STOCHASTIC POPULATION MODELS

EXERCISE 13

13.
Consider the following individual-level processes:

$$(1) \qquad \begin{cases} \mathrm{X} & \xrightarrow{\alpha} & \mathrm{X}+\mathrm{X} & \text{(birth)} \\ \mathrm{X} & \xrightarrow{\beta} & \dagger & \text{(death)} \\ \mathrm{X}+\mathrm{X} & \xrightarrow{\gamma} & \mathrm{X}+\mathrm{R} & \text{(interference)} \end{cases}$$

where X is an active individual and R an individual that has been expelled (=removed) from the population.

(a) Formulate a population model as a stochastic birth-death process assuming that birth, death and interference are independent Poisson processes.

(b) What is the probability of a successful invasion if the population is introduced into a new and hitherto unoccupied territory?

(c) Calculate numerically the quasi-stationary probability distribution of the population size and give the expected time till extinction if we take the quasi-stationary distribution as the initial state of the population.

(d) What population model do we get if we let system size go to infinity?

(e) Give the Fokker-Planck approximation for a semi-large system together with the corresponding SDE. Approximate the quasi-stationary distribution by a Gaussian distribution and give the auto-covariance and spectral density of the corresponding stationary process.