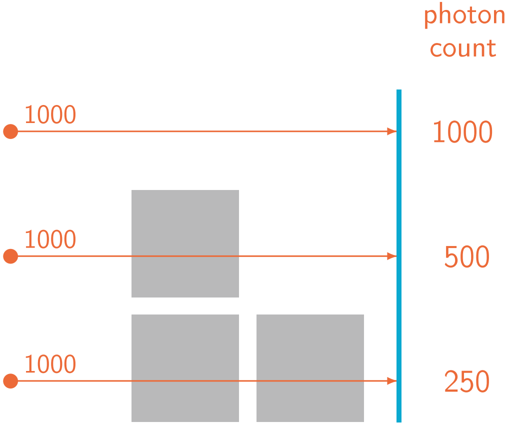
Wilhelm Conrad Röntgen inveted X-rays and was awarded the first Nobel Prize in Physics in 1901



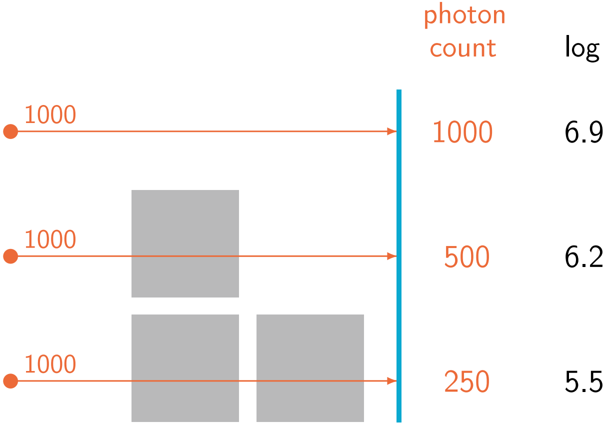
A digital X-ray detector counts how many photons arrive at each pixel



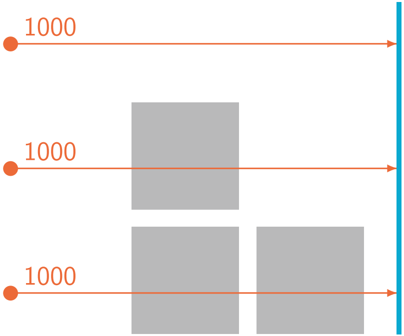
Adding material between the source and detector reveals the exponential X-ray attenuation law



We take logarithm of the photon counts to compensate for the exponential attenuation law



Final calibration step is to subtract the logarithms from the empty space value (here 6.9)



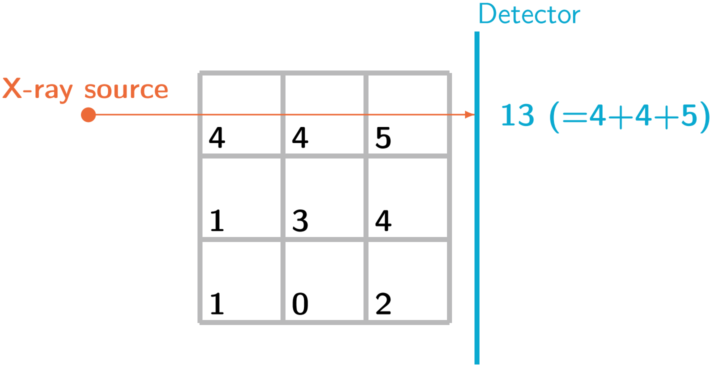
photon count	log	line integral
1000	6.9	**0.0**
500	6.2	**0.7**
250	5.5	**1.4**

Outline

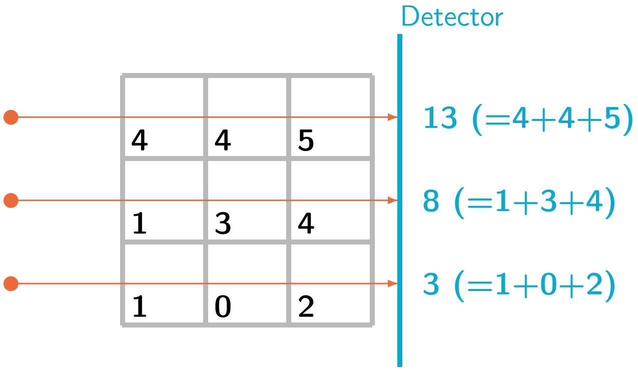
Let us study a simple two-dimensional example of tomographic imaging

4	4	5
1	3	4
1	0	2

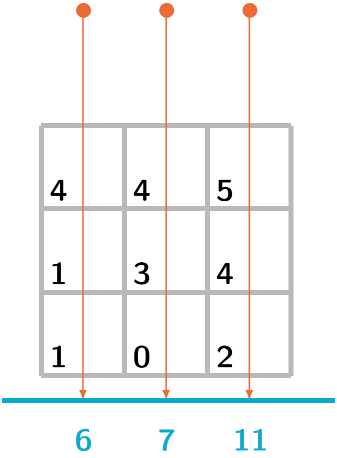
Tomography is based on measuring densities of matter using X-ray attenuation data



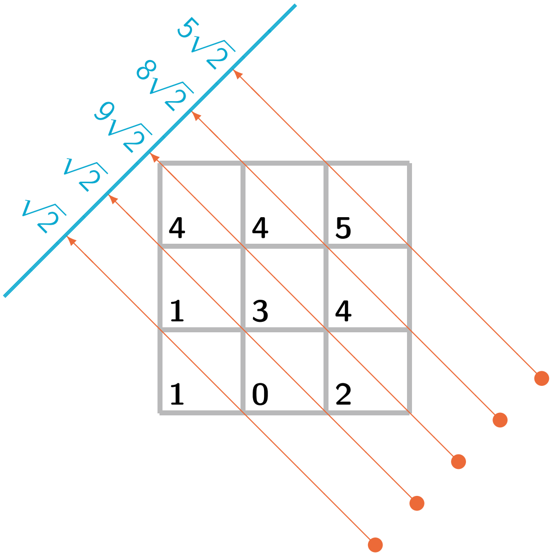
A projection image is produced by parallel X-rays and several detector pixels (here three pixels)



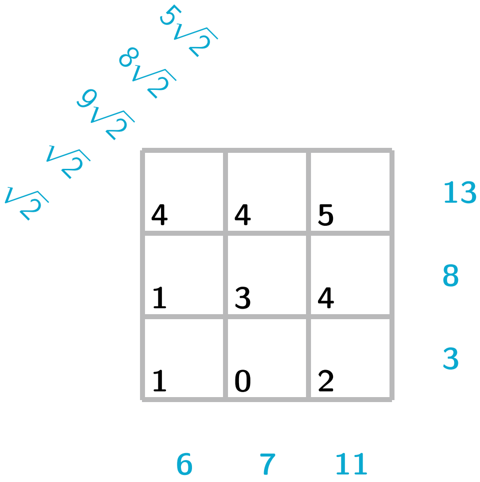
For tomographic imaging it is essential to record projection images from different directions



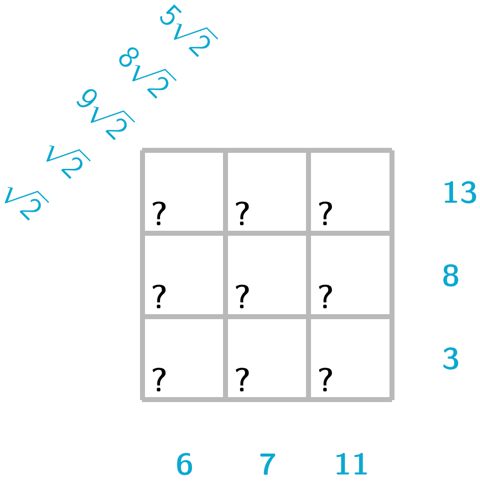
The length of X-rays traveling inside each pixel is important, thus here the square roots



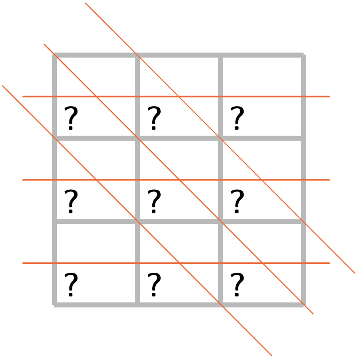
The direct problem of tomography is to find the projection images from known tissue



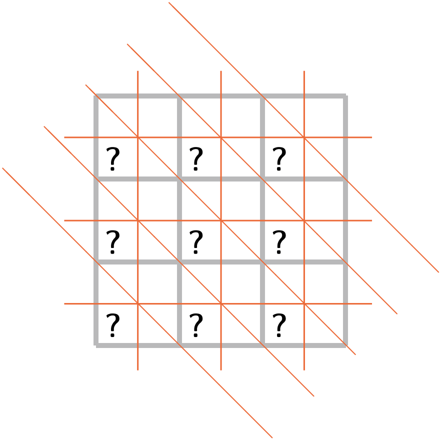
The inverse problem of tomography is to reconstruct the interior from X-ray data



The limited-angle problem is harder than the full-angle problem

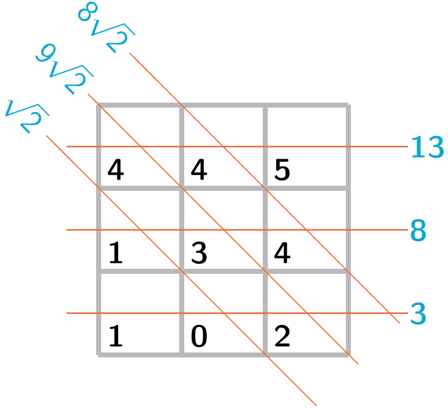


9 unknowns,
6 equations



9 unknowns,
11 equations

In limited-angle imaging, different objects may produce the same data

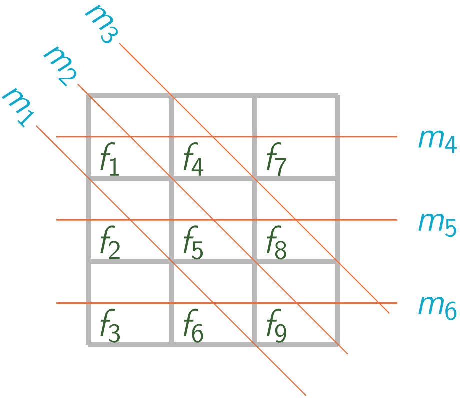


Reconstruction requires additional *a priori* information

5	6	2
1	5	2
4	0	-1

9	1	3
1	0	7
3	0	0

We write the reconstruction problem in matrix form



$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix}, \quad m = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{bmatrix},$$

Measurement model: $m = Sf + \varepsilon$

This is the matrix equation related to the above measurement

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$