

STOCHASTIC POPULATION MODELS

EXERCISES 7-9

7.

Consider the Wiener process $\{W(t)\}_{t \geq 0}$ with $W(0) = 0$. Show that $\mathcal{E}\{W(t)W(s)\} = \min\{t, s\}$

8.

Integrate the linear stochastic differential equation $dX = g(t)dW$ with $X(0) = x_0$. Show that it doesn't matter how we sample the integrand, and so, in particular, it doesn't matter whether we interpret the differential equation as an Ito equation or a Stratonovitch equation.

9.

Integrate the following stochastic differential equations with deterministic initial condition $X(0) = x_0$ for suitable choice of x_0 :

(a) $dX = X^2 dW$ (S)

(b) $dX = e^{-t} X dW$ (S)

(c) $dX = -X dt + X dW$ (I)