Dependence logic

Problems 5

Tuesday 29.4.2014

1. Every sentence $\phi \in \mathcal{D}$ is logically equivalent to a sentence $\psi \in \mathcal{D}$ of the form

$$\forall x_1 \ldots \forall x_n \exists x_1 \ldots \exists x_m (\theta_1 \wedge \theta_2), \tag{1}$$

where θ_1 is a conjunction of dependence atoms and θ_2 is a quantifier-free first-order formula. Is the following generalization of this result possible: Every sentence $\phi \in \mathcal{D}$ is strongly logically equivalent to a sentence as in (1)?

2. Let ϕ be a sentence of dependence logic that is in negation normal-form and does not contain any universal quantifiers. Show that there is a first-order sentence ϕ^* which is logically equivalent to ϕ. (Hint: Use induction on ϕ. Note also that over teams $X = \{s\}$ with just one assignment dependence atoms can be replaced with $\top$.)

3. Show that

$$=\!(x_1, ..., x_n) \equiv x_n \perp_{x_1,...,x_{n-1}} x_n.$$

4. Show that every formula of inclusion logic and independence logic can be translated to Σ^1_1 by extending the translation $\phi \mapsto \tau_{1,\phi}$ between dependence logic and Σ^1_1 by clauses corresponding to independence atoms and inclusion atoms. (see Theorem 6.2 of the course textbook on page 88.)

5. The connective called intuitionistic implication $\phi \twoheadrightarrow \psi$ is defined by

$$\mathcal{M} \models_X \phi \twoheadrightarrow \psi \text{ iff (for all } Y \subseteq X : \text{ if } \mathcal{M} \models_Y \phi \text{ then } \mathcal{M} \models_Y \psi).$$

Let $\mathcal{D}(\twoheadrightarrow)$ be the extension of dependence logic in which $\twoheadrightarrow$ is introduced as a new connective but negation is only allowed in front of atomic formulas ($\wedge$ and $\forall$ are also available). Show that dependence atoms $=\!(t_1, ..., t_k)$ can be expressed in $\mathcal{D}(\twoheadrightarrow)$ using only dependence atoms of the form $=\!(t_i)$.

6. Let ϕ be a sentence of $\mathcal{D}(\twoheadrightarrow)$. Construct a sentence $\psi \in \mathcal{D}(\twoheadrightarrow)$ such that for all $\mathcal{M}$:

$$\mathcal{M} \models \psi \Leftrightarrow \mathcal{M} \not\models \phi.$$

This shows that, for sentences, $\mathcal{D}(\twoheadrightarrow)$ is closed under classical negation.