

Dependence logic  
Problems 2  
Tuesday 1.4.2014

1. Show that the following strong logical equivalences hold:

$$\begin{aligned}\phi \vee \top &\equiv^* \top \\ \neg(\phi \wedge \psi) &\equiv^* (\neg\phi \vee \neg\psi)\end{aligned}$$

2. Show that the following strong logical equivalences hold:

$$\begin{aligned}\exists x_n \exists x_m \phi &\equiv^* \exists x_m \exists x_n \phi \\ \neg \forall x_n \phi &\equiv^* \exists x_n \neg \phi\end{aligned}$$

3. Show the following non-equivalence:

$$\phi \vee \neg\phi \not\equiv^* \top.$$

4. Show the following non-equivalence:

$$\phi \vee \phi \not\equiv^* \phi.$$

5. Show that

$$\phi \wedge \phi \not\equiv^* \phi, \text{ but } \phi \wedge \phi \equiv \phi.$$

6. Show the following non-equivalence:

$$(\phi \wedge \psi) \vee \theta \not\equiv^* (\phi \vee \theta) \wedge (\psi \vee \theta).$$