Differential Equations II
Exercise 5
10.4. 2014 (16-18 o'clock in CK111)

1. Solve the linear homogeneous system of differential equations

$$x_1'(t) = x_1(t) + x_2(t)$$
$$x_2'(t) = x_1(t) + x_2(t).$$

2. Determine a fundamental system of solutions of the homogeneous system

$$\overline{x}'(t) = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix} \overline{x}(t).$$

Note: $\lambda = -1$ is a root of the characteristic polynomial.

3. Find all solutions of the homogeneous DE-system

$$\overline{x}'(t) = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix} \overline{x}(t).$$

Hint: the characteristic polynomial is $p(\lambda) = -(\lambda - 5)(\lambda - 2)^2$.

4. Find all solutions of the system

$$x_1'(t) = 2x_1(t) + x_2(t)$$
$$x_2'(t) = -x_1(t) + 2x_2(t).$$

5. Solve the initial value problem

$$\overline{x}'(t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \overline{x}(t), \quad \overline{x}(0) = (1, 1, 1)^T.$$

6. Solve the non-linear system of differential equations

$$x_1'(t) = x_1(t)$$
$$x_2'(t) = x_1(t)x_2(t) - x_2(t)$$

by first solving $x_1'(t) = x_1(t)$.

Course exam: Monday 28.4 at 13–15 o'clock (simultaneously the exam for the course *Geometria*). An alternative course exam will be arranged if needed. Overview of the course during the final lecture on Wednesday 16.4. Last exercises (Exercise 6) on Thursday 24.4.