University of Helsinki
Department of Mathematics and Statistics
Neuronetworks February 2014
Exercises for week 1

1 Exercise. Linear algebra reminder. A column vector $\bar{x}$ is the *right eigenvalue* of a matrix W if there is $\lambda \in \mathbb{R}$ (called the *eigenvalue*) if $Wx = \lambda\bar{x}$. A row vector $\bar{x}$ is a *left eigenvalue* if there is $\lambda \in \mathbb{R}$ with $\bar{x}W = \lambda\bar{x}$

1. Show that a matrix W has a right eigenvalue if and only if its transpose W^T has a left one.
2. Give an example of a 2×2 matrix without right eigenvalues.
3. Give an example of two different 3×3 matrices such that the set of normalised right eigenvectors (normalised to length $= 1$) is a basis of $\mathbb{R}^3$. We say that the matrix has a *complete* set of (right)eigenvalues.
4. Compute $\bar{x}W$, $W\bar{x}^T$ and $\bar{x}W\bar{x}^T$ where
$$\bar{x} = [\,1 \;\; 2\,] \text{ and } W = \begin{bmatrix} 1 & 0 \\ 2 & -2 \end{bmatrix}.$$
5. Does the matrix W from above have eigenvalues?

2 Exercise. Show that a McCulloch-Pitts unit (definition on page 32, Chapter 2, in the material) can be simulated by a perceptron (Definition 1 on page 60, Chapter 3).

3 Exercise. Suppose that we have a perceptron which separates finite sets A and B of $\mathbb{R}^n$. Show that A and B can be separated by a network of McCulloch-Pitts units (with inhibitory connections allowed). (Hint: show first that this a perceptron can be replaced by another perceptron with only rational weights and then use Proposition 4 of Chapter 2).

4 Exercise. Show that the parity function of $n > 2$ bits cannot be computed by a single perceptron.

5 Exercise. Let $n > 2$. Construct a neural network consisting of perceptrons which computes the parity function. I.e. it takes a vector $\bar{x} \in \{0,1\}^n$ as an input and outputs 1 if the number of 1's in x is odd and otherwise outputs 0.

6 Exercise. Write a computer program in the programming language of your choice which takes as input two sets of 2-dimensional inputs and finds weights for a perceptron (with two inputs) which separates these sets if possible in a bounded number of steps: set maximum number of steps to 100. The program outputs the weights and the number of steps needed to achieve the result. (program terminates if a solution is found).