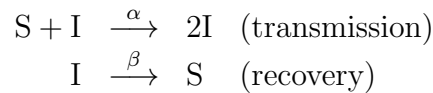


## STOCHASTIC POPULATION MODELS

### EXERCISES 4-6

**4.**

Consider the epidemiological model of the last exercise based on the processes



Give the transfer function  $T(\omega)$  for a seasonally varying infection rate  $\alpha$ . Calculate the maximum gain  $G_m$  and the cutoff frequency  $\omega_c$ . How does the cutoff frequency depend on the average infection rate? Express the cutoff frequency in units of *cycles per mean recovery time* ( $\beta^{-1}$ ). What does this tell you about absence (or presence) of seasonality of the prevalence of the disease in relation to the average length of an infection?

**5.**

Consider the resource-consumer model

$$\begin{cases} \frac{d}{dt}R = R_0 - \alpha R - \beta RC & (\text{resource}) \\ \frac{d}{dt}C = \gamma\beta RC - \alpha C - \delta C & (\text{consumer}) \end{cases}$$

Can you interpret the different terms and parameters? Calculate the equilibria and determine their stability. For the positive equilibrium, give the transfer functions  $T_R(\omega)$  and  $T_C(\omega)$  for, respectively, the resource and the consumer if the  $R_0$  is seasonally varying. Is there peak frequency where the gain is maximal?

**6.** Let  $\tilde{f}$  and  $\tilde{h}$  denote the Fourier transforms of, respectively,  $f$  and  $h$ , and prove that:

(a) the Fourier transform and its inverse are linear operators,

(b)  $\tilde{\tilde{f}}(t) = 2\pi f(-t)$ ,

(c)  $\widetilde{\left(\frac{d}{dt}f\right)}(\omega) = i\omega\tilde{f}(\omega)$ ,

(d)  $\frac{d}{d\omega}\tilde{f}(\omega) = -i\widetilde{(tf)}(\omega)$ ,

(e)  $\tilde{f}_\tau(\omega) = e^{-i\omega\tau}\tilde{f}(\omega)$  where  $f_\tau(t) := f(t - \tau)$ ,

(f)  $\widetilde{(f * h)}(\omega) = \tilde{f}(\omega)\tilde{h}(\omega)$  where  $(f * h)(t) := \int_{-\infty}^{+\infty} f(\tau)h(t - \tau)d\tau$ ,

(g)  $\widetilde{(fh)}(\omega) = (\tilde{f} * \tilde{h})(\omega)$ ,

(h)  $\int_{-\infty}^{+\infty} f(t)\tilde{h}(t)dt = \int_{-\infty}^{+\infty} \tilde{f}(t)h(t)dt$ .