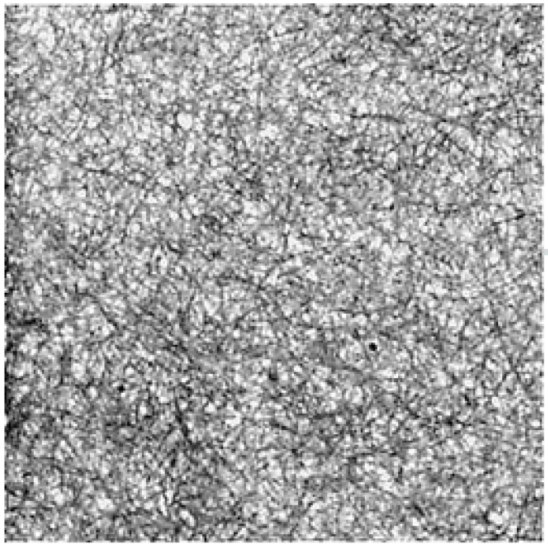
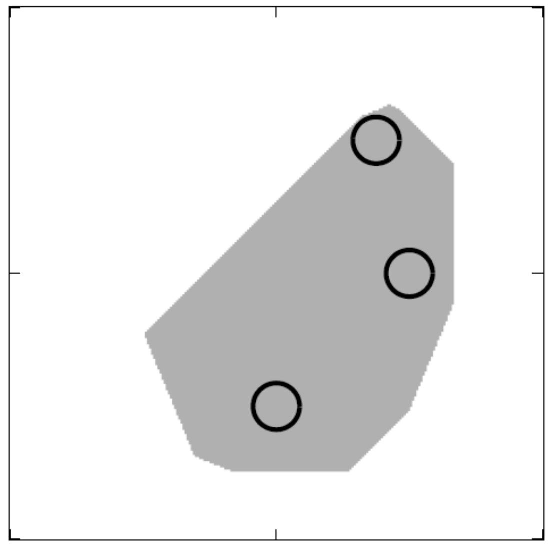
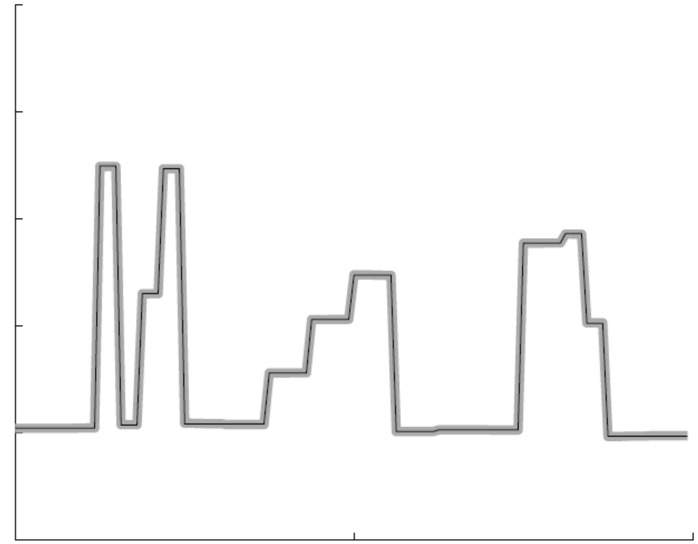
Inverse Problems



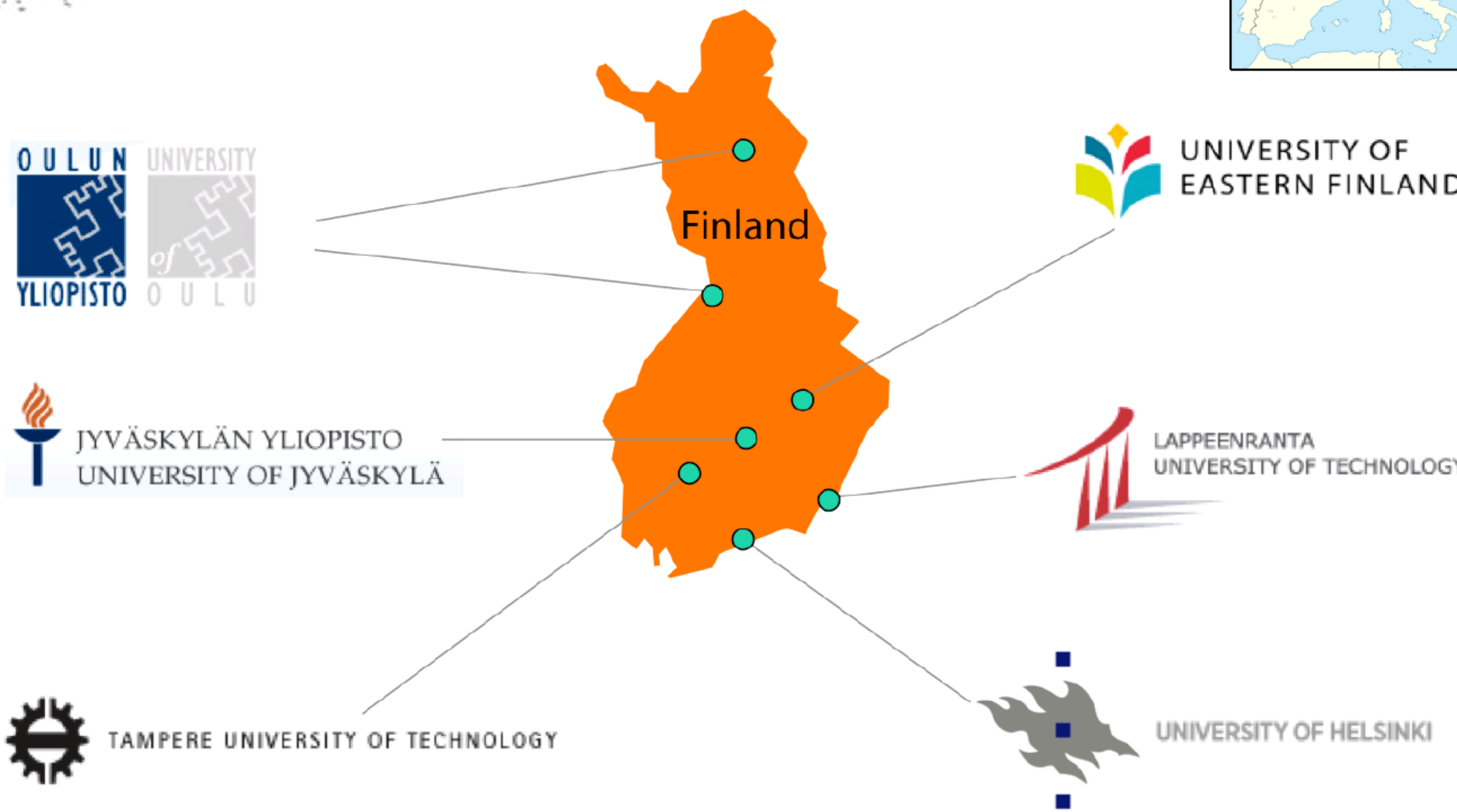




Spring 2013
Department of Mathematics and Statistics
University of Helsinki

Lecturers: **Matti Lassas** (analytic inversion)
Samuli Siltanen (computational inversion)
Assistants: **Hanne Kekkonen** and **Esa Niemi**

Finnish Centre of Excellence in Inverse Problems Research



https://wiki.helsinki.fi/display/inverse/Home

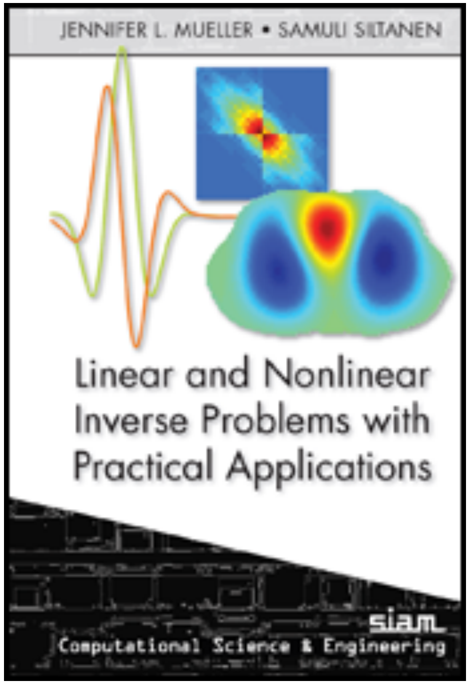
Period III: Lectures and exercises:

Tue 10-12, Exactum D123,
Wed 12-14, Exactum D123,
Fri 12-14, Exactum B120,
Fri 10-12, Exactum C122 (exercises).

Period IV: Project work.

Course material:

Part I of *Linear and Nonlinear Inverse Problems and Practical Applications*
by J.L. Mueller and S. Siltanen (SIAM 2012)



Additional material uploaded to the website.

What are inverse problems?

Inverse problem: Image deblurring

Direct and inverse problem of image deblurring

Direct problem:

Given a sharp photograph, what would the blurred version of the image look like?

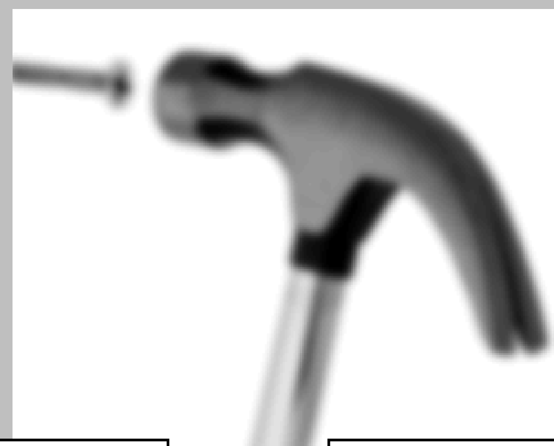
Inverse problem:

Given a blurred photograph, reconstruct the sharp image

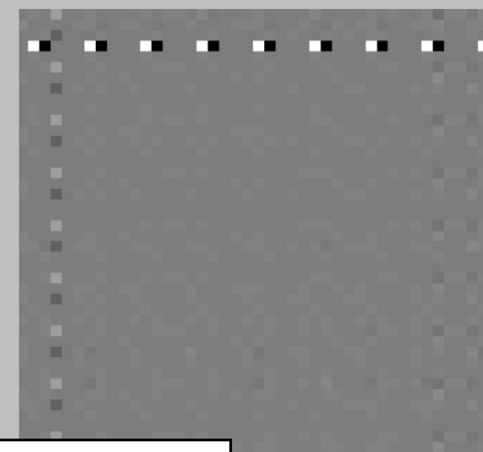
The inverse problem is more difficult

Original image

Blurred image



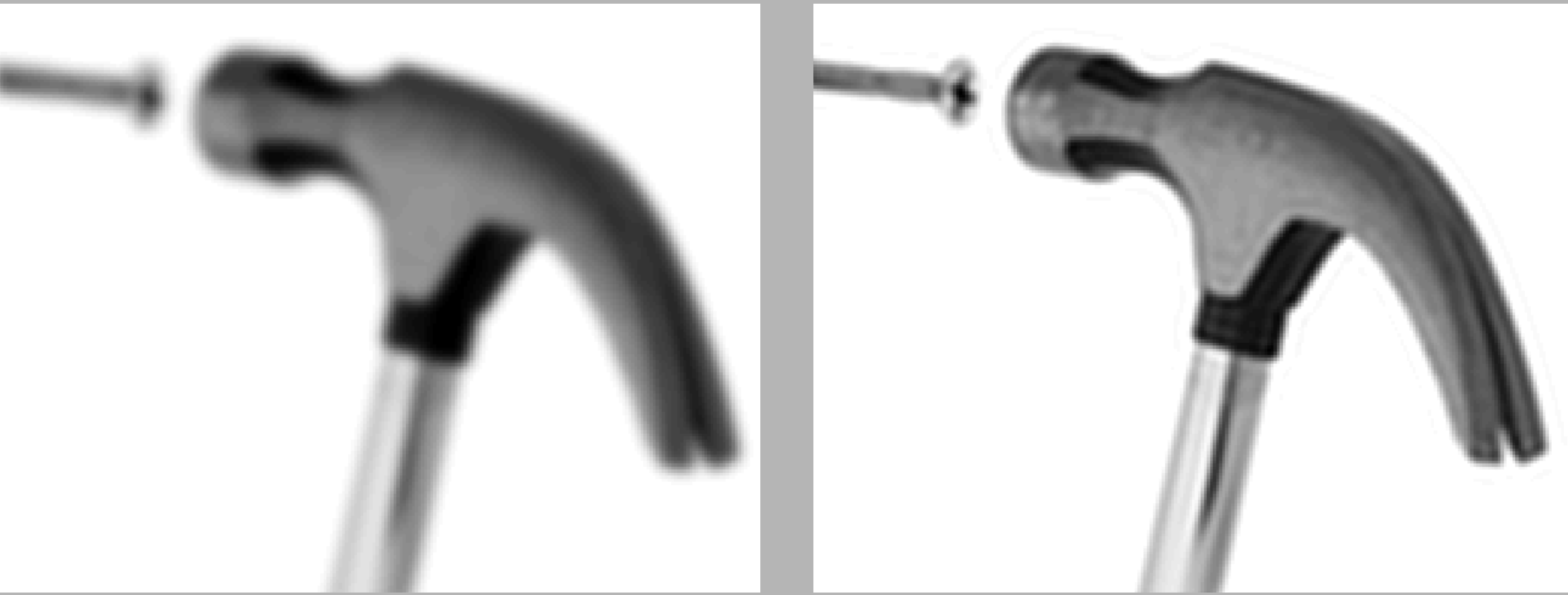
Simple inversion



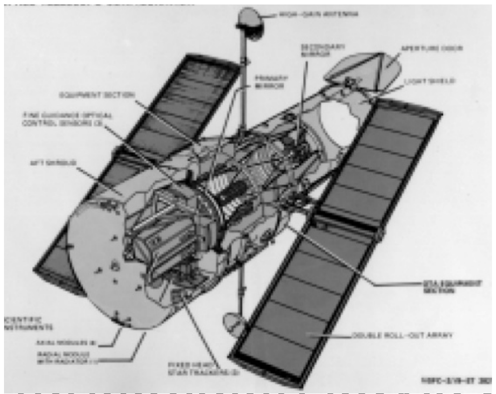
Direct problem

Inverse problem

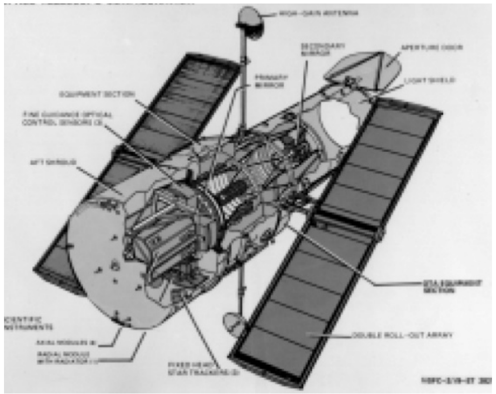
With properly regularized inversion we can sharpen the photograph

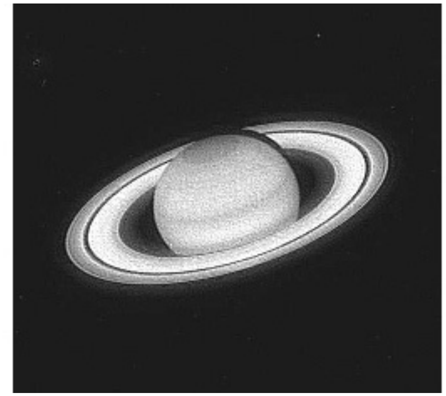


The Hubble space telescope had a flaw in its mirror, resulting in blurred images



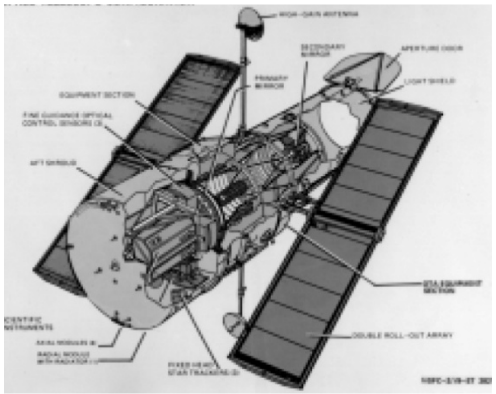
The mirror flaw was compensated by a deconvolution algorithm

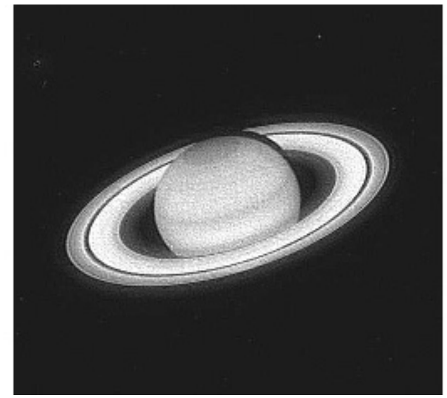




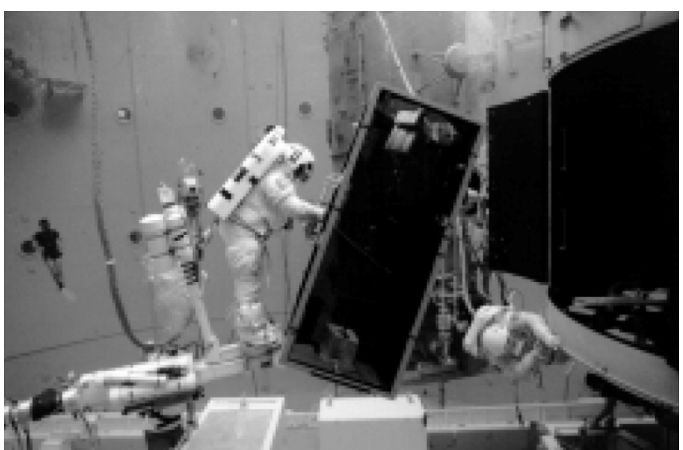
Source: NASA, Quarktet

The mirror flaw was compensated by a deconvolution algorithm





The mirror was replaced in 1993. However, even the new sharp images could be further enhanced with deconvolution!



Source: NASA, Quarktet

Inverse problem: Computerized tomography

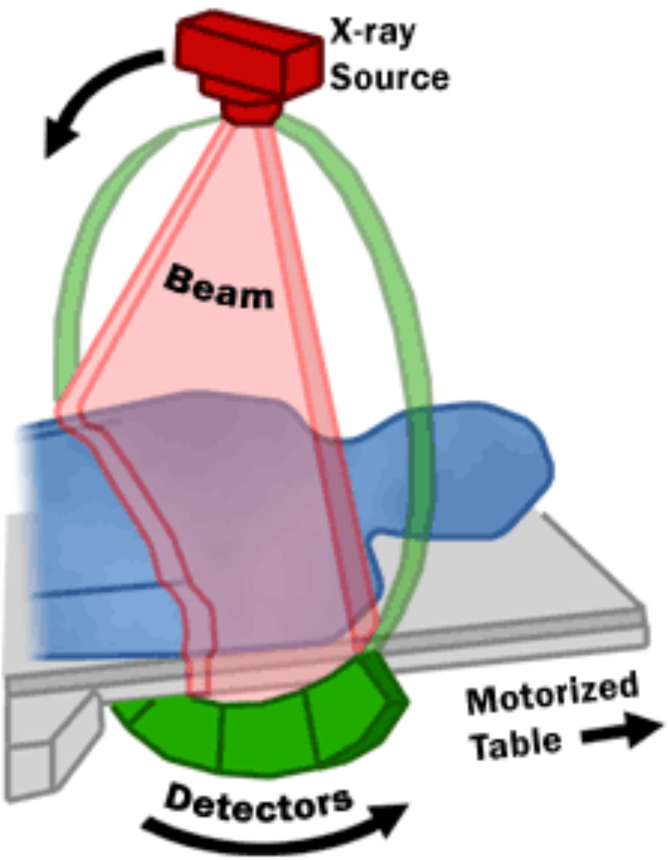
Direct problem:

If the inner structure of a person is known, what would X-ray images of her look like?

Inverse problem:

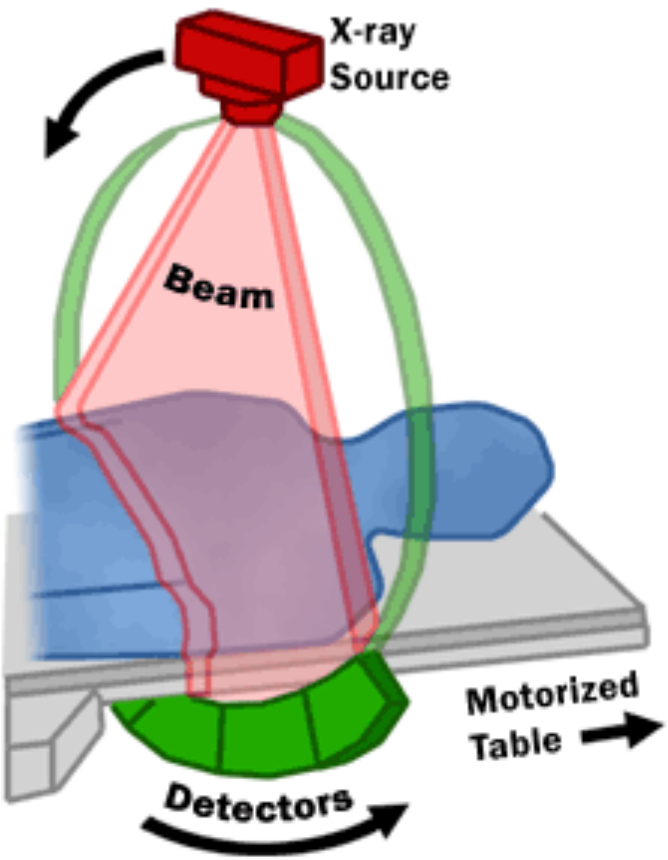
Given X-ray images from all around the body, what is the inner 3-D structure?

Traditionally, CT data is collected slice by slice



Images from http://www.fda.gov/cdrh/ct/what.html

Using a reconstruction algorithm, inner structure in the slice is revealed



Johann Radon (1887-1956)

$$f(x) = \frac{1}{4\pi^2} \int_{S^1} \int_{\mathcal{R}} \frac{\frac{d}{ds}(Rf)(\theta, s)}{x \cdot \theta - s} ds d\theta$$

Filtered back-projection

$$f(x) = \frac{1}{4\pi^2} \int_{S^1} \int_{\mathcal{R}} \frac{\frac{d}{ds}(Rf)(\theta, s)}{x \cdot \theta - s} ds d\theta$$

Filtered back-projection (FBP) is mathematical technology used on a daily basis in hospitals around the world.

The quality of 3D reconstruction using FBP is excellent.

Nobel prize was awarded to Hounsfield and Cormack 1979.

However, a comprehensive data set is mandatory for FBP.

A series of projects started in 2001 aiming for a new type of low-dose 3D imaging

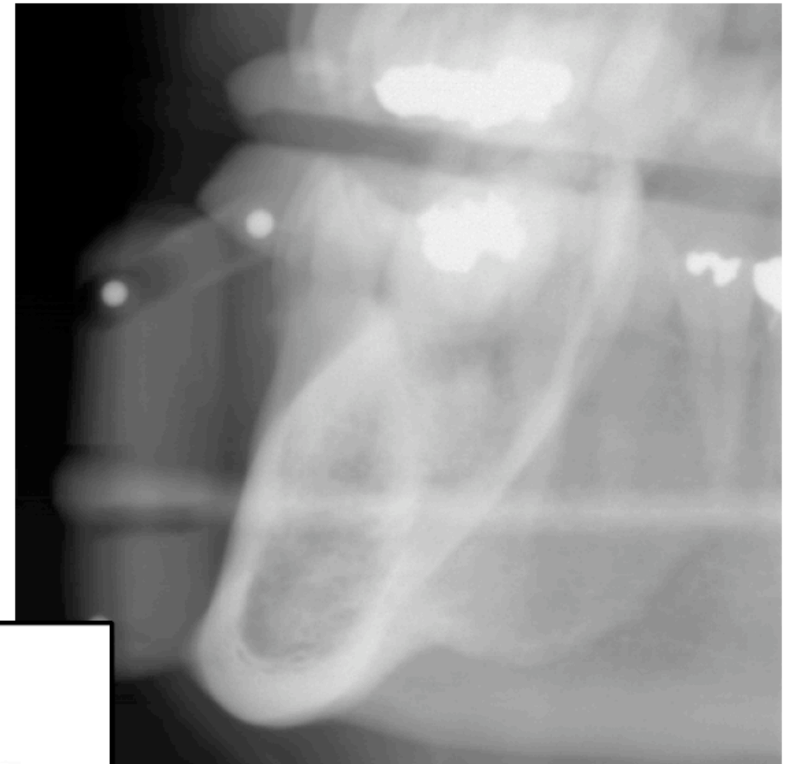
The goal was a mathematical algorithm with

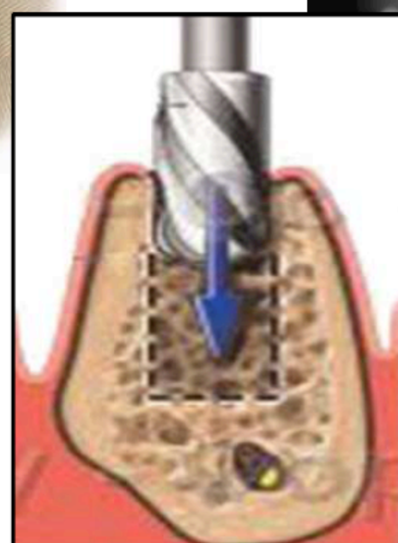
Input: small number of digital X-ray images taken with any X-ray device

Output: three-dimensional reconstruction with quality good enough for the clinical task at hand

Products of Instrumentarium Imaging in 2001:

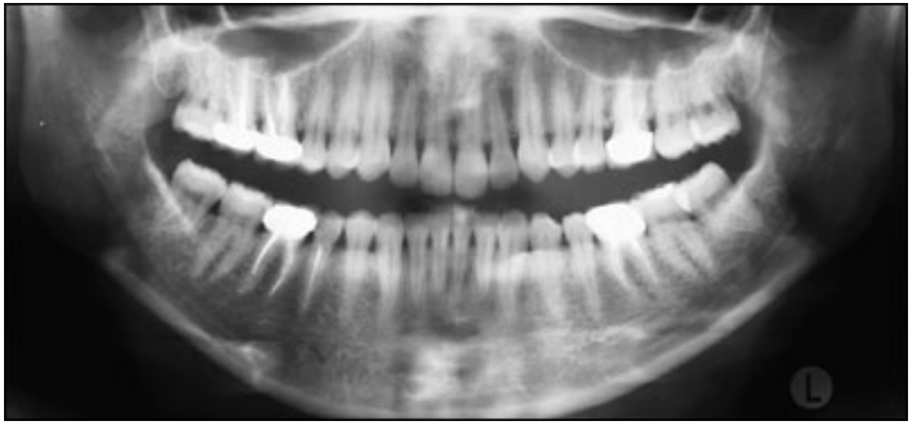
2D projection radiograph is not enough for dental implant planning





Panoramic X-ray device rotates around the head and produces a general picture

Panoramic imaging was invented by Yrjö Paatero in 1950's.



Nowadays a panoramic device is standard equipment at every dental clinic around the world.

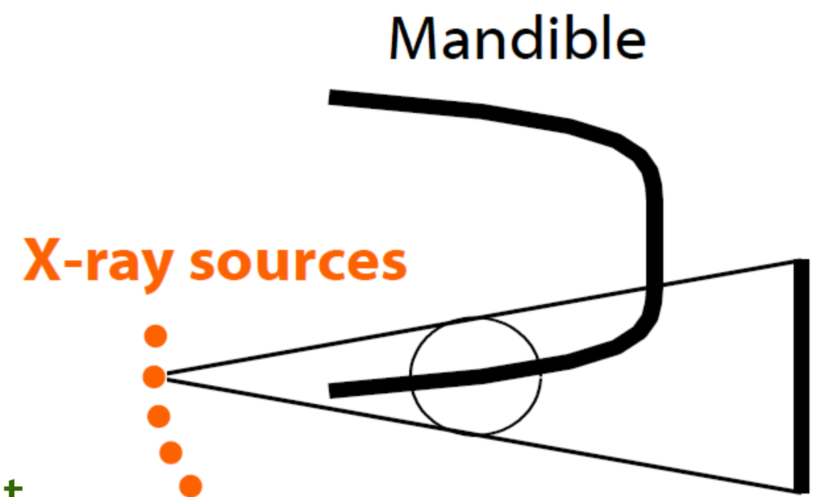
In our project, we reprogrammed the device so that it collects limited-angle data.

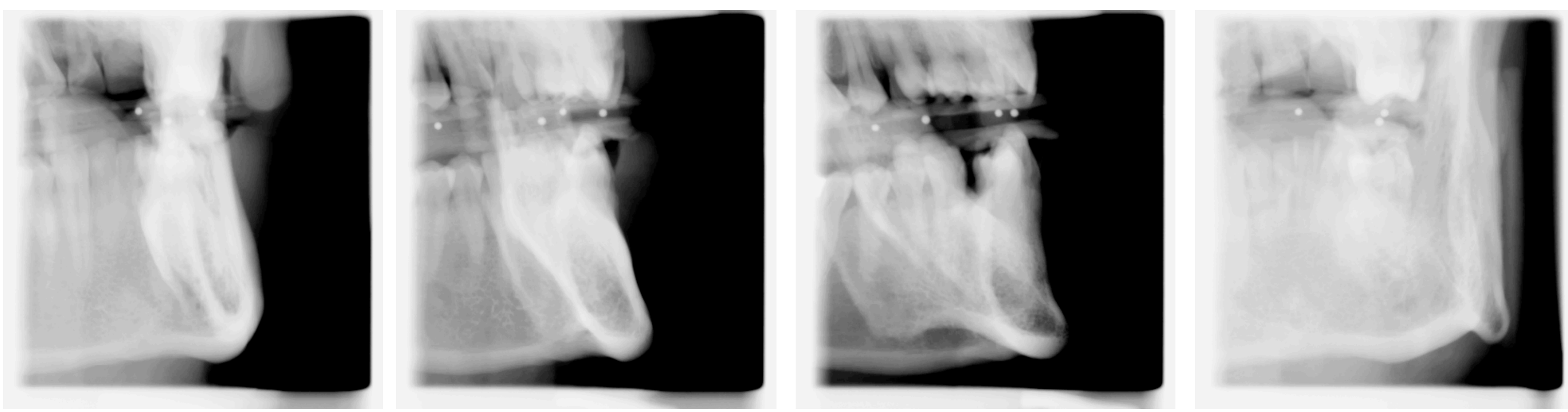
We consider the following limited angle experiment with the panoramic x-ray device:

11 projection images of the mandibular area

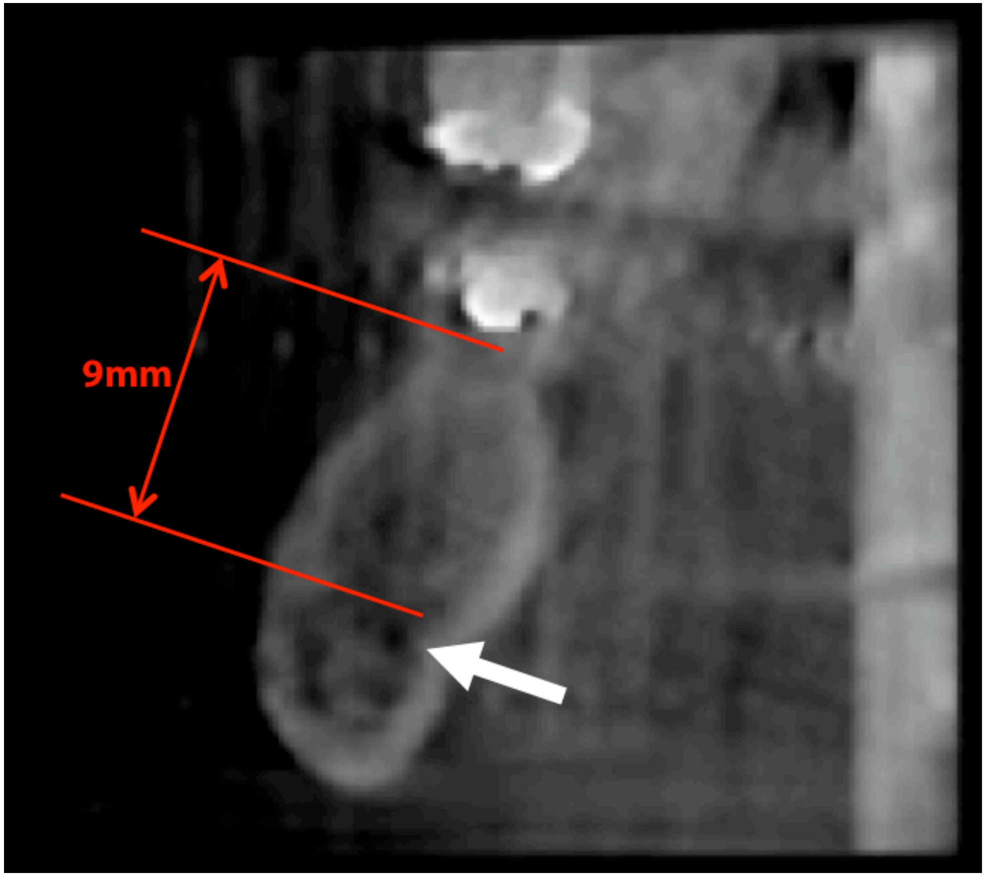
40 degrees angle of view

1000 x 1000 pixels per image, formed by a scanning movement





Limited angle level set reconstruction can be used for locating the mandibular nerve



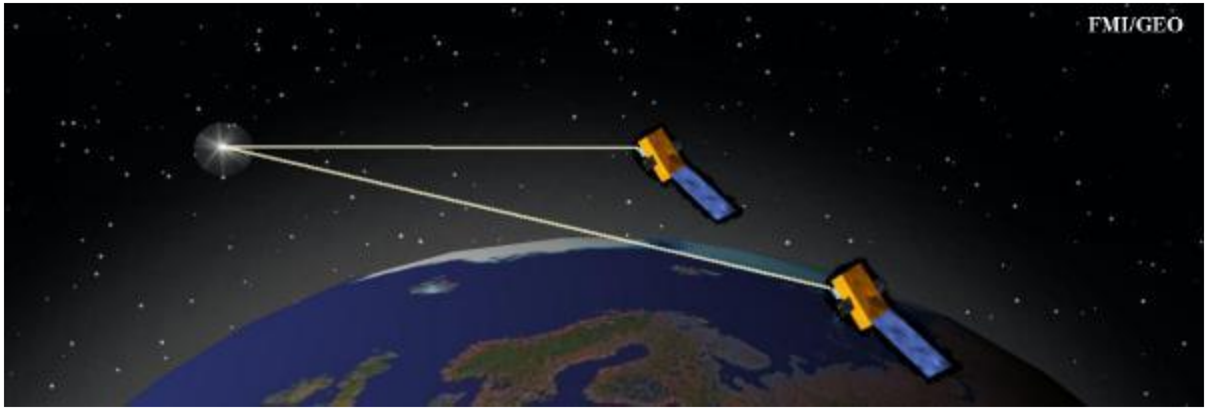
PALODEX GROUP

This is core technology for the PaloDEx Group's VT product that has been in the market since 2007.

Remark that a software update transforms a 2D device into a 3D device.

Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas and Kalke (2006)
Kolehmainen, Lassas and S (2008)
Cederlund, Kalke and Welander (2009)
Hyvönen, Kalke, Lassas, Setälä and S (2010)

Inverse problem: ozone layer tomography



Direct problem:

If the ozone profile of the atmosphere were known, what star occultation measurements would we get?

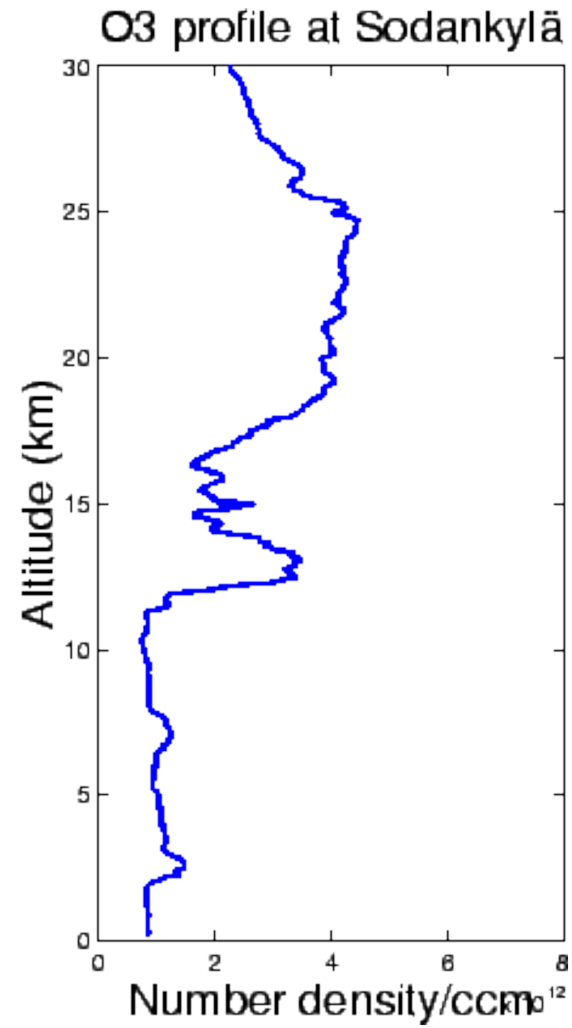
Inverse problem:

Given star occultation measurements , what is the ozone profile?

Show animation of measurement!

http://envisat.esa.int/instruments/gomos/descr/flash.html

As a result we get ozone density as function of altitude



This inverse problem is mathematically the same than the CT problem, except with limited data

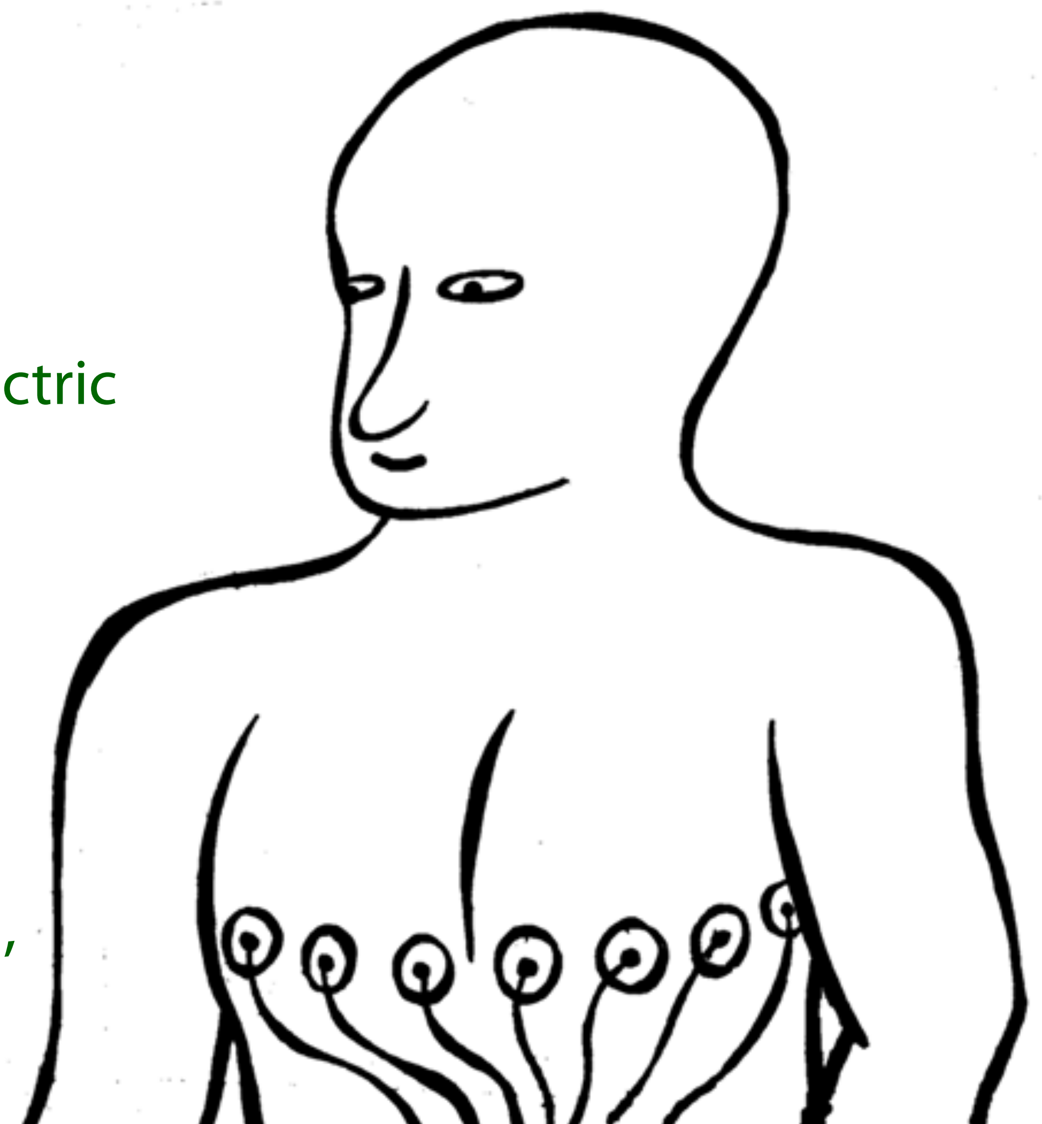
Sources:
European Space Agency
Finnish Meteorological Institute
Envisat and GOMOS projects
http://www.fmi.fi/tutkimus_otsoni/otsoni_26.html
http://envisat.esa.int/handbooks/gomos/CNTR2.htm

Inverse problem:
Electrical impedance tomography

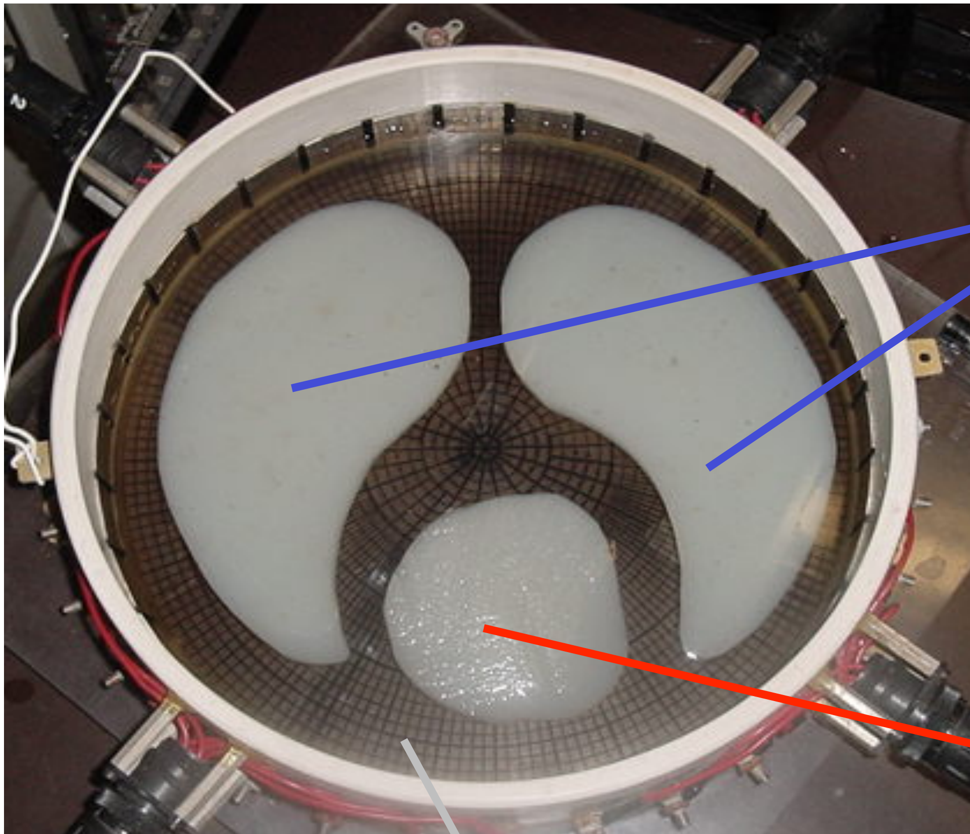
Feed electric currents
through electrodes,
measure voltages

Reconstruct the image of electric
conductivity
in a two-dimensional slice

Applications:
monitoring heart and lungs
of unconscious patients,
detecting pulmonary edema,
enhancing ECG and EEG



At the RPI lab, we construct a chest phantom consisting of saline and agar

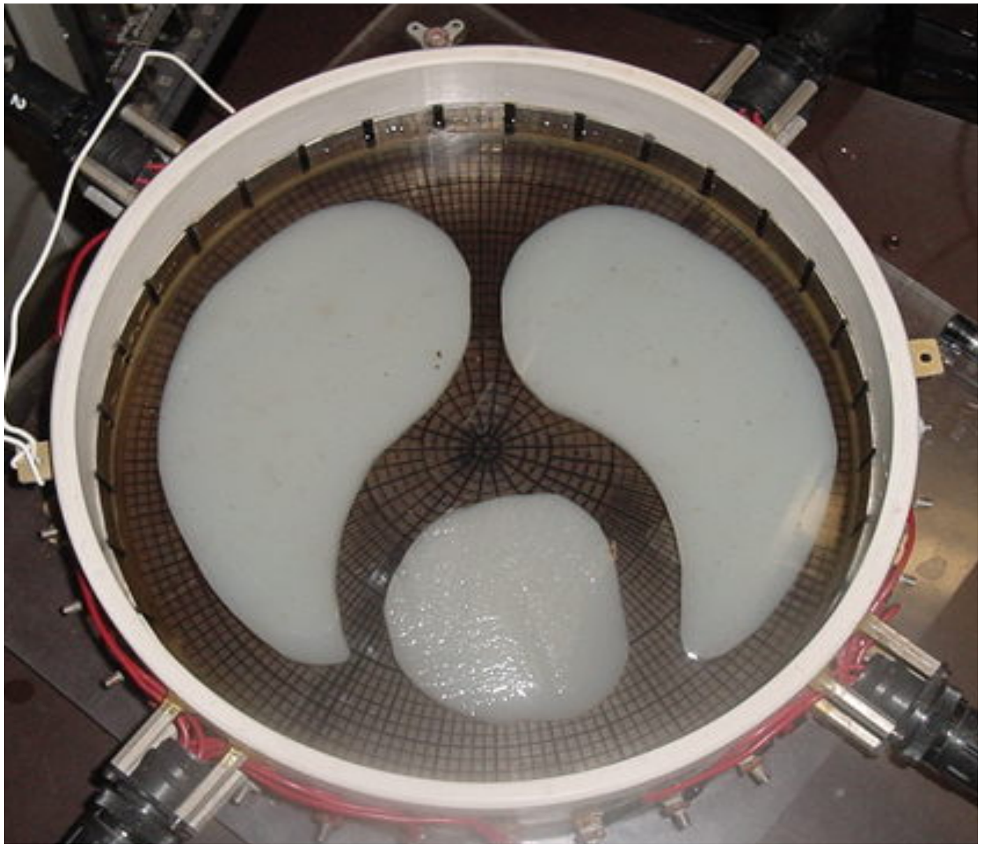


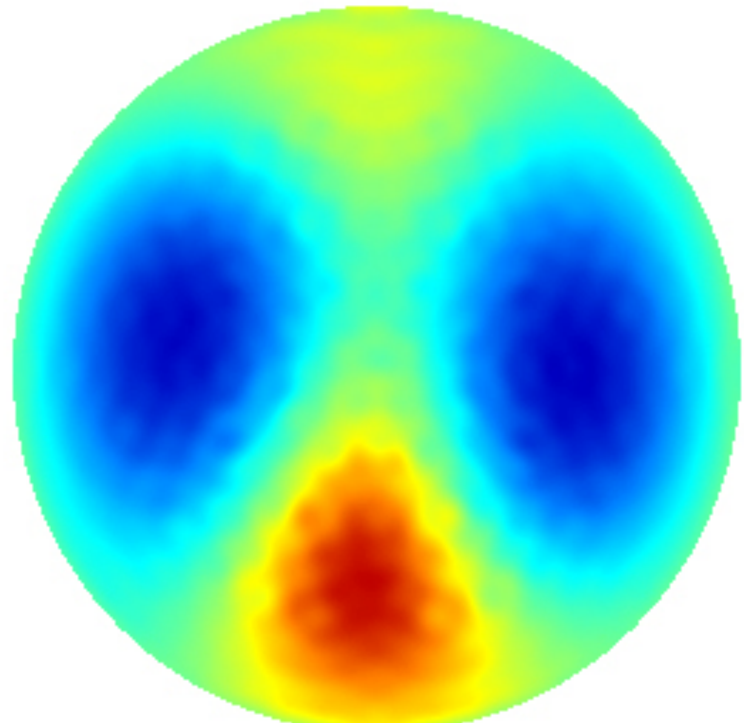
"Lungs" with lower conductivity than background (240 mS/m)

"Heart" with higher conductivity than background (750 mS/m)

Background of salt water, conductivity 424 mS/m.
Diameter of the tank is 30cm.

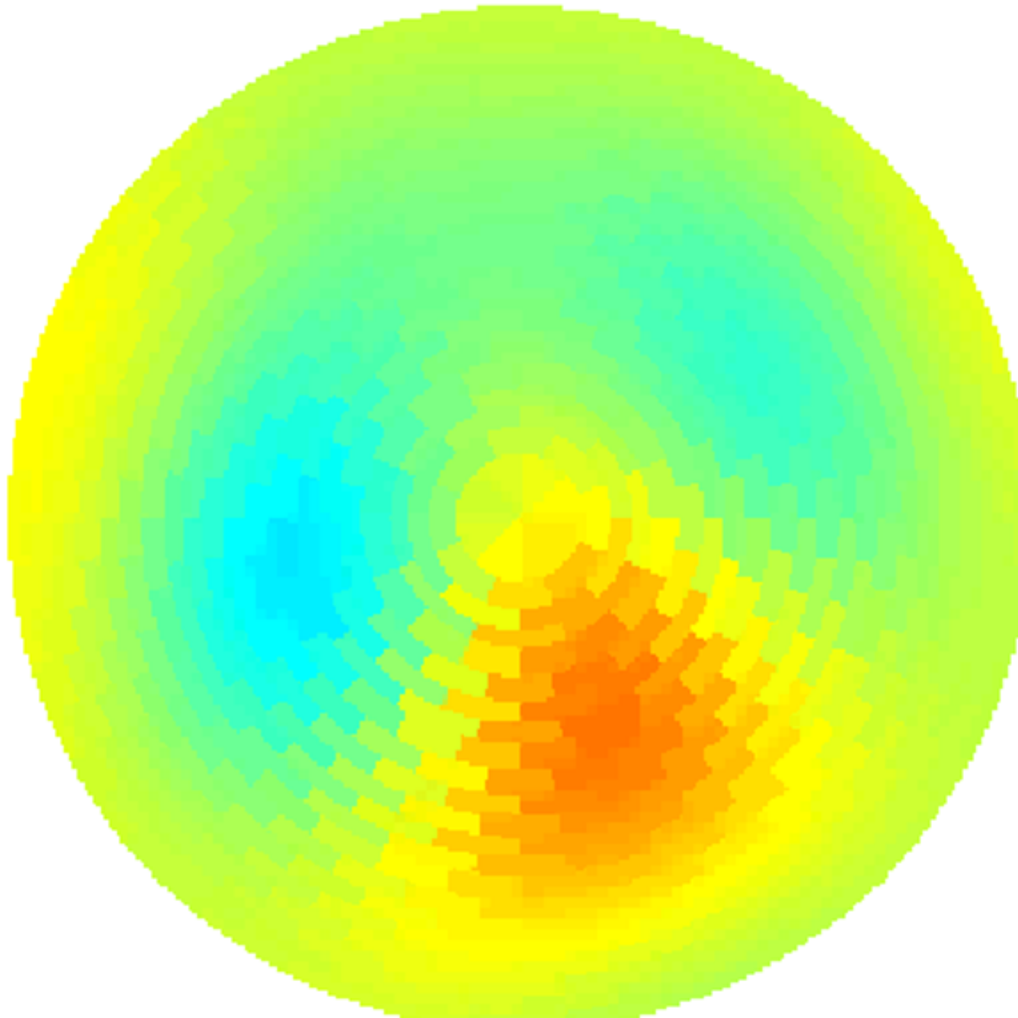
Reconstruction from phantom data



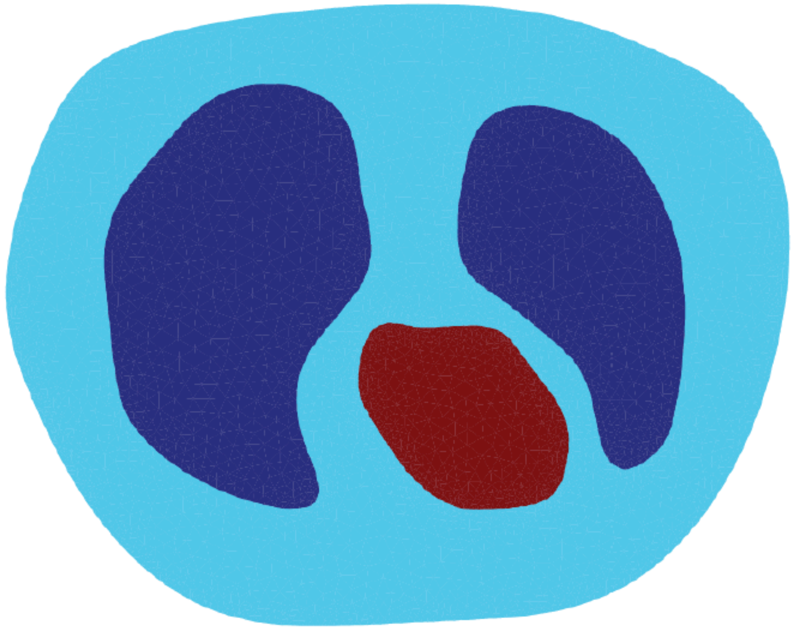
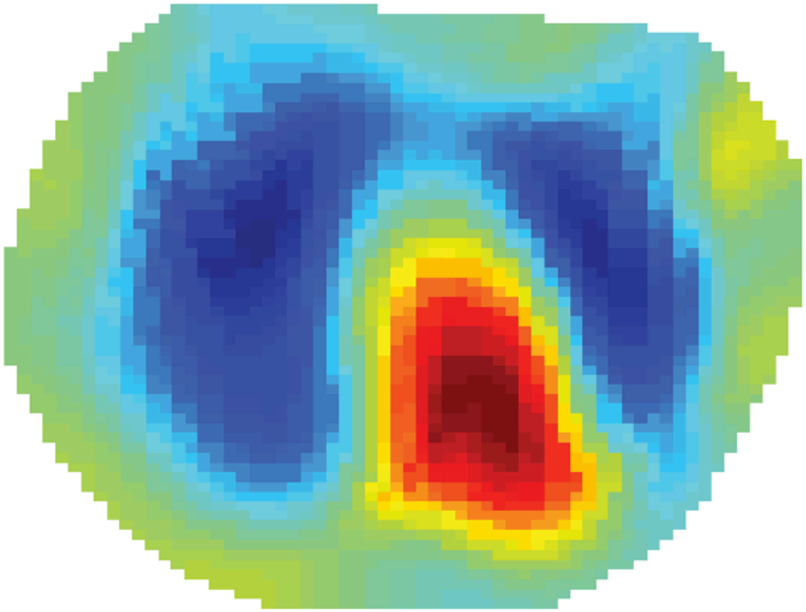


This example is from
Isaacson, Mueller, Newell and Siltanen 2004
IEEE Transactions on Medical Imaging 23, pp. 821-828

Reconstruction from data collected from a living person (Isaacson, Mueller, Newell, S)



Unknown boundary shape can be estimated from EIT data using Teichmüller space methods

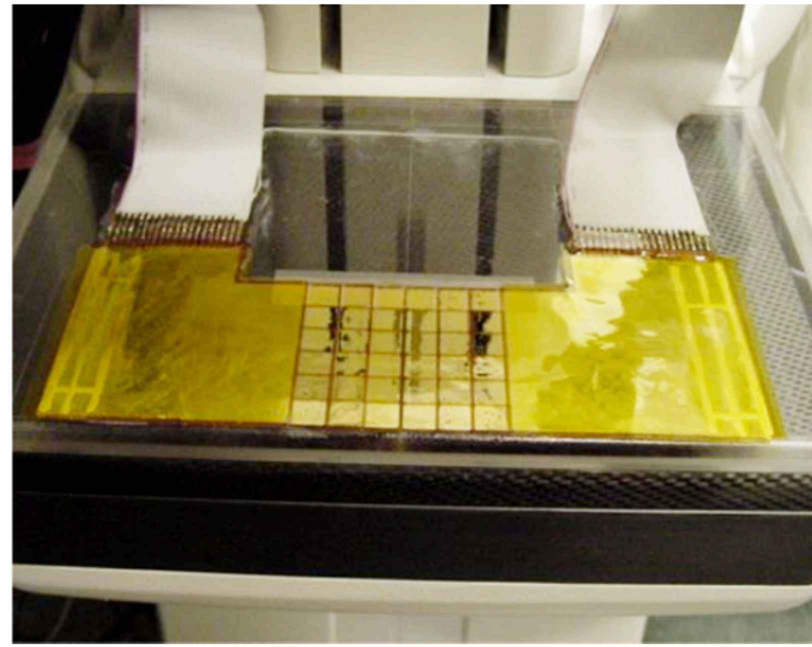
 

[Kolehmainen, Lassas, Ola & S 2012]

The most promising use of EIT is detection of breast cancer in combination with mammography

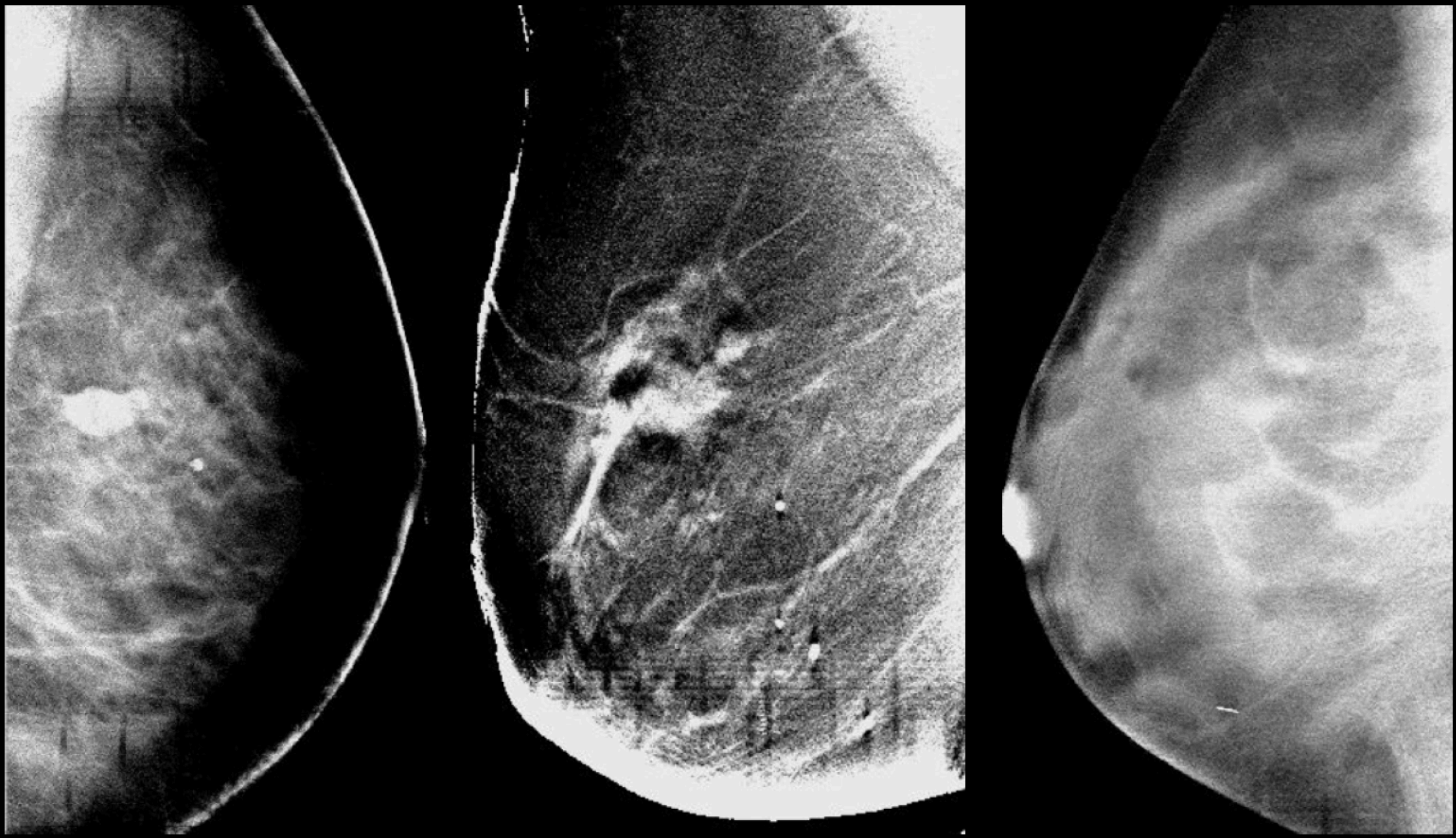
ACT4 and mammography devices

Radiolucent electrodes

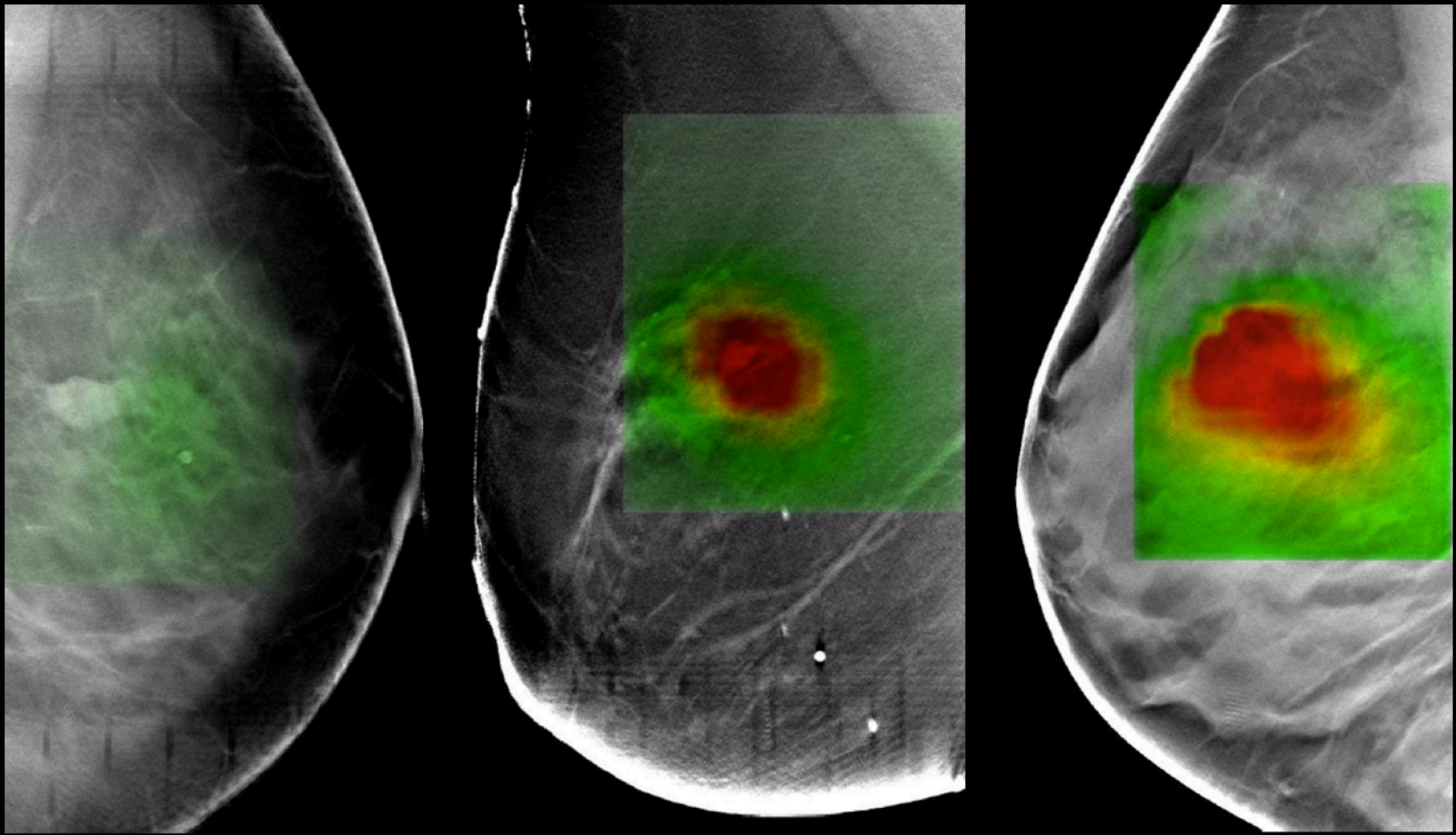


Cancerous tissue is up to four times more conductive than healthy breast tissue [Jossinet 1998]. The above experiment by **David Isaacson**'s team measures 3D X-ray mammograms and EIT data at the same time.

Which of these three breasts have cancer?

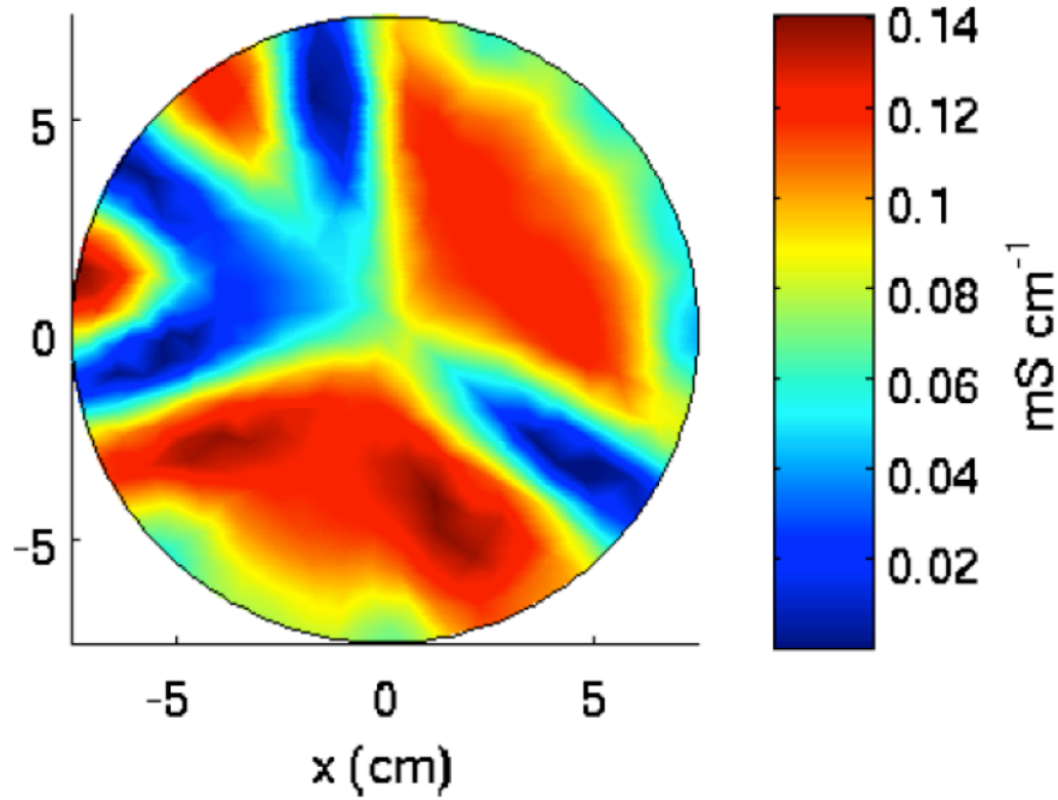


Spectral EIT can detect cancerous tissue



[Kim, Isaacson, Xia, Kao, Newell & Saulnier 2007]

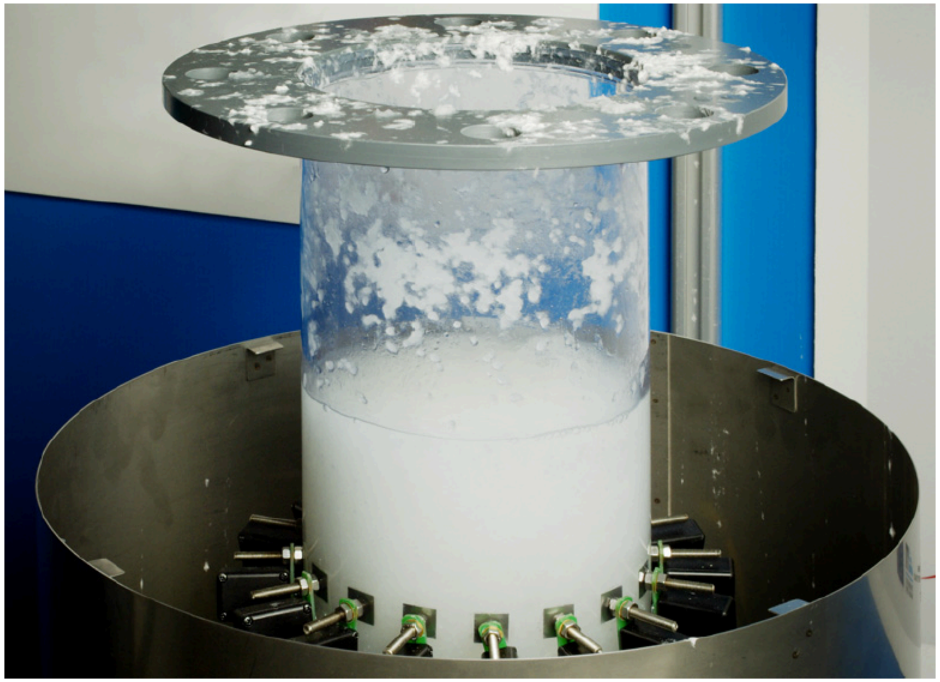
EIT can be used for crack detection in concrete structures

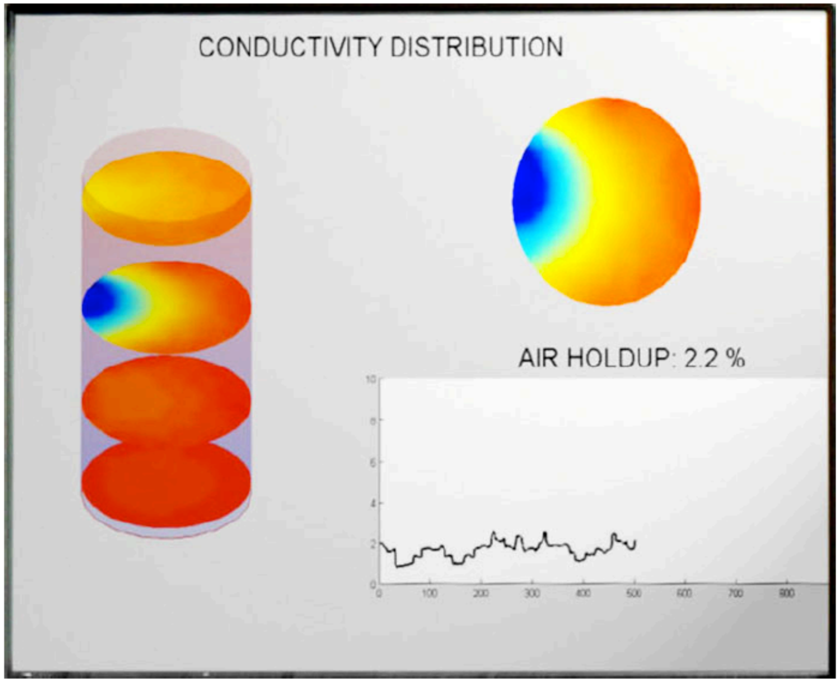


[Karhunen, Seppänen, Lehikoinen, Monteiro, Kaipio, Blunt, Hyvönen]

EIT is useful for industrial process monitoring

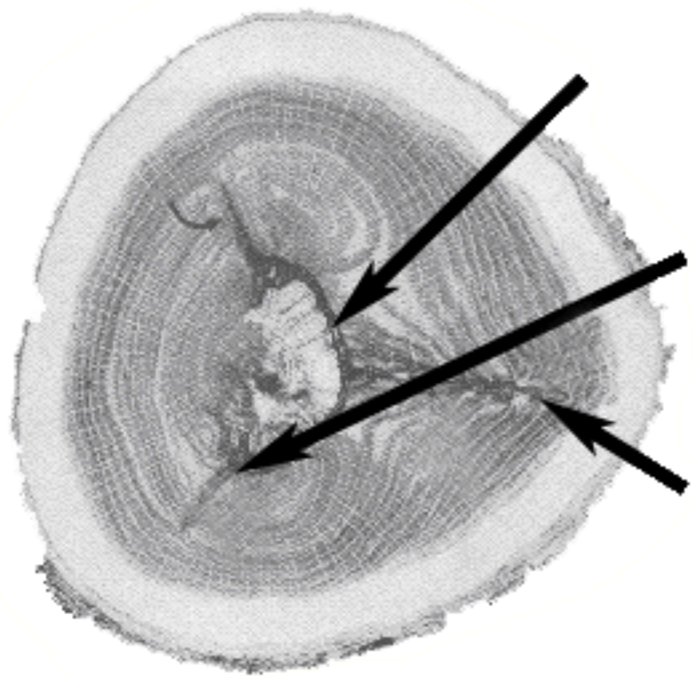
In this industrial application, air holdup in pulp suspension is monitored using EIT





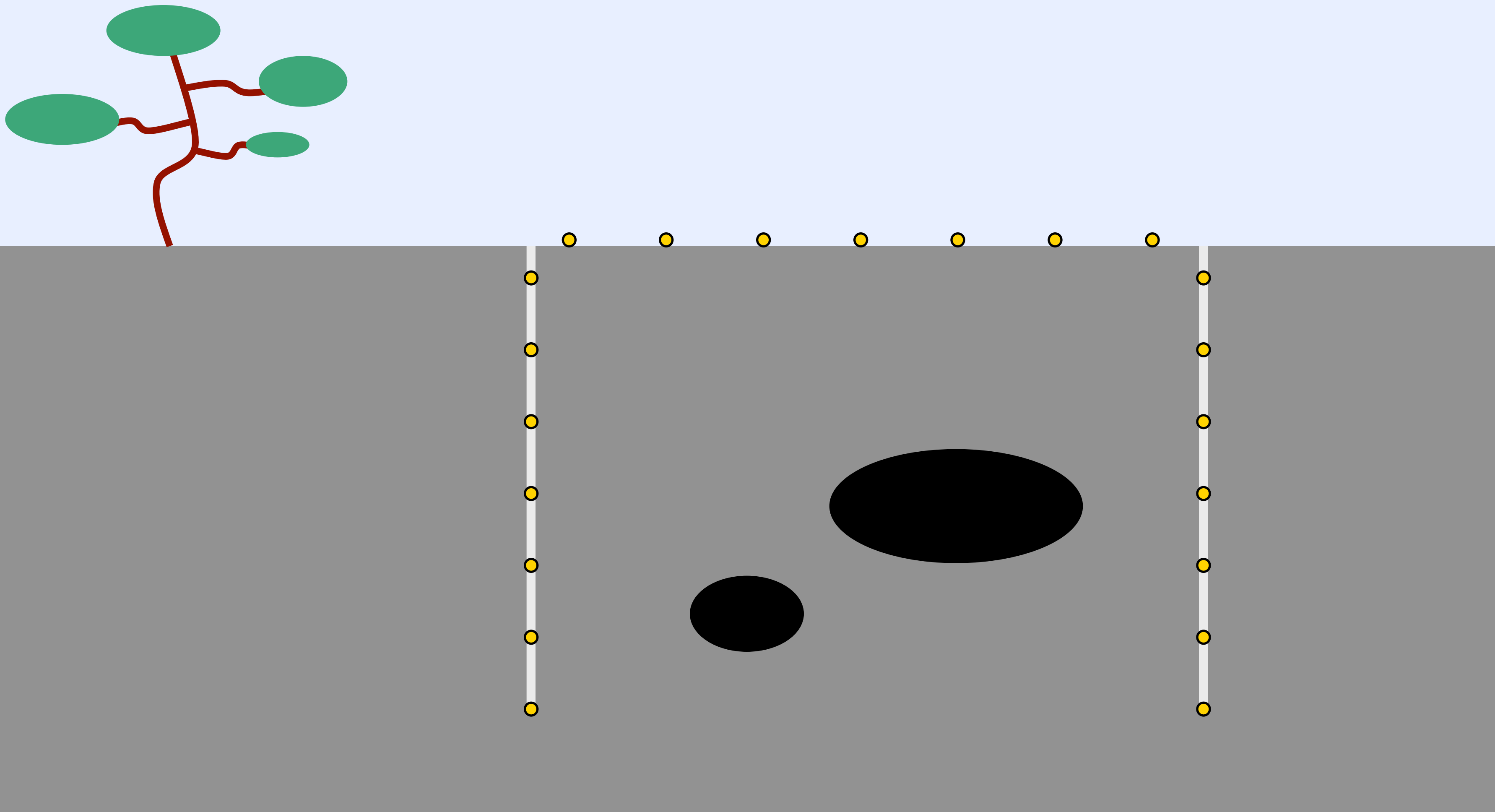
The startup company Numcore commercialized EIT and was acquired by Outotec in March 2012

EIT can be used for finding defects in materials

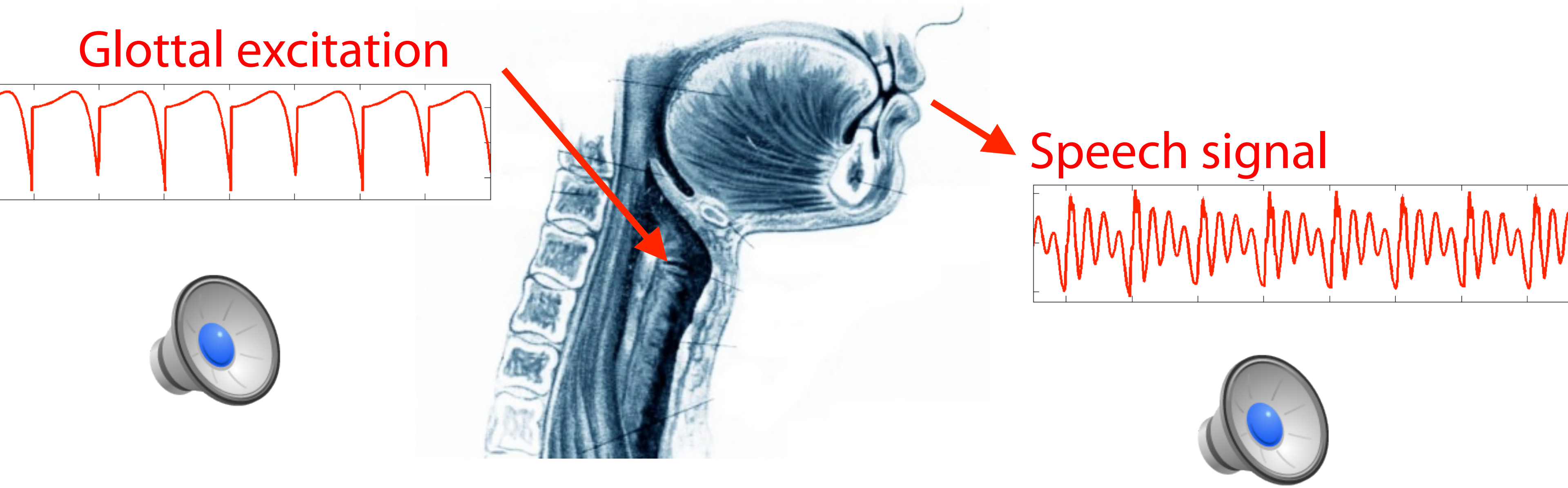


Shigo, A.L., 1983. Tree Defects: A Photo Guide. USDA Forest Service, No. Cent. For. Exp. Sta., GTRNE-82.

Geological sensing of oil or metals is another application of EIT



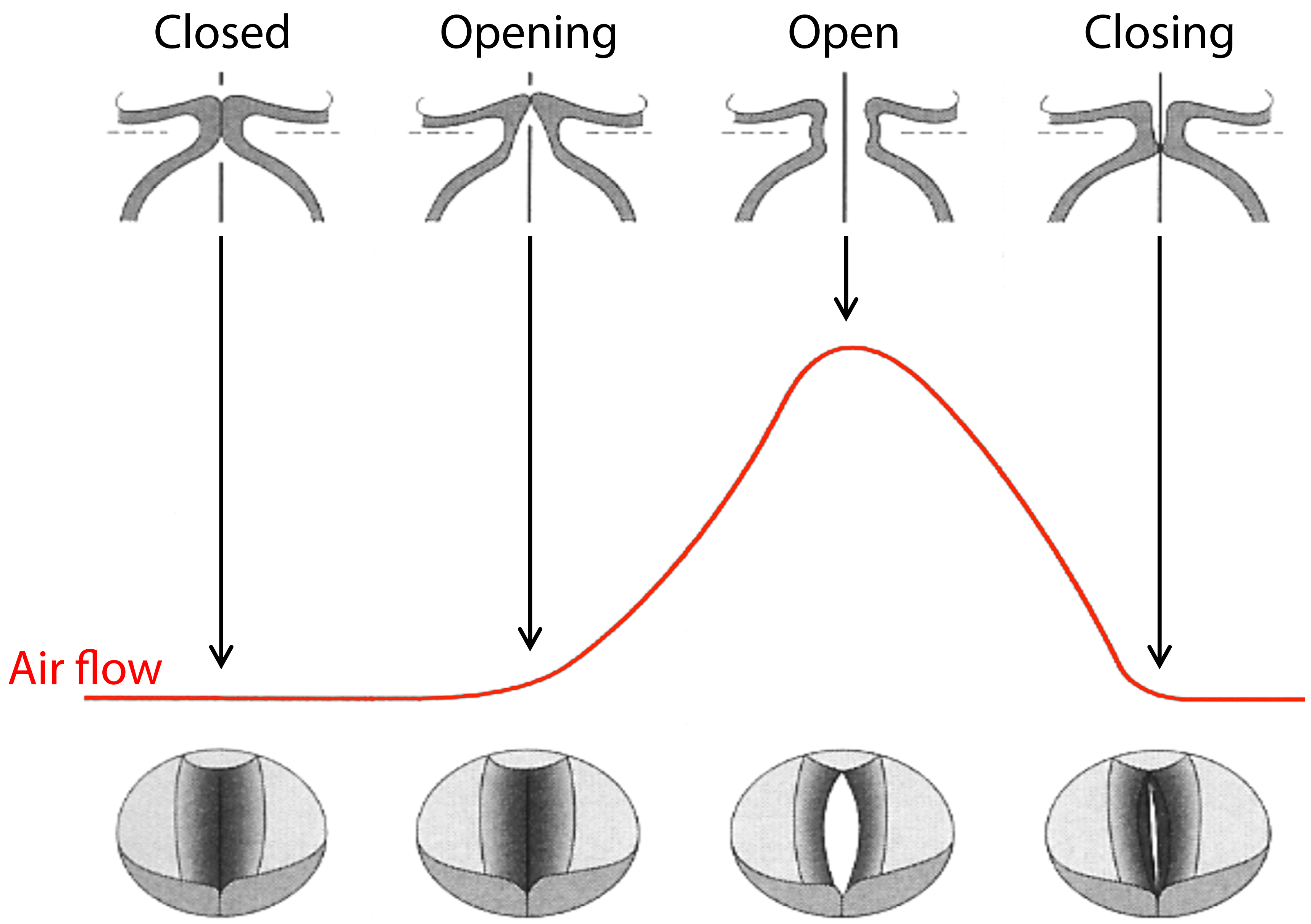
The goal of Glottal Inverse Filtering is to recover the glottal excitation and the vocal tract filter

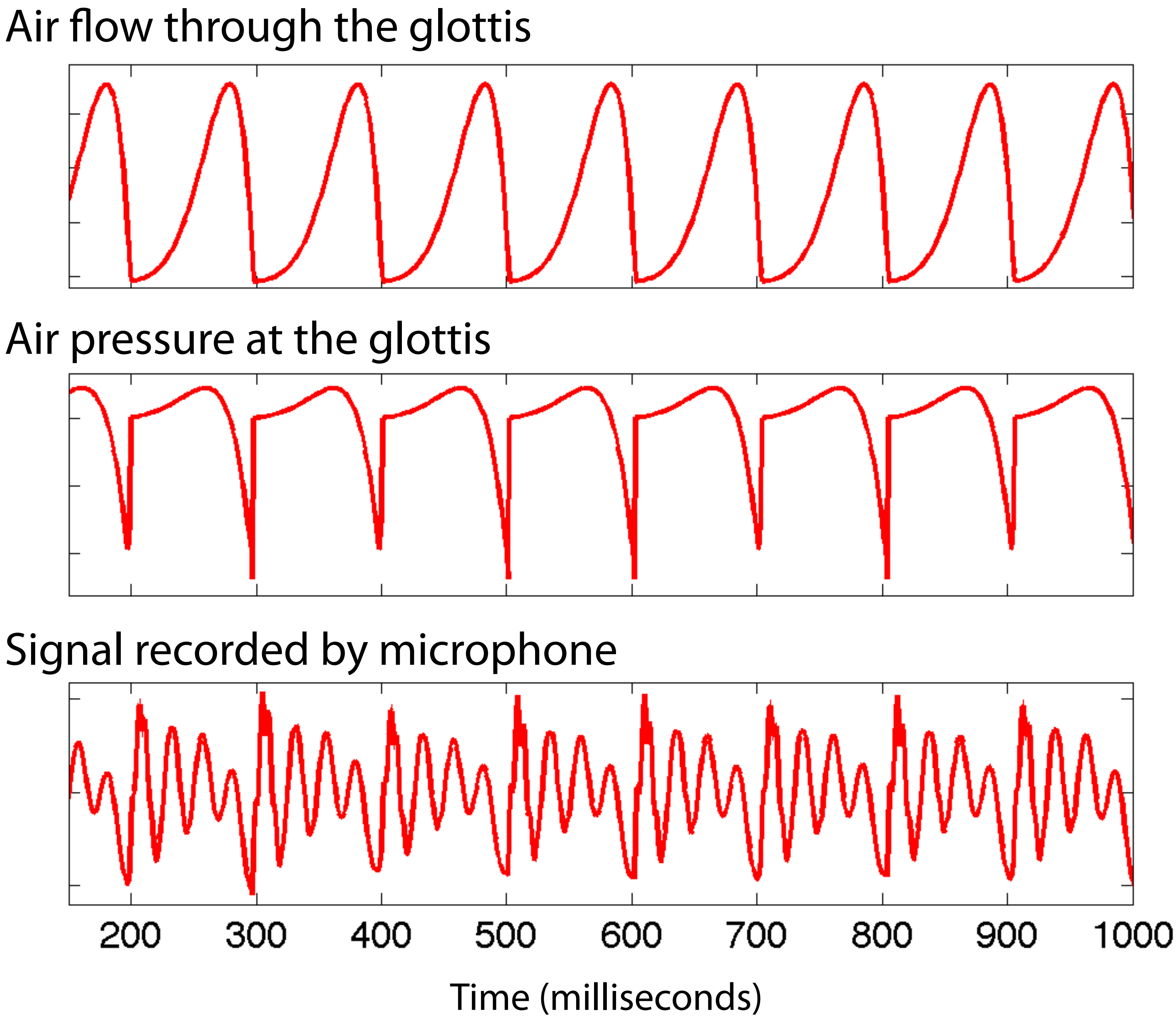


Glottis is the opening between vocal folds.

Direct problem: If we know the glottal excitation signal and the shape of the vocal tract, what does the microphone record?
Inverse problem: Given a speech signal recorded by a microphone, find the glottal excitation and the vocal tract.

Glottal flow as function of time



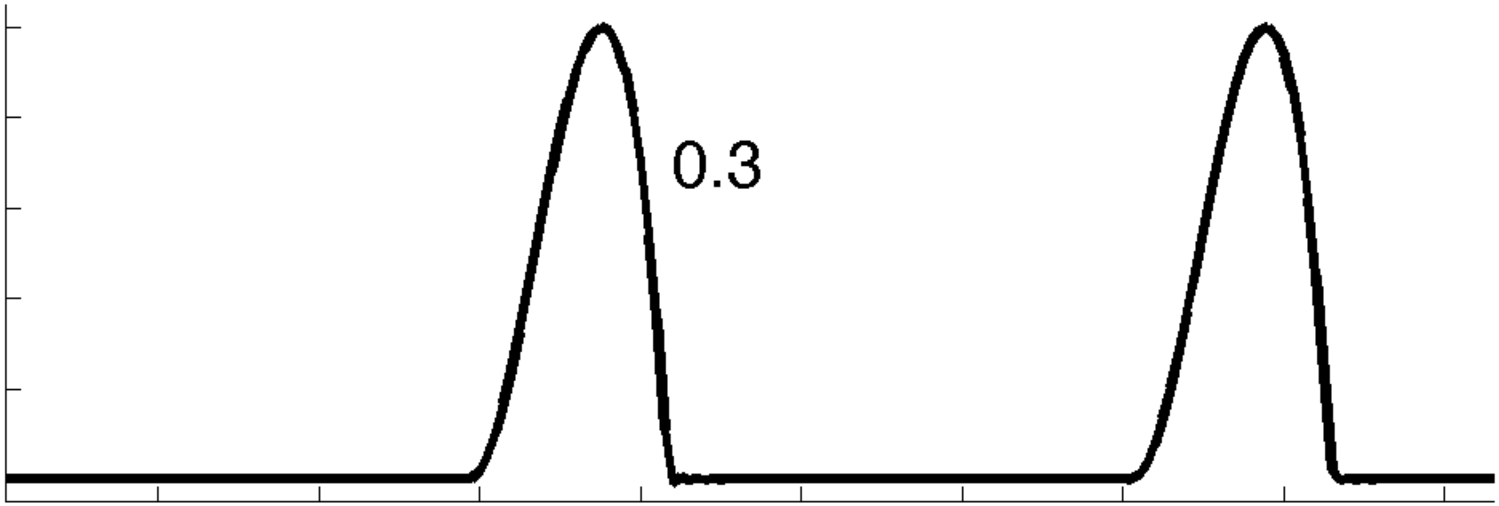

Air flow through the glottis
Air pressure at the glottis
Signal recorded by microphone
200
300
400
500
600
700
800
900
1000
Time (milliseconds)

An improved GIF algorithm has important applications

1. Computational speech synthesis:
Clearer information announcements,
more efficient automatic telephone-based services, and
devices that help handicapped people express emotions.

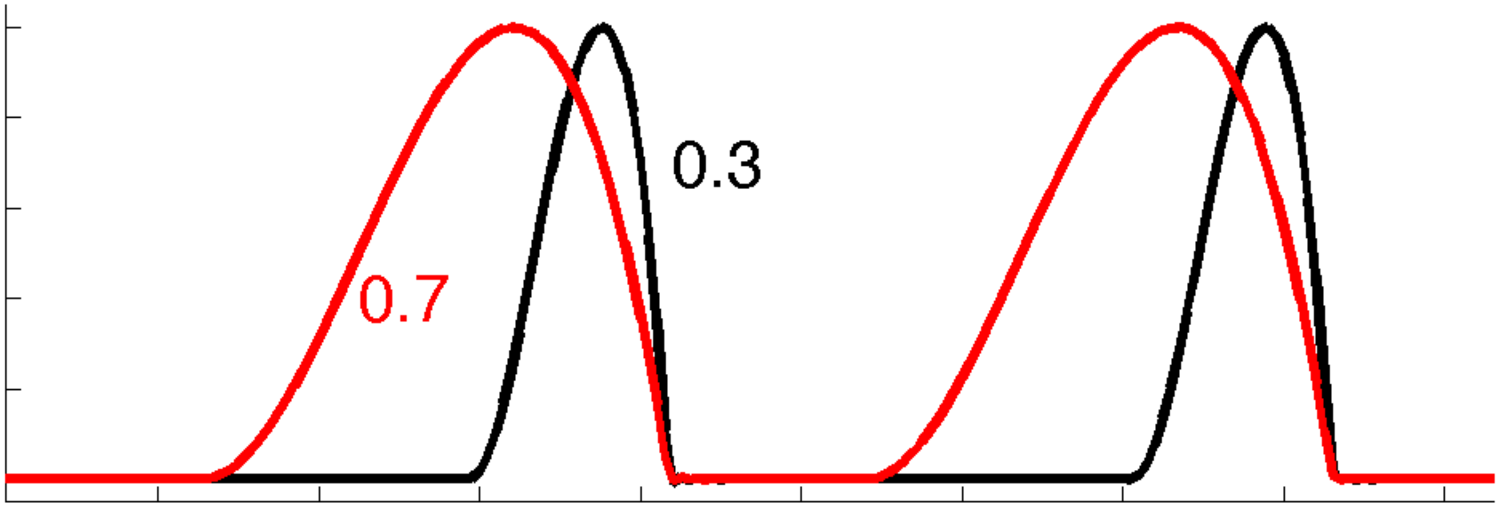
2. Noise-robust automatic speech recognition:
Efficient and reliable man-machine interfaces.

We use the Klatt model for the glottal excitation



The Klatt model has only one parameter called K, with values between zero and one.

We use the Klatt model for the glottal excitation



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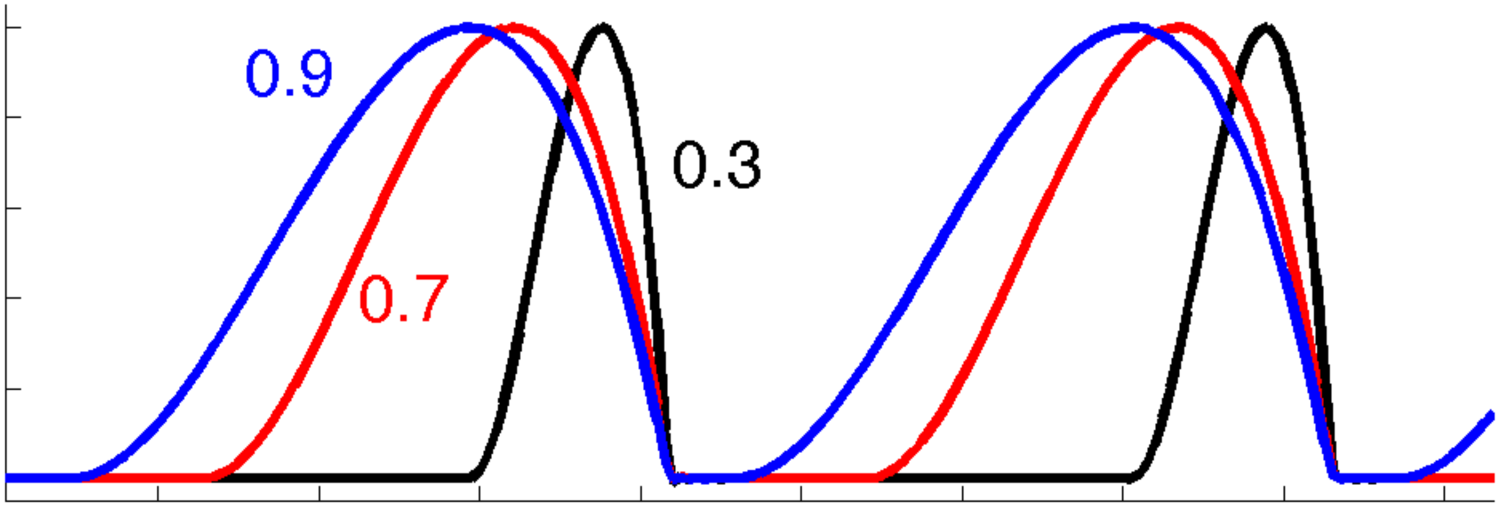
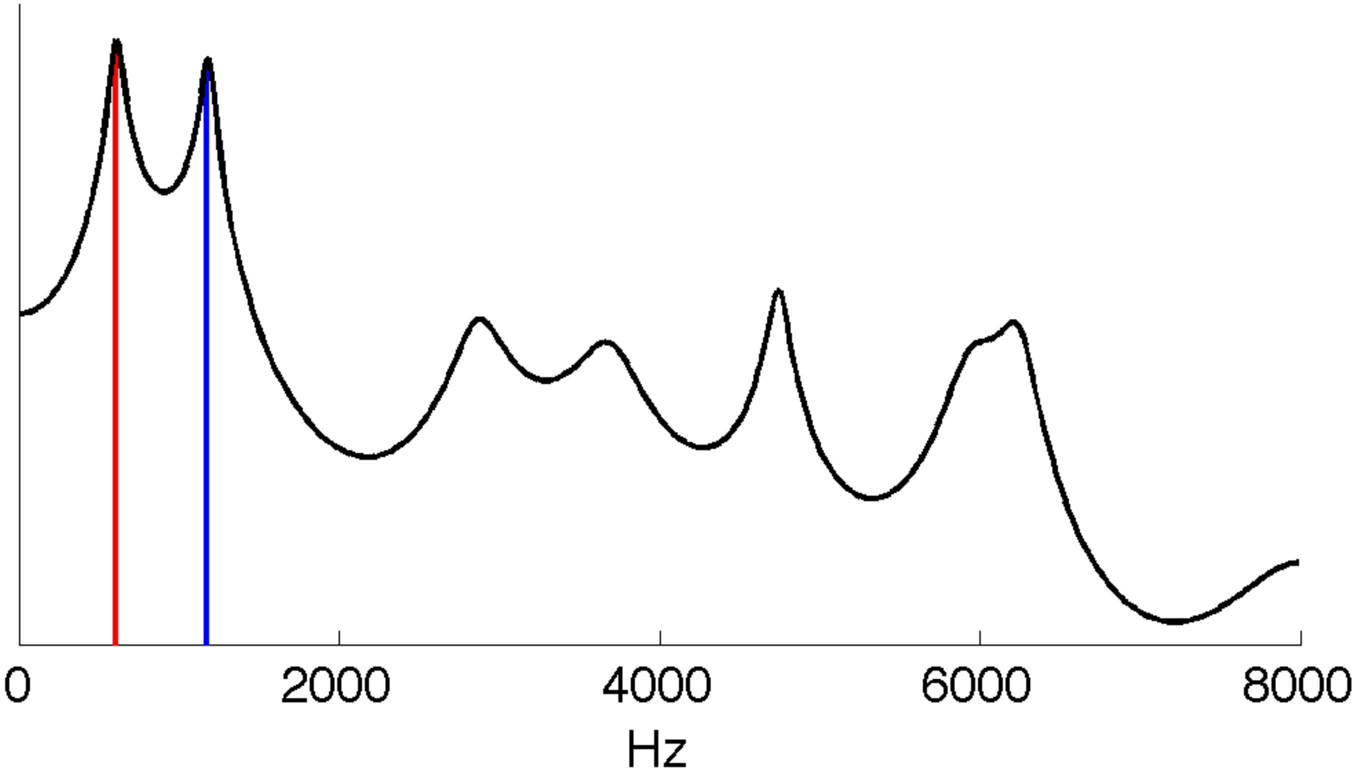
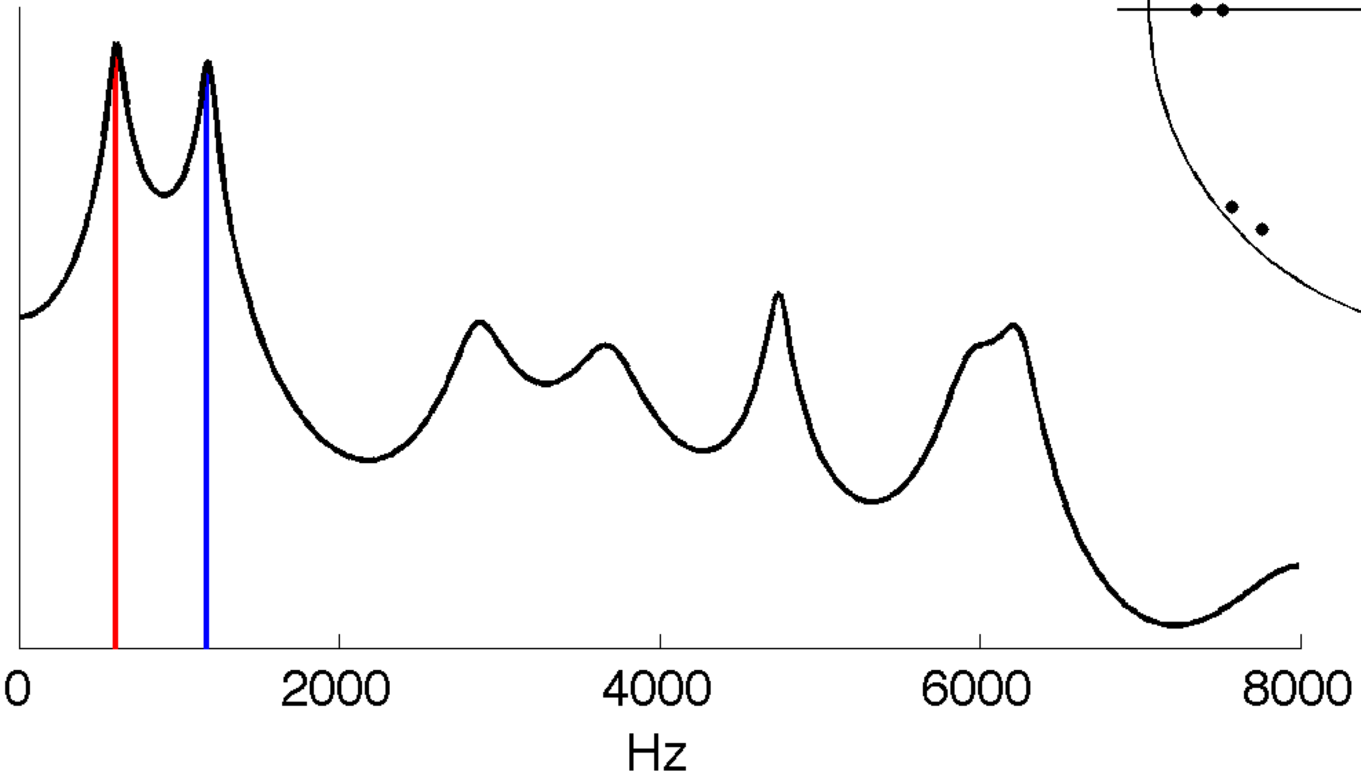


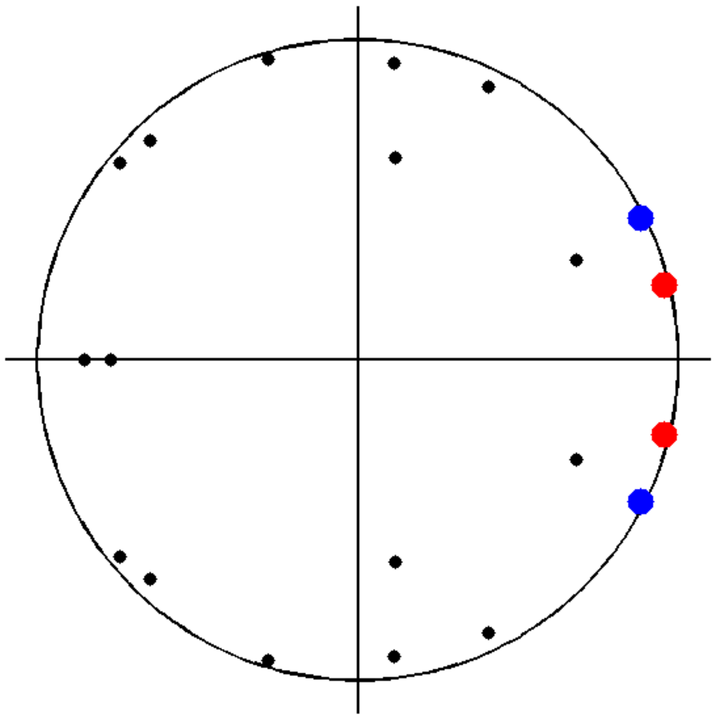
The Klatt model has only one parameter called K, with values between zero and one.

We model the effect of the vocal tract by a linear filter, much like a frequency equalizer

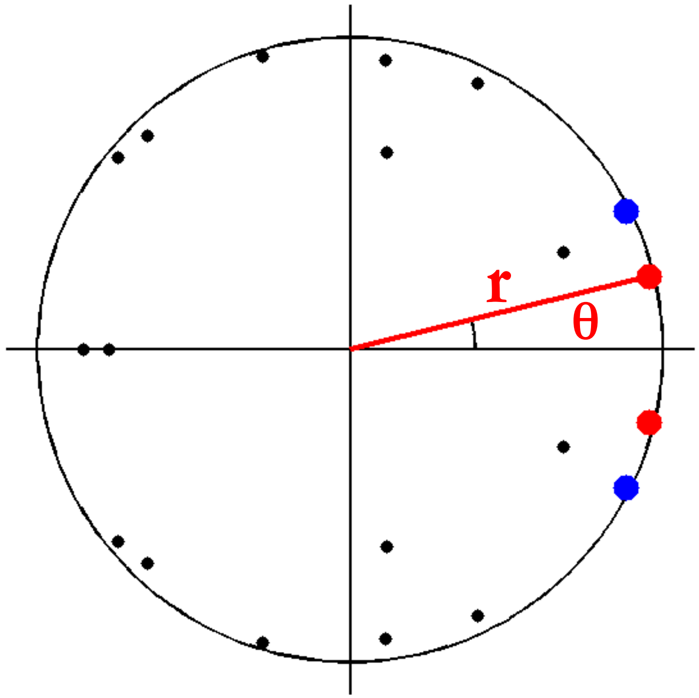


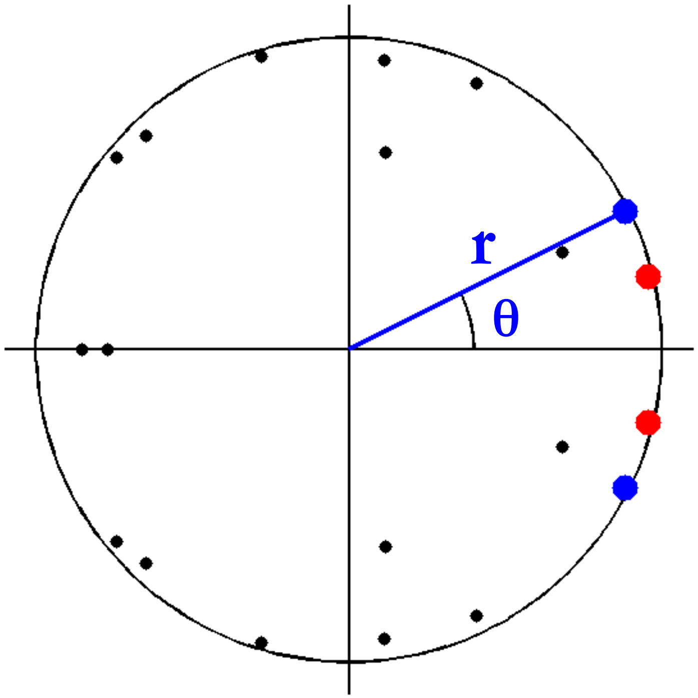
Technically, the frequency response is described by an all-pole filter





Looking at two first formants leads to four parameters for the vocal tract

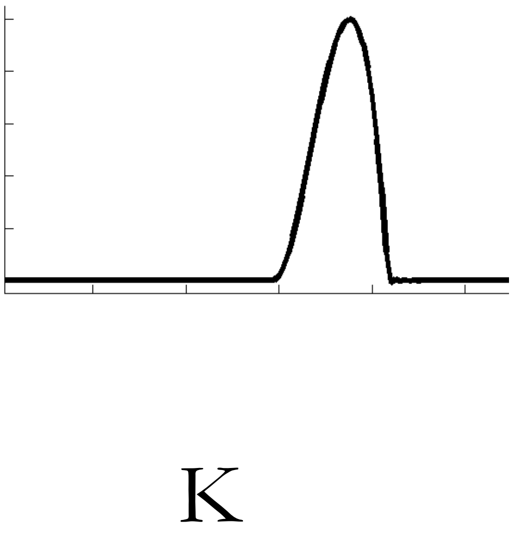


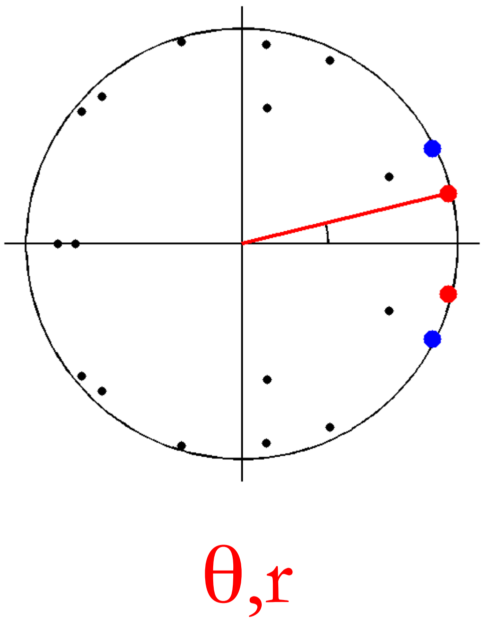


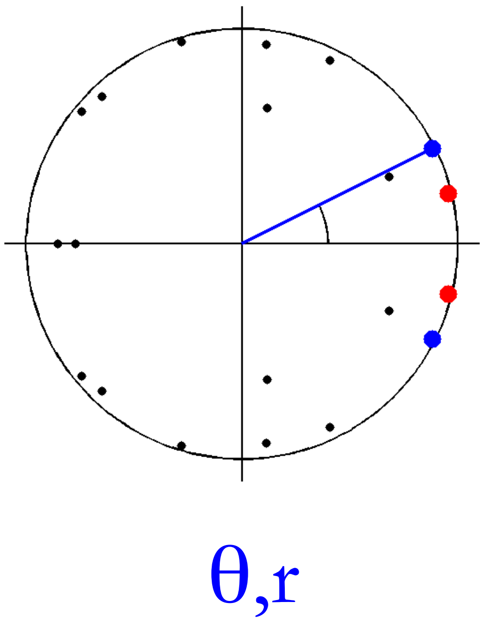
Angle and length of the pole corresponding to the first formant

Angle and length of the pole corresponding to the second formant

In this model, glottal inverse filtering means finding these five numbers:





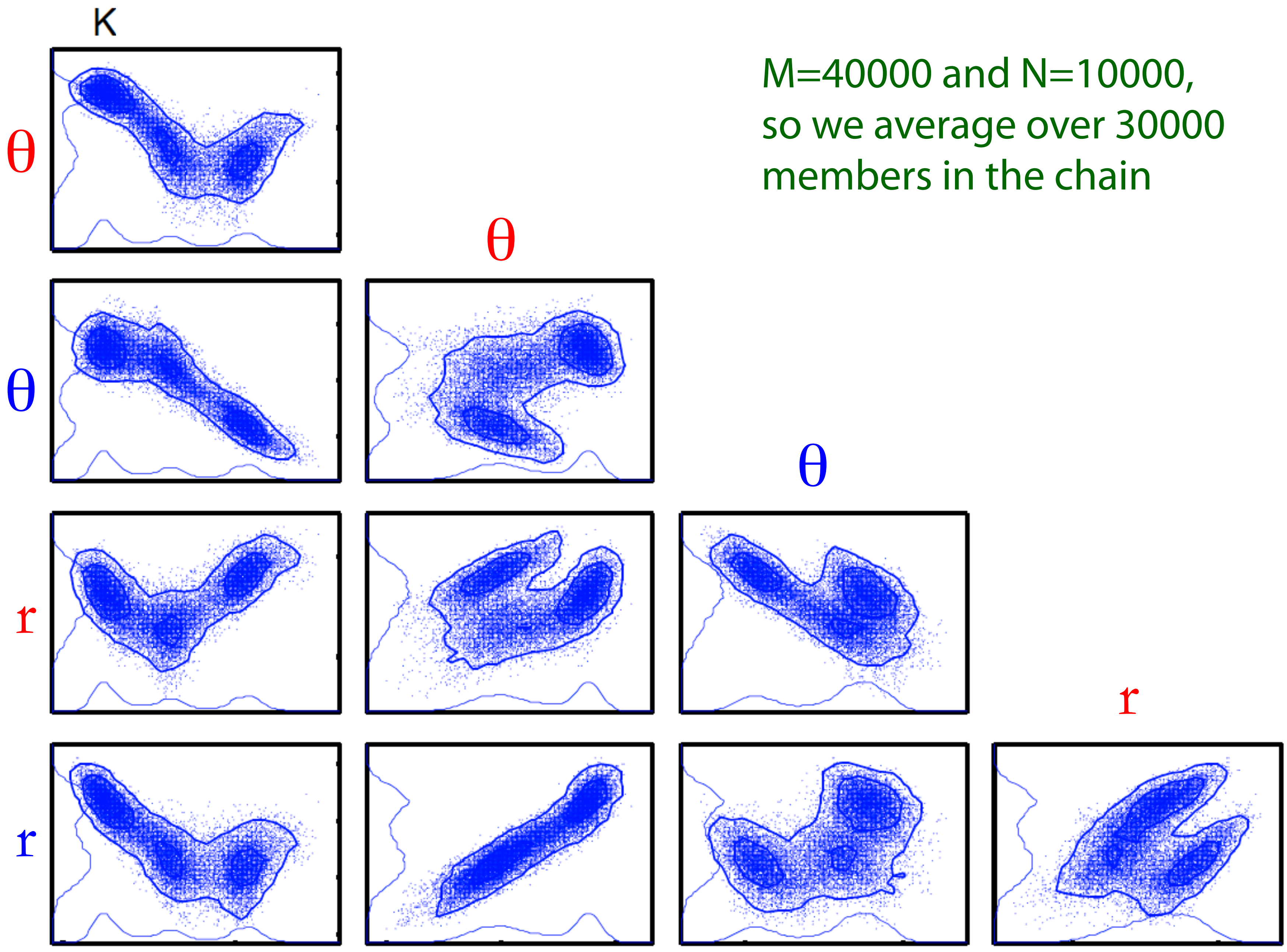


But where does Monte Carlo come in the picture?

The Markov chain Monte Carlo method produces a long sequence of excitations and vocal tracts

$$\underbrace{\begin{bmatrix} K_1 \\ \theta_1 \\ r_1 \\ \theta_1 \\ r_1 \end{bmatrix}, \begin{bmatrix} K_2 \\ \theta_2 \\ r_2 \\ \theta_2 \\ r_2 \end{bmatrix}, \ldots, \begin{bmatrix} K_N \\ \theta_N \\ r_N \\ \theta_N \\ r_N \end{bmatrix}}_{\text{discard}}, \underbrace{\begin{bmatrix} K_{N+1} \\ \theta_{N+1} \\ r_{N+1} \\ \theta_{N+1} \\ r_{N+1} \end{bmatrix}, \begin{bmatrix} K_{N+2} \\ \theta_{N+2} \\ r_{N+2} \\ \theta_{N+2} \\ r_{N+2} \end{bmatrix}, \ldots, \begin{bmatrix} K_M \\ \theta_M \\ r_M \\ \theta_M \\ r_M \end{bmatrix}}_{\text{average}}$$

The choosing of the members in the above chain is done in a controllably random way. As a result, the chain explores all combinations of glottal excitation and vocal tract filter that
(1) Produce closely the measured signal, and
(2) Satisfy our a priori information (for example, if we know the vowel, we have a rough idea where the main formants are located)



M=40000 and N=10000,
so we average over 30000
members in the chain

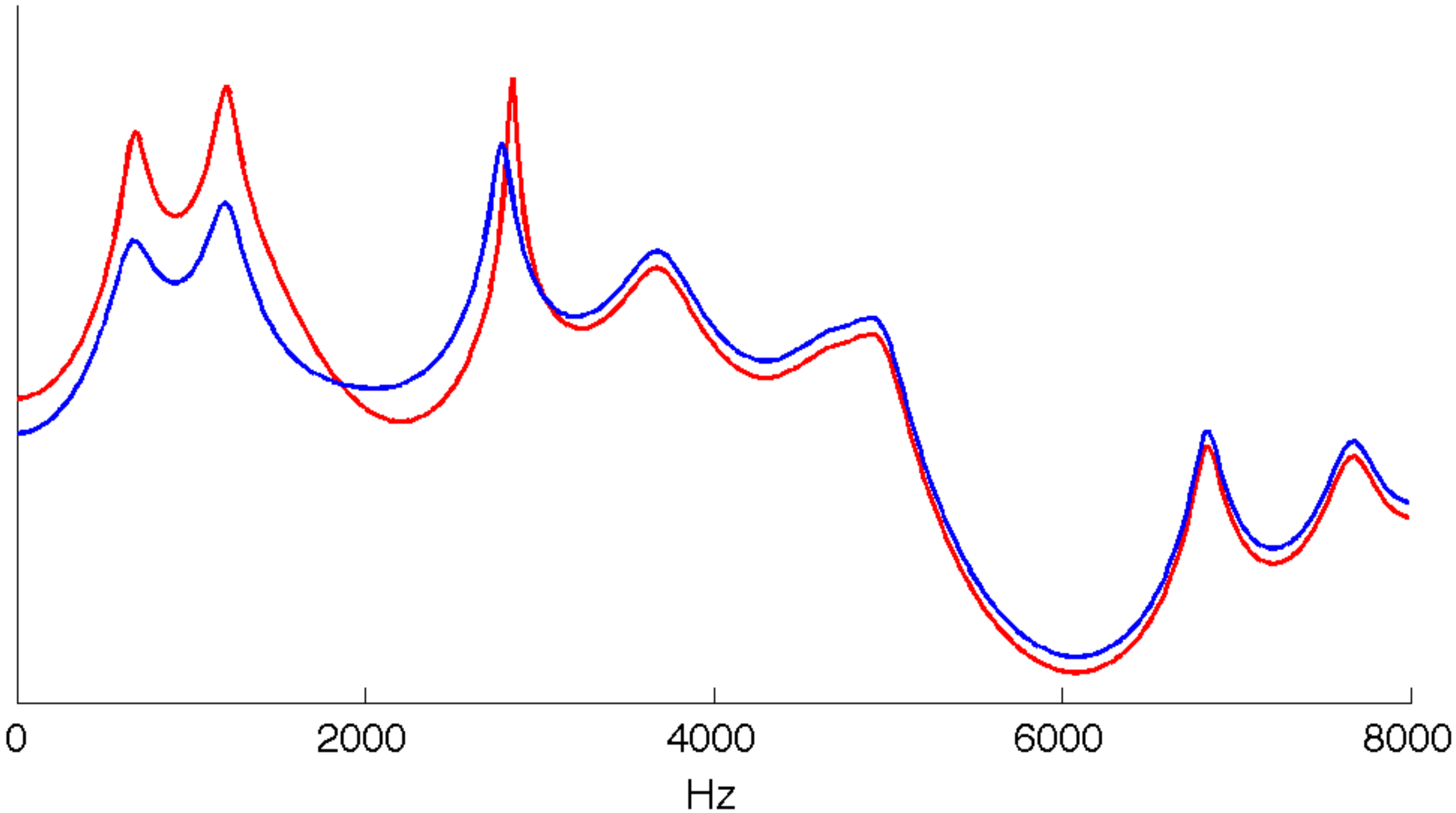
We perform glottal inverse filtering with two methods and compare the results

First, initial estimates of the vocal tract and glottal flow are evaluated by Iterative Adaptive Inverse Filtering (IAIF), see [**Paavo Alku** 1992].

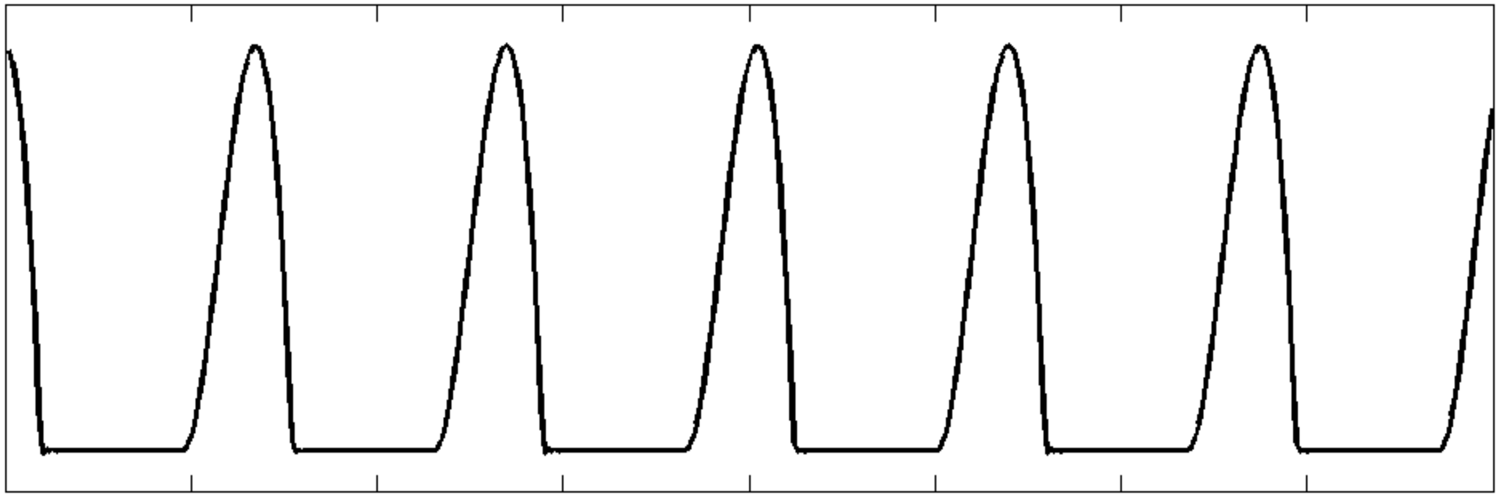
Starting with the IAIF result as an initial point, MCMC refines the GIF model parameters in order to get optimal inverse filtering.

We apply the method to sustained vowel signals collected from Finnish female students.

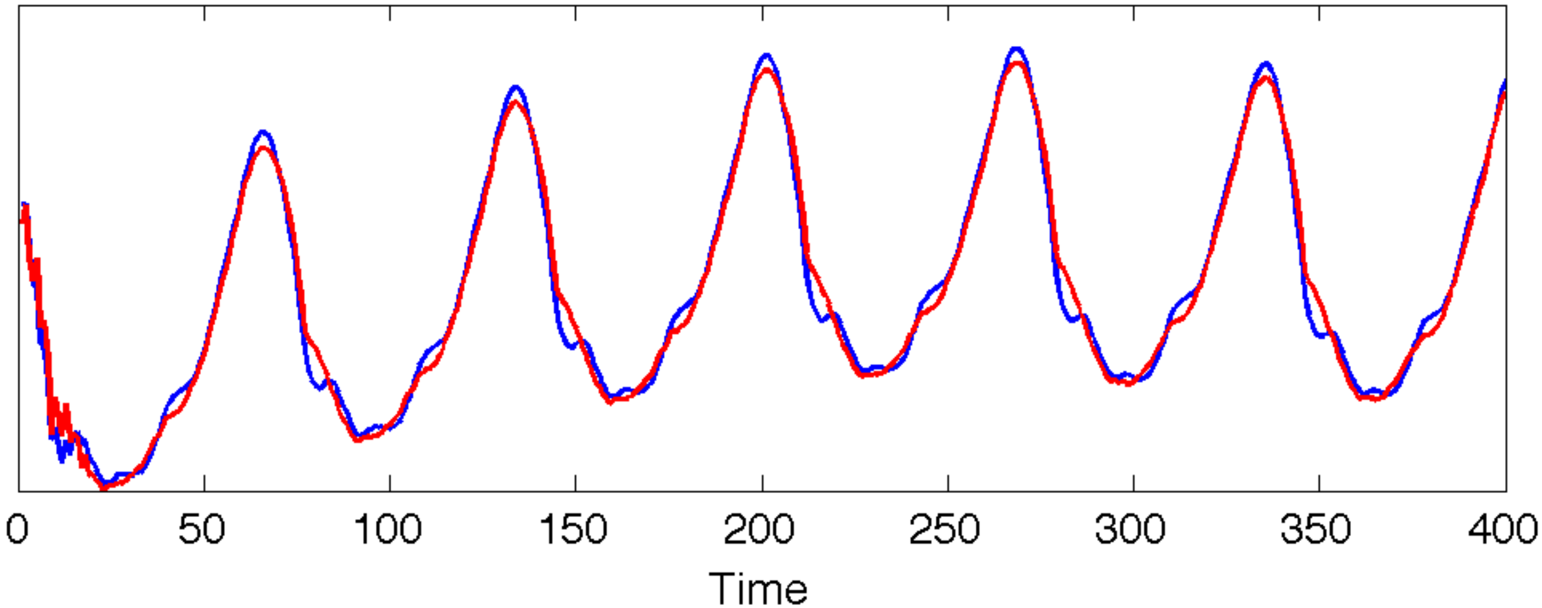
Case 1 (Julia): f0=237, frequency response by **IAIF** and **MCMC-GIF**



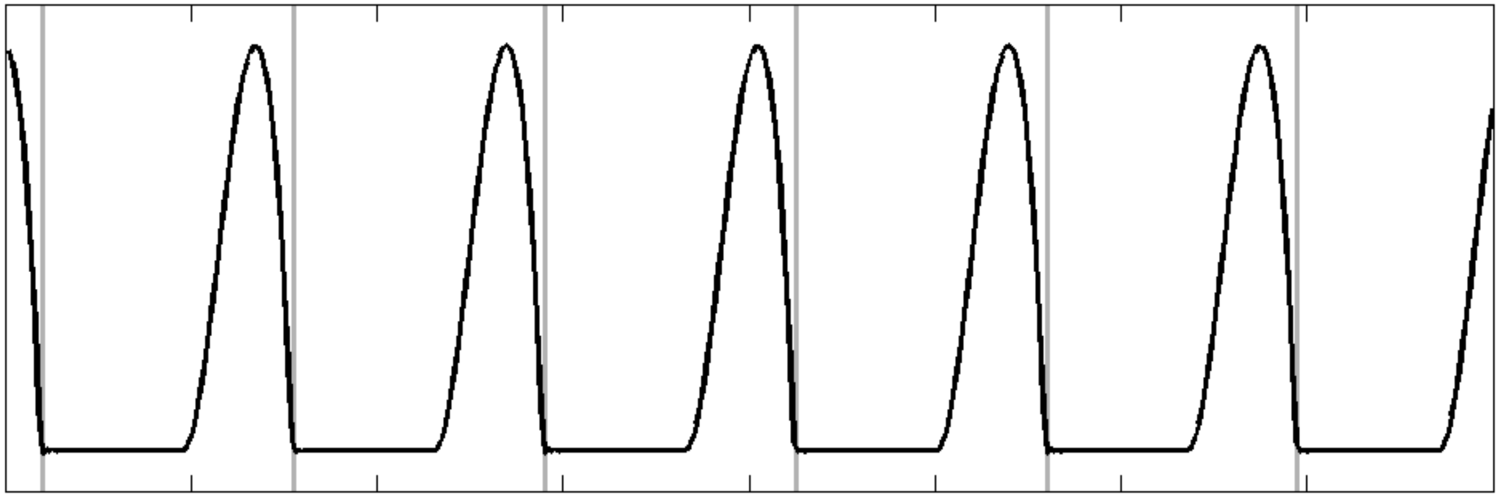
Case 1 (Julia): f0=237, Klatt pulse recovered by MCMC-GIF



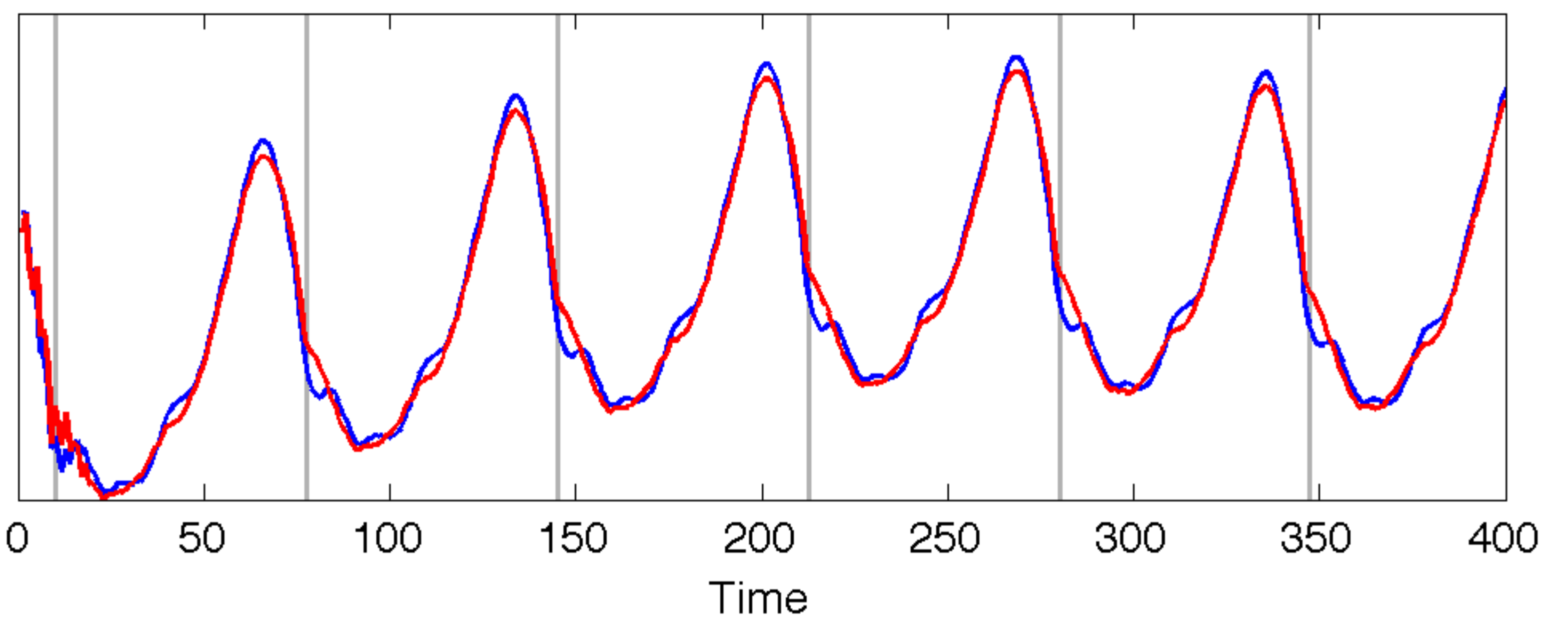
Inverse filtered results by **IAIF** and **MCMC-GIF**



Case 1 (Julia): f0=237, Klatt pulse recovered by MCMC-GIF



Inverse filtered results by **IAIF** and **MCMC-GIF**



Improved glottal inverse filtering can increase the quality of synthetic speech

Let us compare examples of synthetic speech.

Baseline

HMM modeling

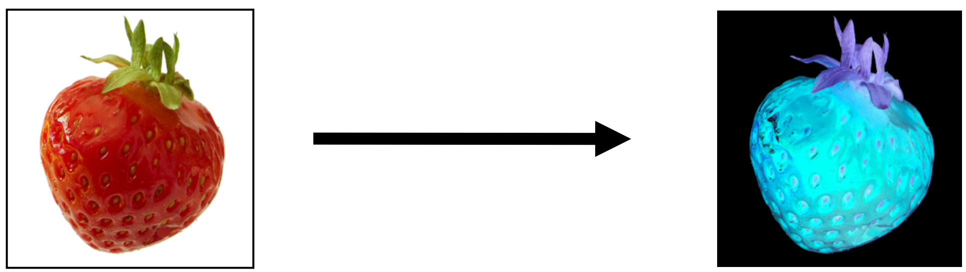
Raitio, Suni, Yamagishi, Pulakka, Nurminen, Vainio, Alku (2010)

What are *not* inverse problems?

Example of a non-inverse problem: Inverting a photograph

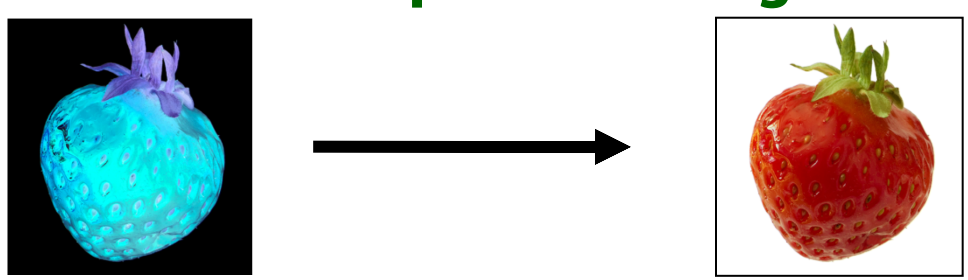
Direct problem:

Given a photograph, determine the negative image



"Inverse problem":

Given a negative, determine the positive image



Hadamard's definition of a "well-posed problem" has three parts

(H1) A solution exists

(H2) The solution is unique

(H3) The output depends continuously on the input

A problem is called "ill-posed", or inverse problem, if (H1), (H2) or (H3) fails.

Jacques Salomon Hadamard (1865-1963)

The mathematical model of EIT is the inverse conductivity problem introduced by Calderón

Let $\Omega \subset \mathbb{R}^2$ be the unit disc and let conductivity $\sigma : \Omega \to \mathbb{R}$ satisfy

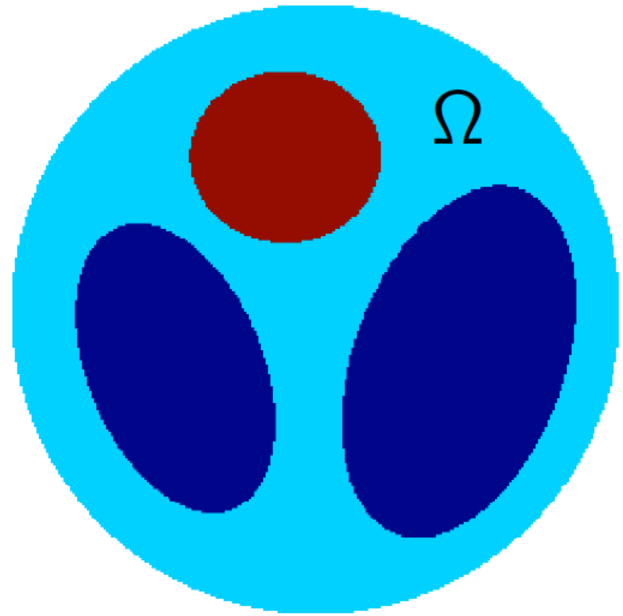
$$0 < M^{-1} \leq \sigma(z) \leq M.$$

Applying voltage f at the boundary $\partial\Omega$ leads to the elliptic PDE

$$\begin{cases} \nabla \cdot \sigma \nabla u = 0 \text{ in } \Omega, \\ u|_{\partial\Omega} = f. \end{cases}$$

Boundary measurements are modelled by the Dirichlet-to-Neumann map

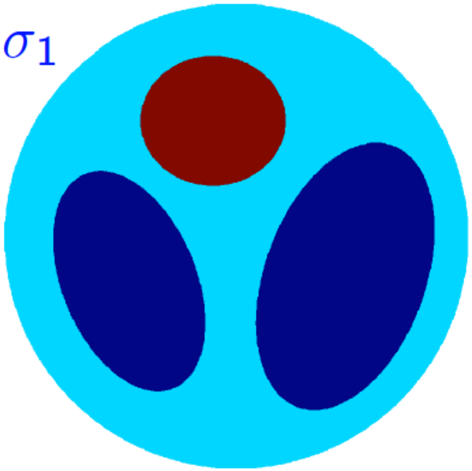
$$\Lambda_\sigma : f \mapsto \sigma \frac{\partial u}{\partial \vec{n}}|_{\partial\Omega}.$$



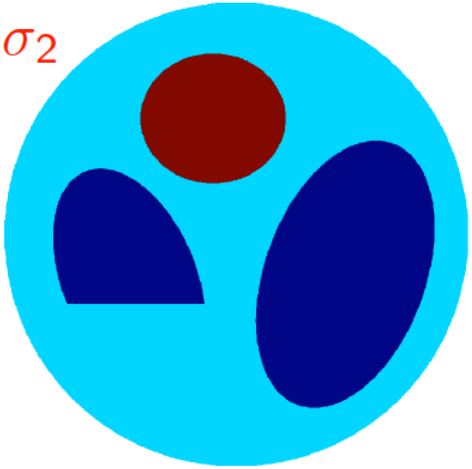
Calderón's problem is to recover σ from the knowledge of Λ_σ. It is a nonlinear and ill-posed inverse problem.

We illustrate the ill-posedness of Calderón's problem using a simulated example

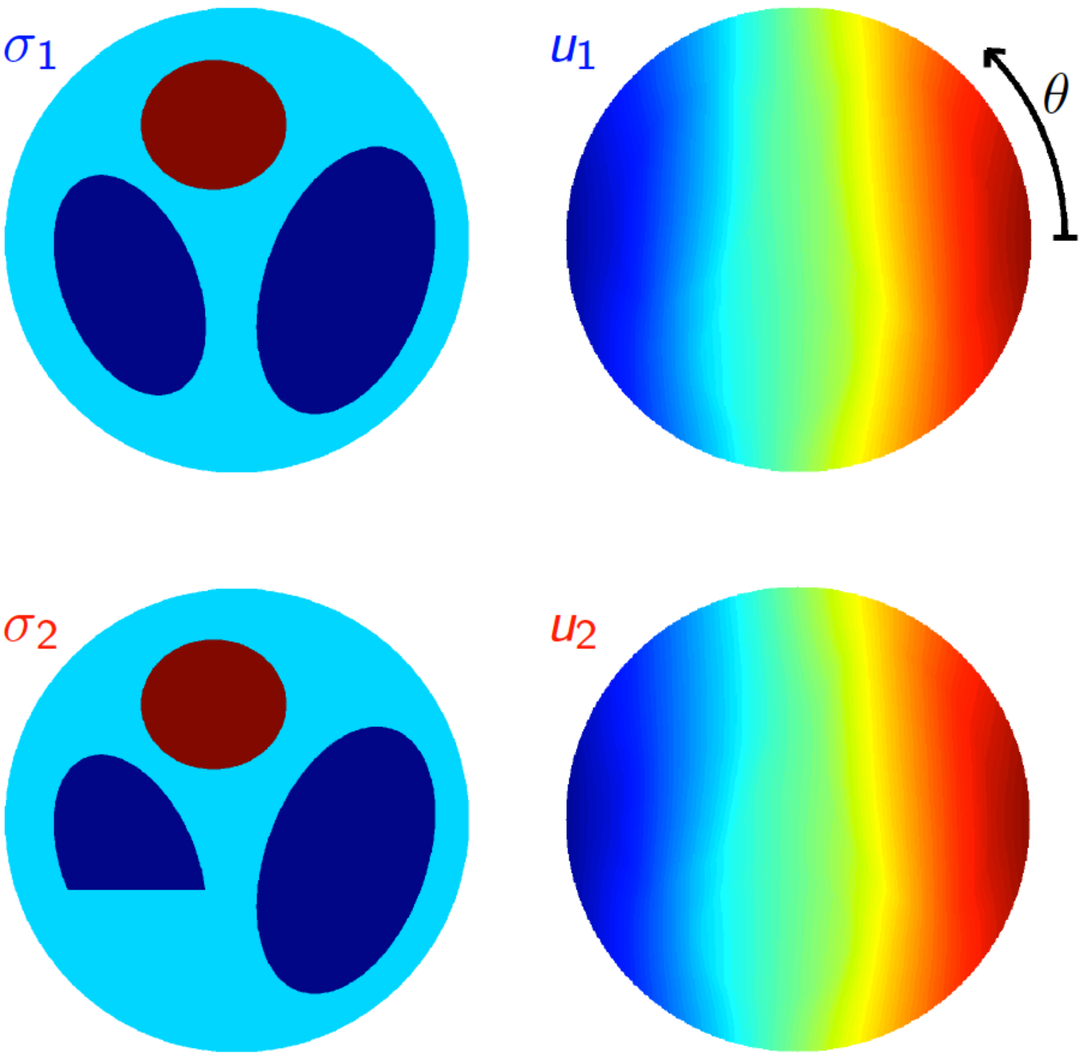
σ_1



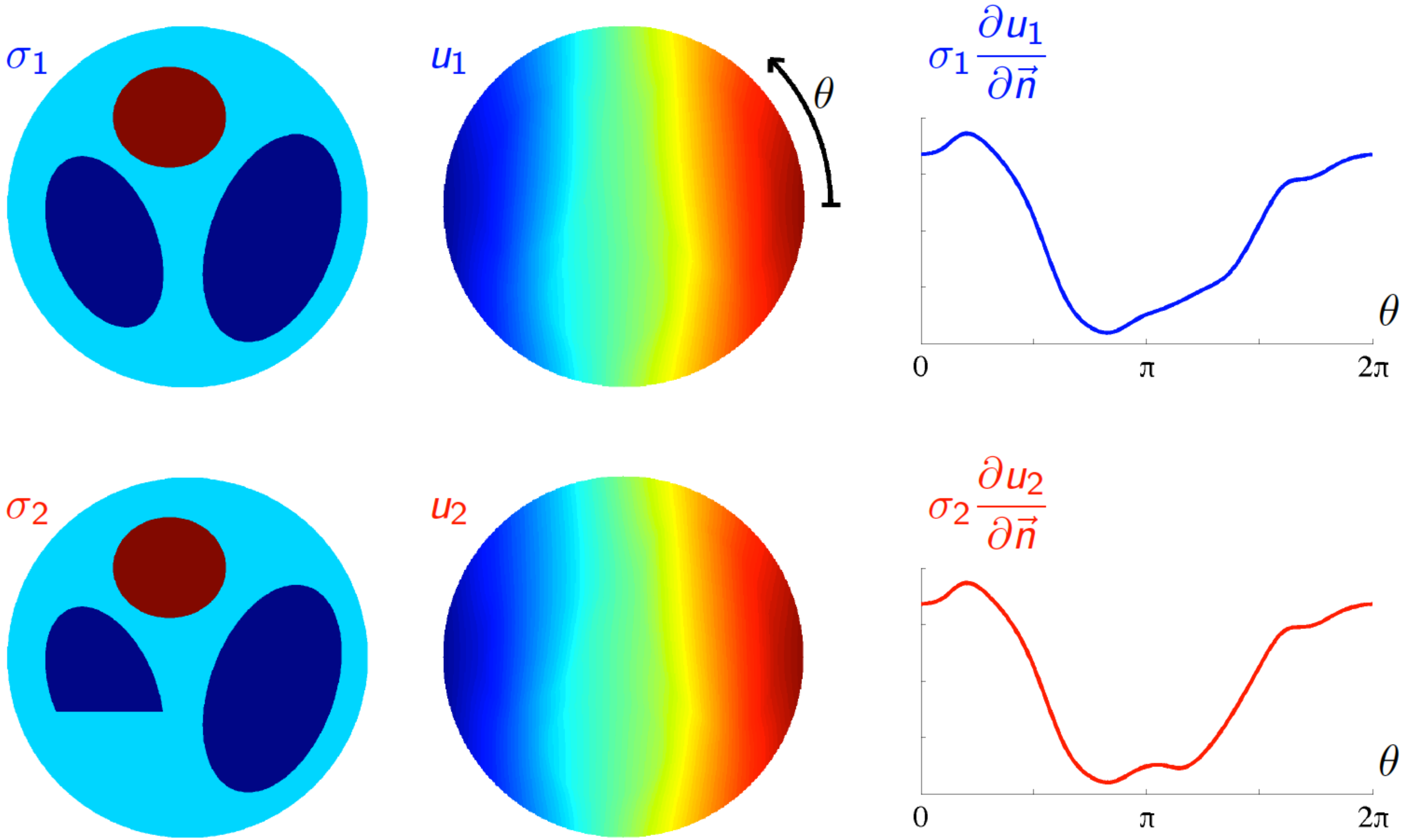
σ_2



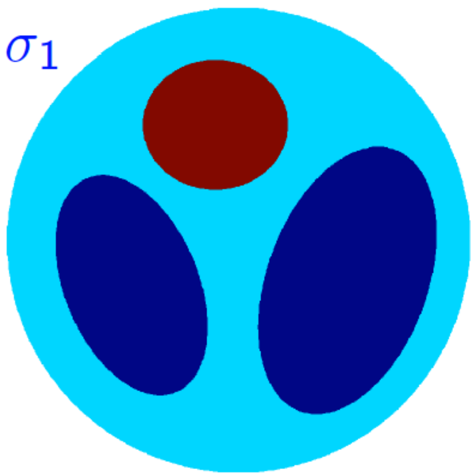
We apply the voltage distribution $f(\theta) = \cos\theta$ at the boundary of the two different phantoms

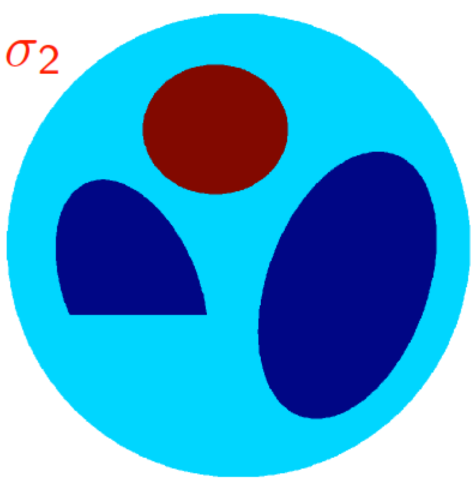


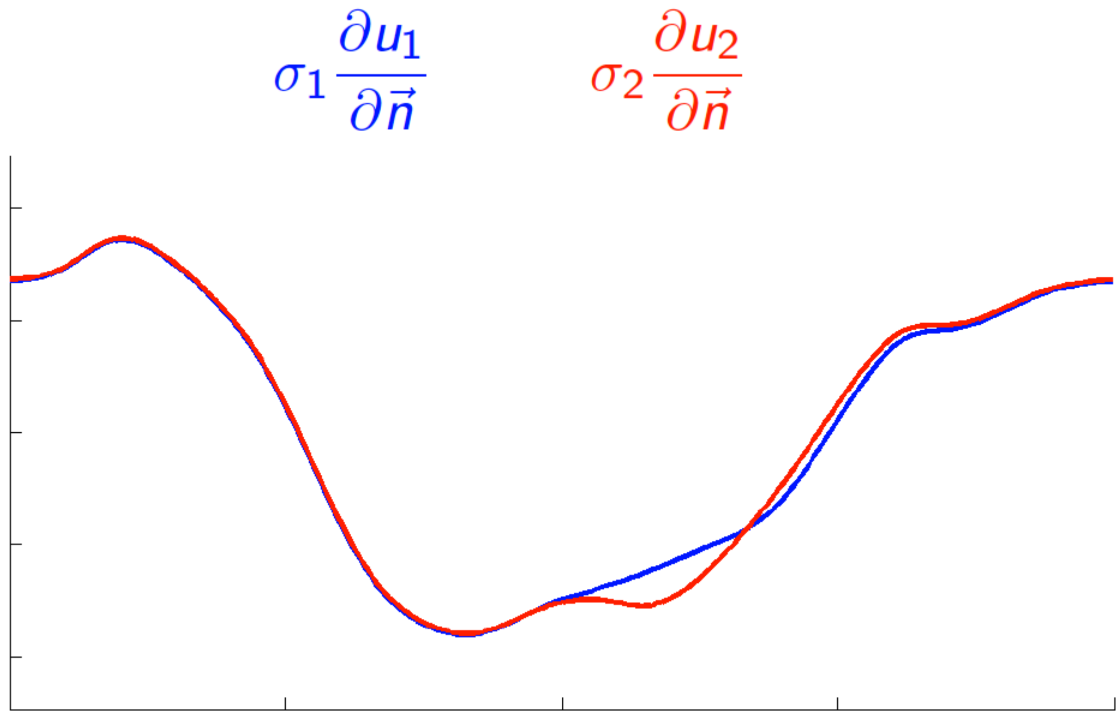
The measurement is the distribution of current through the boundary



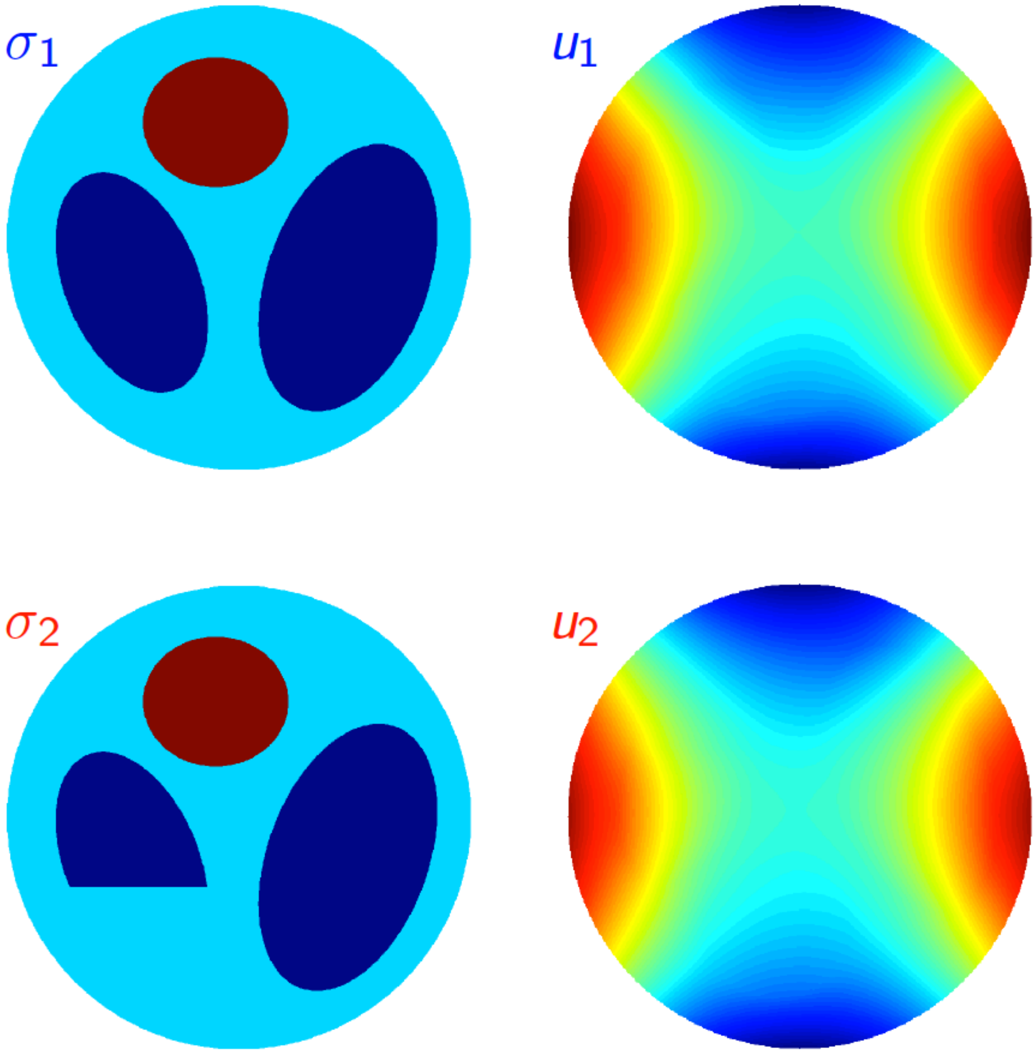
The measurements are very similar, although the conductivities are quite different



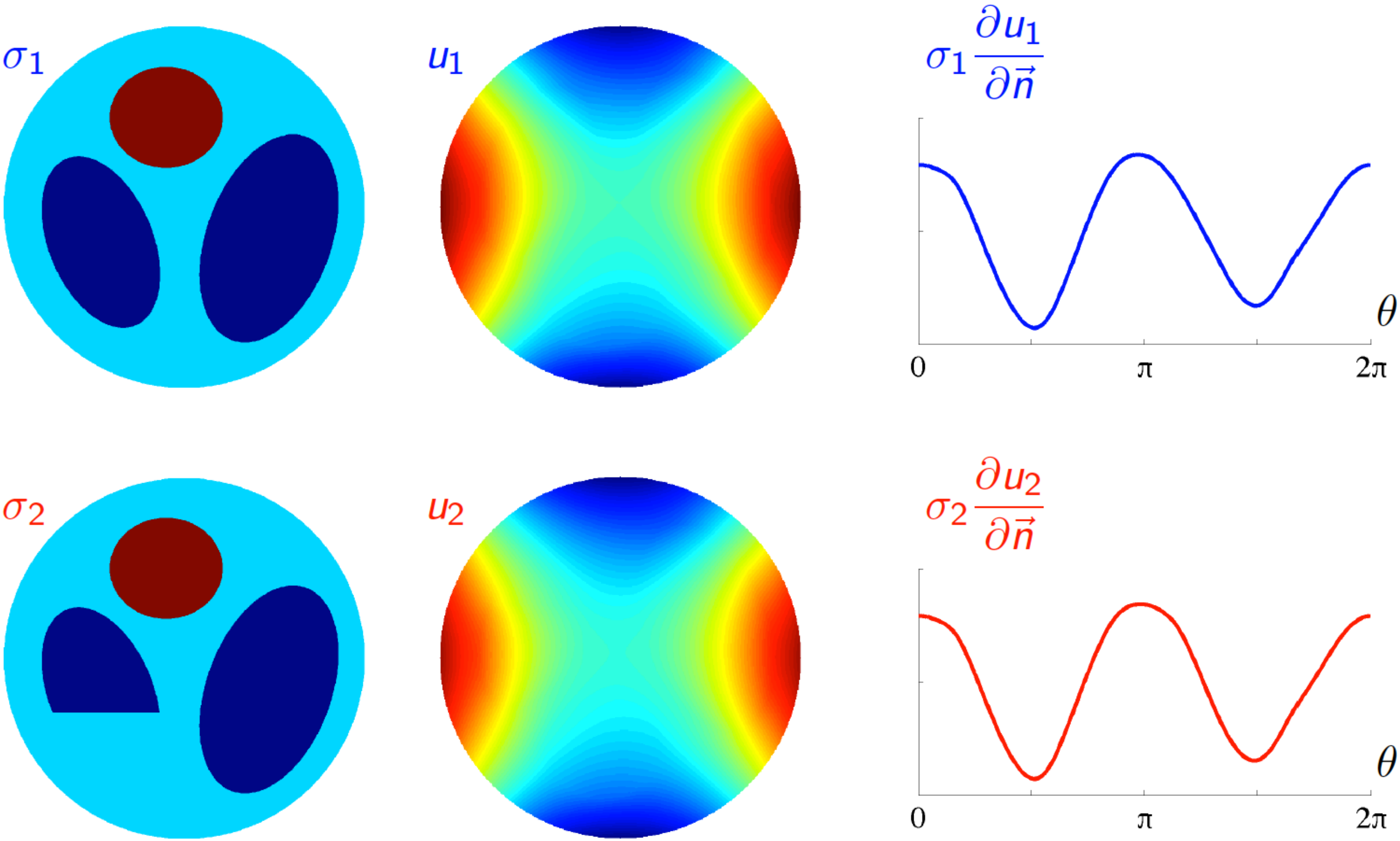




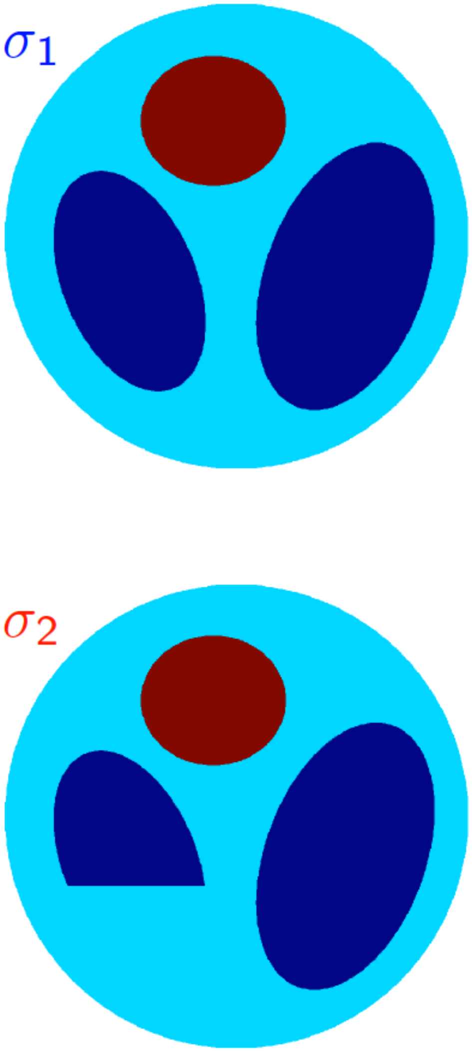
Let us apply the more oscillatory distribution $f(\theta) = \cos 2\theta$ of voltage at the boundary

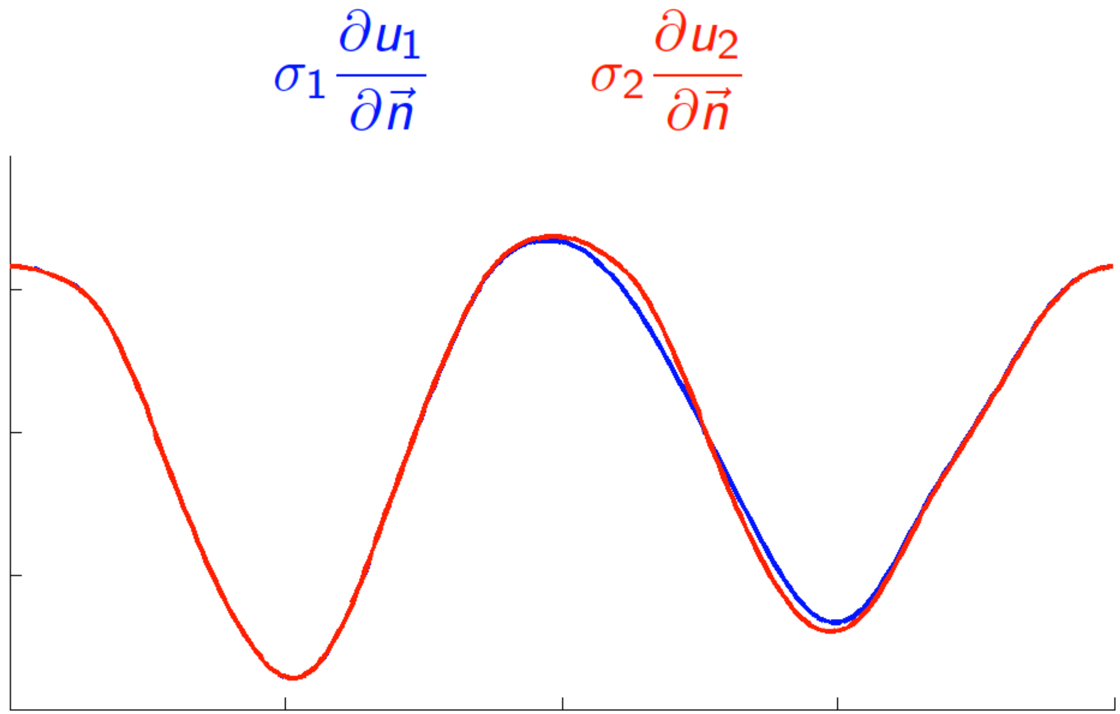


The measurement is again the distribution of current through the boundary

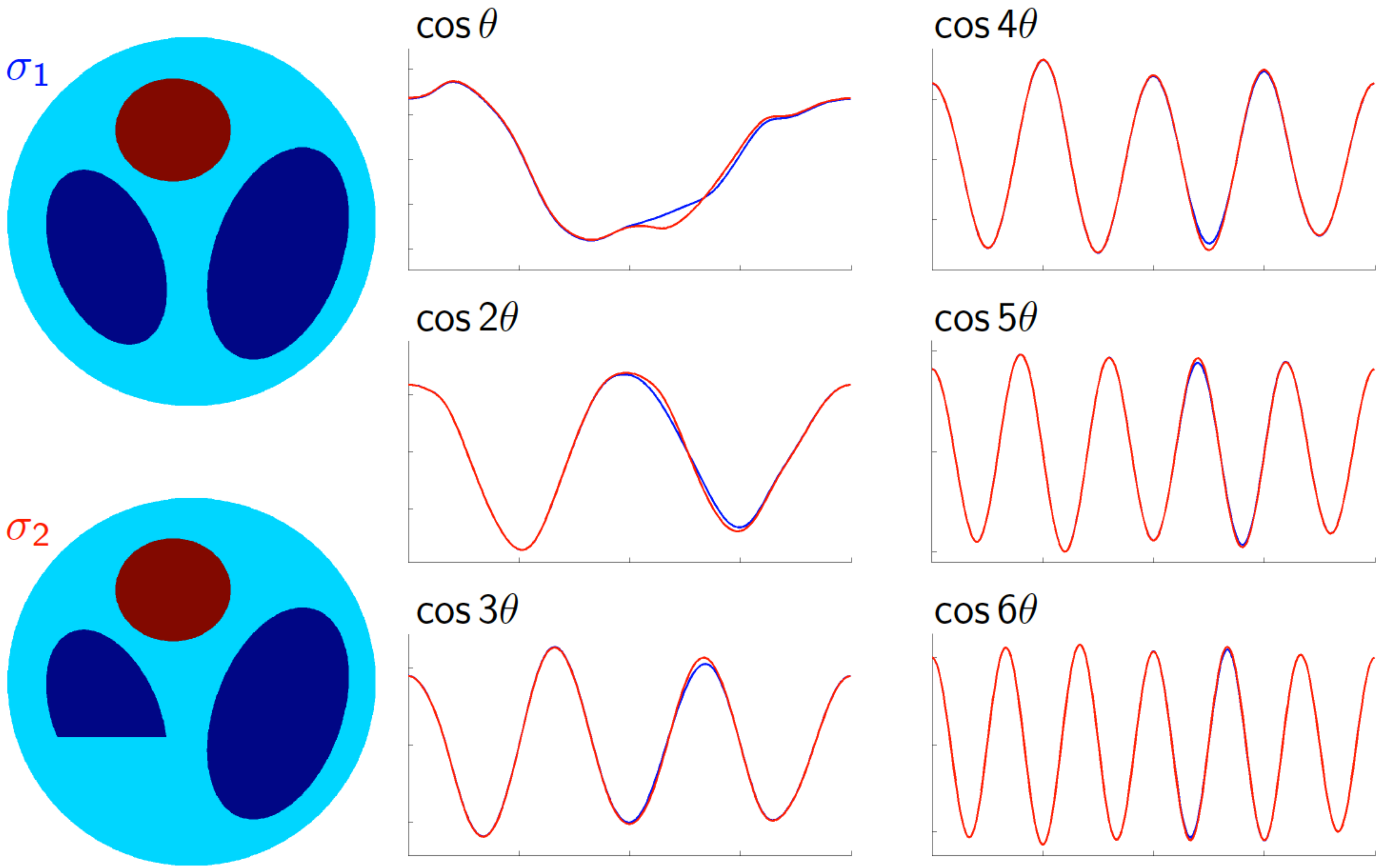


The current distribution measurements are again very similar

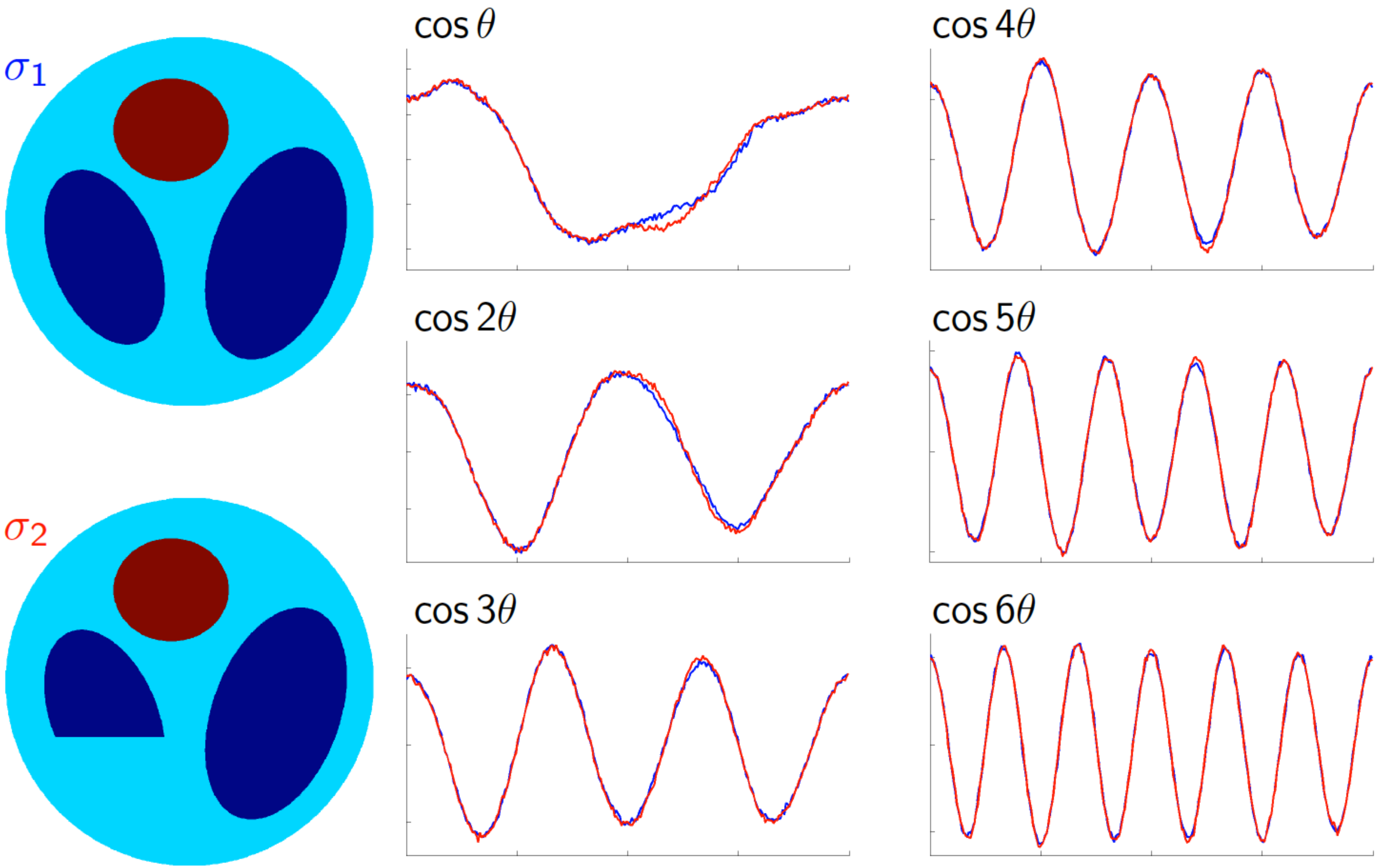




EIT is an ill-posed problem: big differences in conductivity cause only small effect in data



EIT is an ill-posed problem: noise in data causes serious difficulties in interpreting the data



What is this course all about?

Goals of the computational part of this course:

1. **Learn how to write a practical inverse problem in matrix form: $m=Af+\varepsilon$**

2. **Learn how to detect ill-posedness from a matrix A using Singular Value Decomposition**

3. **Learn how to overcome ill-posedness by regularization**

4. **Acquire skills to solve practical inverse problems using Matlab**

5. **Learn to report your scientific findings in writing**

Good to know

Matrix algebra

Least squares solution of linear systems

Basic Matlab programming (do you have access to Matlab and Image Processing Toolbox?)

How to pass the course?

Return solutions to exercise problems and pass final exam: 10 credit units

Complete project work: 5 credit units