

Differential Equations II

Exercise 6

24.4. 2014 (16-18 o'clock in CK111)

1. Show that the matrix

$$X(t) = \begin{pmatrix} 1 & e^{2t} \\ -1 & e^{2t} \end{pmatrix} \in \mathbf{R}^{2 \times 2}$$

is invertible for all $t \in \mathbf{R}$, and compute the inverse matrix $X(t)^{-1}$.

2. Solve the system of differential equations

$$\bar{x}'(t) = A\bar{x}(t), \quad t \in \mathbf{R}, \quad \text{where } A = \begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix}.$$

Note: the matrix A has a double eigenvalue $r = 3$. Determine a second solution $\bar{x}^2(t) = e^{3t}(\bar{v} + t(A - 3I)\bar{v})$ corresponding to the eigenvalue $r = 3$ for the fundamental system of solutions, where $\bar{v} \in \mathbf{R}^2$, $\bar{v} \neq \bar{0}$, satisfies $(A - 3I)^2\bar{v} = \bar{0}$.

3. Solve the system of differential equations

$$\begin{aligned} x_1'(t) &= x_1(t) - x_2(t) \\ x_2'(t) &= 5x_1(t) - 3x_2(t) \end{aligned}$$

with the help of the elimination method.

4. Solve the non-homogeneous system of differential equations

$$\bar{x}'(t) = \begin{pmatrix} 0 & 2 \\ -1 & 3 \end{pmatrix} \bar{x}(t) + \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix}$$

using the elimination method.

5. Solve the linear non-homogeneous system of differential equations

$$\begin{aligned} x_1'(t) &= x_1(t) + x_2(t) + e^{-t} \\ x_2'(t) &= x_1(t) + x_2(t) + e^t \end{aligned}$$

with the help of the formula for the variation of parameters. *Hint:* the corresponding homogeneous system of differential equations was solved in exercise 5:1 and the inverse $X(t)^{-1}$ of a fundamental matrix $X(t)$ is computed in exercise 6:1.

6. Solve the linear non-homogeneous system of differential equations

$$\begin{aligned}x_1'(t) &= x_1(t) + x_2(t) + \sin t \\x_2'(t) &= x_1(t) + x_2(t) + \cos t\end{aligned}$$

with the ansatz $t \mapsto (\sin t)\bar{a} + (\cos t)\bar{b}$, where $\bar{a}, \bar{b} \in \mathbf{R}^2$ are unknown vectors.

Course exam: Monday 28.4. at 13-15 o'clock in Exactum (simultaneously the course exam for the course *Geometria*). *Recall: you are allowed to bring a one-sided memory helper of size A4 to the course exam.*

Topics of the exam: non-linear DEs of second order*, linear DEs of higher order with constant coefficients*, the local existence and uniqueness theorem for first order DEs, linear DE-systems of the first order, solving linear DE-systems of the first order with constant coefficients. *Note: see also the course notes DE 2011 in C326 for topics marked **