EVOLUTION AND THE THEORY OF GAMES

Model solutions 11-4-2013

20. Consider the Iterated Prisoners Dilemma. Write out the path of play (lets say, at least the first 5 time-steps, i.e. for $t = 1, 2, 3, 4, 5$) of the following strategy profiles

(a) (3 points) $s = (allC, allD)$

(b) (3 points) $s = (Pavlov, TFT)$

(c) (3 points) $s = (TFT, TFT)$

Solution:
(a) The first five rounds are

$$(a_1^s, a_2^s, a_3^s, a_4^s, a_5^s) = ((C,D),(C,D),(C,D),(C,D),(C,D))$$

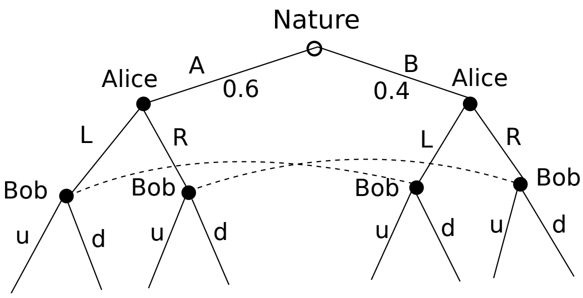
(b)

$$(a_1^s, a_2^s, a_3^s, a_4^s, a_5^s) = ((C,C),(C,C),(C,C),(C,C),(C,C))$$

(c)

$$(a_1^s, a_2^s, a_3^s, a_4^s, a_5^s) = ((C,C),(C,C),(C,C),(C,C),(C,C))$$

21. (4 points) Describe how does the game, depicted in the game-tree below, proceed. (Who moves when and what the players know at the time of the move?)



Solution: The first move is done by Nature, which represents a random action. With probability 0.6 nature chooses move A. For example, with probability 0.6

Alice is of type A (e.g. weak or strong etc. see signaling games), or, with probability 0.6 the weather is of type A, and therefore the following subgame may be different than if the weather would be of type B. Alice knows what move nature makes, i.e., she knows at which node she is. Next it is Alices turn to move, and she can choose either L or R. Bob knows whether Alice chose L or R, but he doesn't know whether Nature played A or B. This is exactly like in signaling games.

22. Consider the following game: There are two players and player 1 receives a book which, with probability p is a small game theory pocket reference, and with probability $1-p$ is a Star Trek data manual. The player sees the book, wraps it up, and decides whether to offer it to player 2 as a gift (hence player 1 has two moves: give a present or not to give a present). Player 2 hates Star Trek and is currently suffering in a graduate game theory course, so she would prefer to get the game theory references but not the Star Trek manual. Unfortunately, she cannot know what is being offered until she accepts it.

If the gift is accepted, then player 1 gets a positive payoff, say 1, because everyone likes when their gifts are accepted. Player 1 hates the humiliation of having a gift rejected, so the payoff is -1. If no gift is given both receive a 0 payoff.

Player 2 strictly prefers accepting the game theory book to not accepting it (hence accepting the game theory book yields a payoff 1 to player 1); she is indifferent between not accepting this book and accepting the Star Trek manual (hence rejecting the game theory book as well as accepting the start trek manual yields payoff 0), but hates rejecting the Star Trek manual more than the game theory book because while dissing game theory is cool, dissing Star Trek is embarrassing (rejecting star trek manual yields payoff -1).

(a) (3 points) Construct a game-tree.

(b) (3 points) Find all pure Nash equilibria.

(c) (3 points) Are there any non-credible Nash equilibria? If yes, which one(s)?

Solutions:
(a) See Figure 1.

(b) Let us transform the extensive form to a normal form by defining Player 1's strategy as "if at the left node, then play x, if at the right node play z". More formally we can set that a game starting at the left node has a strategy set $X_1 = \{N, G\}$ and at the right node $X_2 = \{N, G\}$, so that the strategy set for the whole game for player 1 is $X = X_1 \times X_2 = \{GG, GN, NG, NN\}$. Player 2's strategy set is $Y = \{y, n\}$. The payoffs are calculated in a straightforward manner. For example $\pi_1(GG, y) = p \cdot 1 + (1-p) \cdot 1 = 1, \pi_2(GG, y) = p \cdot 1 + (1-p) \cdot 0 = p$.

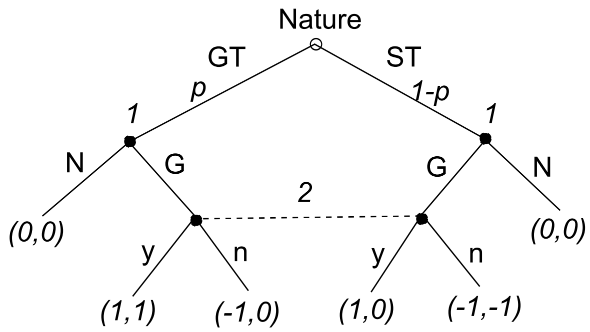


FIGURE 1.

	y	n
GG	1, p	-1, $p-1$
GN	p, p	$-p$, 0
NG	$1-p$, 0	$p-1$, $p-1$
NN	0, 0	0, 0

From the table we see that: (i) (GG,y) is a pure NE for all p (ii) (NN,n) is a pure NE for all p (iii) (NG,n) is a pure NE for $p = 1$ (iv) (GN,n) is a pure NE for $p = 0$.

(c) (NN,n) is non-credible, because if Player 2's information set is ever reached he/she should play y (and not n). The same holds for NE in (iii) and (iv).

23. Consider an iterated prisoners dilemma $\Gamma(\delta)$. Argue that

(a) (3 points) a strategy $(allD, allD)$ is a (symmetric) Nash equilibrium for a maximal strategy set (it not need to be necessarily non-terminal, i.e. we allow for strategies that terminate the game after a fixed number of rounds).

(b) (3 points) every strategy has infinitely many neutral mutants.

Solutions:
(a) In a one-shot game no strategy can do better against a defection than defection. As this is true every round, the maximum payoff for any strategy can be at most as great as when allD plays against allD.

(b) As there is always a new round with probability δ, and as we can change a strategy at any round at an off-path of play (to get a neutral mutant), we get infinitely many neutral mutants.

24. (4 points) Consider a non-terminal and maximal iterated prisoners dilemma $\Gamma(\delta)$. Show that a Nash equilibrium $allD$ can be invaded indirectly.

Solution:
(a) We show the claim is true by finding a neutral mutant of allD and that it can be invaded by a third strategy. For example, a strategy allD' which plays D until the opponent plays C and then it will switch to C for the rest of the game is a neutral mutant of allD. The path of play for a strategy profile (allD,allD') is $((D,D),(D,D),(D,D),\dots)$.
Now, if game goes on long enough allD' can be invaded by a strategy allC', which plays C only the first round and then plays D the rest of the game. The condition for the necessary expected number of rounds can be found by solving $\pi_1(allC', allD') = S + \frac{T}{1-\delta} > \pi_1(allD', allD') = \frac{P}{1-\delta}$. We have that if the discounting factor satisfies $\frac{P-S}{T-S} < \delta < 1$, the expected number of rounds allows the strategy allD' be invaded by allC'.