Introduction to LaTeX
Exercise Sheet 2 (Group 5)

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The `.tex` file containing your solutions to this exercise sheet should be emailed to `clifford.gilmore@helsinki.fi` before 15:00 on 11th April. The subject line of the email should be *Latex Exercises 2* and the file name should be of the form `SurnameExercise2.tex`, e.g `GilmoreExercise2.tex`.

The produced document should contain enough text to fill two pages. If you can't think of anything to write then you can find random text from Lorem Ipsum at http://www.lipsum.com/

Note, in the exercises where you define a `newtheorem` environment, the theorem number will be automatically generated in your document by LaTeX. So the theorem numbers do not need to match the numbers shown in the exercise, i.e it is okay if the theorem's number is different in your solution!

1. Create a document titled, *LaTeX Solutions 2*, with you as the author.

2. Add the following to your document:

   (a) A section titled *Miscellaneous Mathematics*.

   (b) Inside the section add a subsection titled *Text Style Maths*.

   (c) Inside this subsection include the following text, where the mathematics appears inline with the text:

      (i) Pythagoras states for a right angled triangle with side lengths a, b, c, then $a^2 + b^2 = c^2$.

      (ii) Euler's identity states that $e^{i\pi} + 1 = 0$.

3. Add a new subsection to your document with the title *Display Style Maths*. Inside this subsection include the following, where the mathematics appears in the `equation` environment

(a)

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{k^2}=\frac{\pi^2}{6} \tag{1}$$

(b) For $n, k \in \mathbb{N}$ and $k \leq n$

$$(x+y)^n=\sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k \tag{2}$$

(hint: you will need the `amsmath` package and the `\binom` command here)

(c)

$$\int_0^{\frac{\pi}{2}}\sin x\mathrm{d}x$$

(d) For a separable Banach space X, a bounded linear operator T is hypercyclic if there exists a vector $x \in X$ such that its orbit under T is dense in X, i.e.

$$\overline{\{T^n x : n \geq 0\}} = X.$$

(hint: you need the `\overline` command here)

4. Add a new subsection, titled *Aligning Equations*, and using the `align` environment include the following equations:

$$\begin{aligned}\sum_{j=k+1}^{\infty}2^{n_k-n_j}\lVert y^{(j)}\rVert &\leq \sum_{j=k+1}^{\infty}2^{-j}\\ &= 2^{-k-1}+2^{-k-2}+\cdots\\ &= 2^{-k}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)\\ &= 2^{-k}\sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^i = 2^{-k}.\end{aligned}$$

(hint: you need the `\lVert` and `\rVert` commands here)

5. Create a new section titled, *Linear Chaos.*

Define your own Theorem environment with the `newtheorem` command and then use it to state the below theorem.

Theorem 2.1 (Hypercyclicity Criterion)**.** Let X be a separable Banach space and $T \in \mathcal{L}(X)$. If there exist dense subsets X_0, $Y_0 \subset X$, an increasing sequence $(n_k)_k$ of positive integers and maps $S_{n_k}\colon Y_0 \to X$, $k \geq 1$, such that for any $x \in X_0$, $y \in Y_0$

(i) $T^{n_k}x \to 0$,

(ii) $S_{n_k}(y) \to 0$,

(iii) $T^{n_k}S_{n_k}(y) \to y$

as $k \to \infty$, then T is hypercyclic.

(hint: you need the `\mathcal` command here)

6. Using the `proof` environment accessed through the `amsthm` package, add a proof for the above theorem. (You don't need to give the real proof, any paragraph of text will do)

7. Create a *Lemma* environment using the `newtheorem` command. The numbering of the lemma should be in the same sequence as the above Theorem. Add the following lemma.

   Lemma 2.2. For a separable Banach space X, let $T \in \mathcal{L}(X)$. Suppose for each $x \in X$ there exists a nonzero $x^* \in X^*$ such that the set $\{\langle x^*, T^n x\rangle : n \geq 0\}$ is not dense in $\mathbb{C}$. Then T is not hypercyclic.

   (hint: you need the `\langle` and `\langle` command here)