EVOLUTION AND THE THEORY OF GAMES

Solutions 18-4-2013

25. Consider a game of two players depicted in Figure 1.

(a) (3 points) What are the (pure) Nash equilibria of the game? Hint: write the strategies for player 1 as "if at the first node play x, if at the third node play y".

(b) (3 points) How many subgames does the game have?

(c) (3 points) What are the SPNE?

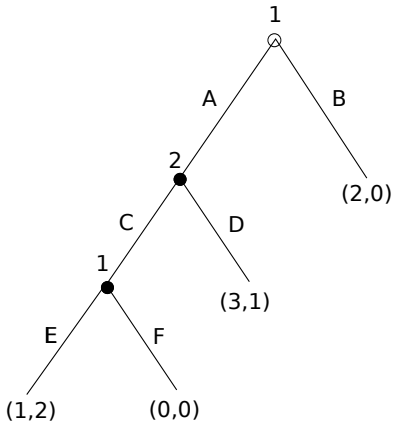

FIGURE 1.

Solutions: **(a)** The normal form is

	y	n
AE	1, 2	3, 1
AF	0, 0	3, 1
BE	2, 0	2, 0
BF	2, 0	2, 0

From the normal form we get that the pure NE are (BE,C) and (AF,D).

(b) The game has three subgames: the whole game, one subgame starting at the player 2's decision node and one subgame starting at second decision node of player 1 (the third decision node from above).

(c) SPNE is a NE which prescribes a NE-behavior in all the subgames. At the smallest subgame (starting at the third decision node) player 1's NE behavior is to play E. At the subgame starting at the second decision node player 2 should play C, because then player 1 plays E and hence Player 2 gets a payoff of 2 instead of 1 what he would get by playing D. At the first decision node then player 1 should play B, cause he gets 2 instead of 1 what he would get if playing A. We have then that playing B at the first decision node, playing C at the second decision node and playing E at the third decision node, i.e. (BE,C) is a SPNE.

26. (4 points) Consider the game-tree depicted in Figure 2. In the lecture we calculated the pure Nash equilibria and the THPNE. Are there any FSPNE?

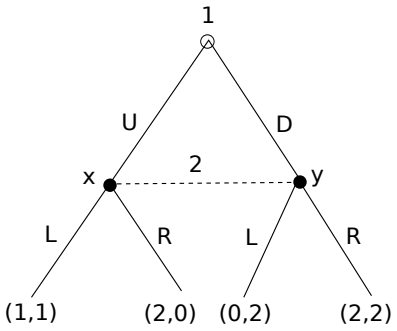

FIGURE 2.

Solution: The question is whether the NE (U,L) and (D,R) are also FSPNE. If (U,L) is played, we assign to the fuzzy subgame starting at Player 2's information set probabilities as follows: as the player 2's information set is reached with probability 1, we assign probability 1 from Natures node to the node x and probability 0 to node y. In this fuzzy subgame player 2 should play L, and so (U,L) is a FSPNE.
Similarly, if (D,R) is played, then in the fuzzy subgame starting at player 2's information node R is a NE move and hence (D,R) is a FSPNE.

27. Consider the game depicted in Figure 3.

(a) (3 points) What are the pure Nash equilibria?

(b) (3 points) How many subgames there are?

(c) (3 points) What are the SPNE?

(d) (3 points) What are the FPNE?

(e) (3 points) What are the TPNE?

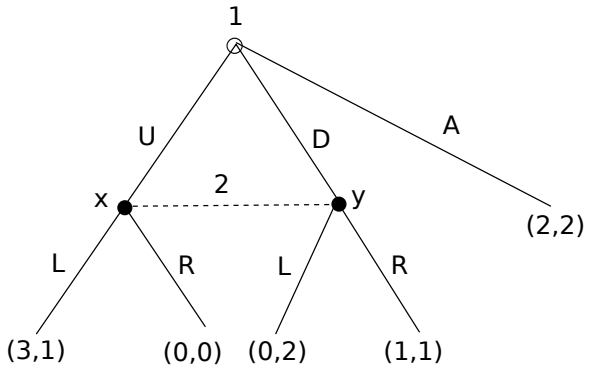

FIGURE 3.

Solutions: **(a)** The normal for is

	L	R
U	3, 1	0, 0
D	0, 2	1, 1
A	2, 2	2, 2

From the normal form we get that (U,L) and (A,R) are pure NE.

(b) The only subgame is the entire game.

(c) Since there is only one subgame, the entire game, then all the NE of the entire game are also SPNE.

(d) (U,L) is a FSPNE, because player 2's information set is reached with probability 1 and so we assign in the fuzzy subgame probability 1 from the Nature node to the node x. In this fuzzy subgame L is indeed the NE behavior for player 2 and hence (U,L) is a FSPNE.

(A,R) is not a FSPNE: Because player 2's information set is not reached under this strategy profile, we assign Nature in this fuzzy subgame a mixed strategy $z = (p, 1-p)$, where p is the probability to play x. Recall, that for a NE to be

FSPNE it has to prescribe a NE behavior in all the fuzzy subgames, in particular, it has to prescribe a NE behavior at least for some p. However, we have $\pi_2(z, L) = 2-p > \pi_2(z, R) = 1-p$ for all p and hence in this fuzzy subgame the NE behavior is to play L and not R.

(e) (A,R) is not a THPNE: Suppose player 1 makes a mistake and plays U with probability ε and D with probability δ. The best response for player 2 to this strategy is L because $\pi_2(z', L) = 2 - \varepsilon > \pi_2(z', R) = 2 - 2\varepsilon - \delta$ for all ε, δ and where $z' = (\varepsilon, \delta, 1 - \varepsilon - \delta)$ is the player 1's strategy.

(U,L) is a THPNE: Suppose player 1 makes a mistake and plays D with probability ε and A with probability δ. The best response for player 2 to this strategy is L, because $\pi_2(z, L) = 1 + \varepsilon + \delta > \pi_2(z, R) = \varepsilon + 2\delta$ for all small ε, δ and where $z = (1 - \varepsilon - \delta, \varepsilon, \delta)$ is the player 1's strategy. Now suppose player 2 makes a mistake and plays R with probability ε. Because $\pi_1(U, z'') = 3 - 3\varepsilon$, $\pi_1(A, z'') = 2$ and $\pi_1(D, z'')$, where $z'' = (1 - \varepsilon, \varepsilon)$ is player 2's strategy, the best reply for player 1 is U for all small ε. Because U and L are best responses when the opponent makes mistakes with small probability, (U,L) is a THPNE.