Inverse problems course, spring 2013 Exercise 1 (25.1.2013)
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Please complete the theoretical exercises (marked with T) before the exercise session and be prepared to present your solution there.

T1. Define a function $g : \mathbb{R} \to \mathbb{R}$ by

$$g(x) = \begin{cases} 1 & \text{for } 0.4 \le x \le 0.6, \\ 0 & \text{otherwise.} \end{cases}$$

Compute the function $g * g$ analytically (by hand), where

$$(g * g)(x) = \int_{-\infty}^{\infty} g(x')g(x - x')\, dx'.$$

Outside which interval $[a, b] \subset \mathbb{R}$ is $(g * g)(x) = 0$?

T2. Let the discrete point spread function $p \in \mathbb{R}^5$ and the vector $f \in \mathbb{R}^{10}$ be defined by

$$\begin{aligned} \widetilde{p} &= [\widetilde{p}_{-2}, \widetilde{p}_{-1}, \widetilde{p}_0, \widetilde{p}_1, \widetilde{p}_2]^T = [1, 1, 1, 1, 1]^T, \\ f &= [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}]^T = [0, 0, 0, 0, 1, 1, 0, 0, 0, 0]^T. \end{aligned}$$

Compute the discrete convolution vector $(\widetilde{p} * f) \in \mathbb{R}^{10}$ by

$$(\widetilde{p} * f)_j = \sum_{\ell=-2}^{2} \widetilde{p}_\ell f_{j-\ell},$$

where $f_{j-\ell}$ is defined using periodic boundary conditions for the cases $j-\ell < 1$ and $j - \ell > n$.

T3. Take $\Delta x = \frac{1}{10}$ and compute the normalized point spread function

$$p = \Big(\Delta x \sum_{j=-2}^{2} \widetilde{p}_j\Big)^{-1} \widetilde{p}.$$

Compute the discrete convolution vector $(\widetilde{p} * f) \in \mathbb{R}^{10}$ with vector $f \in \mathbb{R}^{10}$ as in exercise T2 except that $f_1 = 2$. Be careful with the periodic boundary condition!

You can work on these Matlab exercises (marked with M) in the exercise session.

M1. Compute numerically the function $(g * g)(x)$ as defined above in Exercise T1. Plot the functions $g(x)$ and $(g * g)(x)$ in the same picture with different colors. You can do this by modifying the flies `DC_convmtx.m`, `DC1_cont_data_comp.m` and `DC1_cont_data_plot.m` available at the course webpage.

Save the plot in Encapsulated PostScript format with filename convplot1.eps. The Matlab command you need is
`>> print -depsc convplot.eps`

M2. Let $g(x)$ be as in Exercise T1. Define a normalized point spread function $p(x) := Cg(x)$. Namely, compute numerically such a constant $C > 0$ that

$$\int_{-\infty}^{\infty} p(x)\, dx = 1.$$

Now compute numerically the function $(p * g)(x)$. Plot the functions $g(x)$ and $(p * g)(x)$ in the same picture with different colors. What is the difference with the result of Exercise M1?

You can work on these LaTeX exercises (marked with L) in the exercise session, or you can complete them beforehand.

L1. Download the file `reportdraft1.tex` from the course webpage. Run it with LaTeX; the result should be a pdf document. This document will serve as the template for the project work of this course.

L2. Let us try including a picture in the LaTeX document. Add the command

```
\usepackage{graphicx}
```

to the second row of `reportdraft1.tex`. Next use this series of commands to add the picture to the Section *Results*:

```
\begin{picture}(320,250)
\put(0,0){\includegraphics[height=8cm]{convplot1.eps}}
\end{picture}
```

Try changing the numbers 320,250,0 and 8 above one by one and rerunning LaTeX. Can you figure out the meaning of each of the numbers?