EVOLUTION AND THE THEORY OF GAMES

Solutions 25-4-2013

28. (4 points) In the section *iterated prisoners dilemma with mistakes* (see Lecture notes) we calculate the matrix of the graph A_ε in the case TFT $\times$ TFT (the first example). Calculate the equivalent matrix for the second example with GTFT $\times$ TFT, i.e. the matrix $A_{\varepsilon,\gamma}$ where γ represents the probability that a GTFT player forgives a defection of the opponent.

Solution:

$$\mathbf{A}_{\varepsilon,\gamma} = \begin{pmatrix} ((1-\varepsilon)+\varepsilon\gamma)(1-\varepsilon) & (\varepsilon+\gamma(1-\varepsilon))(1-\varepsilon) & ((1-\varepsilon)+\varepsilon\gamma)\varepsilon & (\varepsilon+\gamma(1-\varepsilon))\varepsilon \\ ((1-\varepsilon)+\varepsilon\gamma)\varepsilon & (\varepsilon+\gamma(1-\varepsilon))\varepsilon & ((1-\varepsilon)+\varepsilon\gamma)(1-\varepsilon) & (\varepsilon+\gamma(1-\varepsilon)^2)(1-\varepsilon) \\ \varepsilon(1-\gamma)(1-\varepsilon) & (1-\varepsilon)^2 & \varepsilon^2(1-\gamma) & (1-\varepsilon)\varepsilon \\ \varepsilon^2(1-\gamma) & (1-\varepsilon)\varepsilon & \varepsilon(1-\gamma)(1-\varepsilon) & (1-\varepsilon)^2 \end{pmatrix}$$

29. (8 points) In the lecture notes we present the game Beer & Quiche. Show that $(\sigma(t_w), \sigma(t_k)) = (b, b)$ is part of a PBNE.

Solution: See Lecture notes for more details as the solution follows very closely the worked out example.
The beliefs of the Receiver (R) after observing beer, i.e. at I_b are $\mu(t_w|b) = 0.1, \mu(t_k|b) = 0.9$. Given this, R should play n: $\pi_R(t_w, b, n)\mu(t_w|b) + \pi_R(t_k, b, n)\mu(t_k|b) = 0.9 > \pi_R(t_w, b, f)\mu(t_w|b) + \pi_R(t_k, b, f)\mu(t_k|b) = 0.1$.
We can't use Bayes rule at the information set I_q, hence we may choose freely (if we can) the beliefs such that S won't want to deviate from (b, b). As R playing n at I_q will make t_w player want to deviate from q to b (getting payoff 3 instead of 2) we want to choose beliefs such that the expected payoff for R is greater by playing f: $\pi_R(t_w, q, f)\mu(t_w|q) + \pi_R(t_k, q, f)\mu(t_k|q) > \pi_R(t_w, q, n)\mu(t_w|q) + \pi_R(t_k, q, n)\mu(t_k|q) \iff \mu(t_w|q) > \frac{1}{2}$. The PBNE is then $((b, b), (n, f); ((0.1, 0.9), (\gamma, 1-\gamma)))$, for $\gamma > \frac{1}{2}$.

30. (6 points) In the lecture notes we present the game Beer & Quiche. Show that $(\sigma(t_w), \sigma(t_k)) = (b, q)$ is not part of a PBNE.

Solution: The beliefs are $\mu(t_w|b) = 1 = \mu(t_k|q)$ and $\mu(t_k|b) = 0 = \mu(t_w|q)$ and hence at I_b R will play f and at I_q R will play n. But then t_w should play q instead of b cause his payoff would increase from 2 to 3. We have then that given those beliefs and the best replies for R, (b,q) is not a best reply for S.

31. (6 points) In the lecture notes we present the game Beer & Quiche. If we modify the payoffs slightly (see Figure 1), show that $(\sigma(t_w), \sigma(t_k)) = (q, b)$ is part of a PBNE.

Solution: This is similar to the example in the lecture notes, but now S won't want to deviate from q as the payoff would be only $\frac{1}{2}$ instead of 1. Furthermore, t_k player doesn't want to deviate from b (payoff 3 as opposed to 0) and we have that $((q, b), (f, n); ((1, 0), (0, 1)))$ is a PBNE.

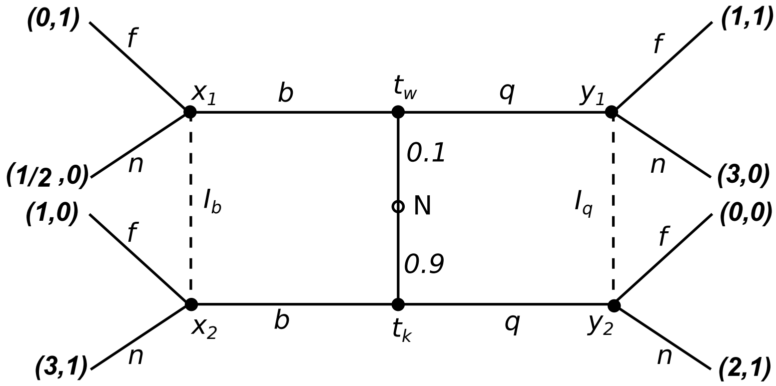

FIGURE 1.