EVOLUTION AND THE THEORY OF GAMES

Exercises 21-3-2013

18. Consider the Hawk-Dove game with payoff matrix

	H	D
H	$\frac{1}{2}(R-C)$, $\frac{1}{2}(R-C)$	$R, 0$
D	$0, R$	$\frac{1}{2}R$, $\frac{1}{2}R$

(a) (3 points) Analyze the *iterated* Hawk-Dove game for the strategies 'always dove' (allD) and 'always hawk' (allH) , i.e. calculate the payoff matrix of the iterated game and determine whether and when any of the two strategies is an ESS.

(b) (3 points) Idem (i.e. do as above: calculate the payoff matrix and give ESS conditions) for allH, allD and Bully (B), which starts with H in the first round, but then does the opposite of what its opponent did in the previous round. (Note that the payoff matrix is a 3×3-matrix)

(c) (3 points) Idem for allH, allD and Retaliator (R), which starts with D in the first round, but then does the same as what its opponent did in the previous round. (as above, the payoff matrix is a 3×3-matrix)

(d) (3 points) Idem for the strategies Bully (B) and Retaliator (R).

19. Consider two bulls fighting over the right to mate with the females of the herd. The game goes like this: First there is a lot of display of power and intention. Next, the two males run to one another on a collision course. Each can either swerve (S) or not swerve (NS). If neither swerves, both males end up with a terrible headache (or worse), and neither one wins. If both swerve, then there is no winner either, but at least no-one got hurt, and the game may be repeated again some other time. If only one of the males swerves, the other is the winner and gains the right to mate with all the females.

The prize of winning is R; the cost of display and charging is D; the cost of the headache (or worse) is C. the payoff matrix of the game thus is:

	S	NS
S	$-D$, $-D$	$-D$, $R-D$
NS	$R-D$, $-D$	$-C-D$, $-C-D$

(a) (3 points) Analyze this game as a single shot (i.e. not repeated) game. Calculate the mixed ESS? What is the role of the "participation fee" D?

(b) (3 points) Analyze the game as an iterated game in which the game is repeated with a probability $\delta \in (0, 1)$ and *only if both males swerved*. Consider only the pure strategies swerve (S) (or 'allS'), not swerve (NS) (or 'allNS') and the mixed strategy (M) found in (a). What is the role of the "participation fee" D now?

(c) (3 points) Do you think that there are better mixed strategies than M? Analyze a game similar to a game in (b), but now only with the pure strategies swerve (S) and not swerve (NS) (and as above, game is repeated only if both males swerved). Find a mixed ESS (G) and compare it to (M).