Topology I

Exercise 6, spring 2012 (week 11)

1 (6:16). A set $A \subset X$ is a *retract* of the space X, if there exists a continuous function $f : X \to A$ (the induced metric in A) such that its restriction $f|A$ maps identically in A, i.e. $f(x) = x$ for all $x \in A$. Show that a retract always is a closed set in X.

Tips. (1) Open complement: Find for a point $x \in X \setminus A$ and its image $f(x)$ such distinct neighborhoods U and V in X that $fU \subset V$. (2) You can also use the result of Problem 3 in Exercise 5.

2. Let

$$A = \{(x, y) \in \mathbf{R}^2 \,|\, xy \geq 0 \text{ ja } x \geq 0\}.$$

Determine the sets $intA$, ∂A and $\bar{A}$ in the space $X = \mathbf{R}^2$. Quite detailed arguments.

3. Suppose A is as in the previous problem. Determine $intA$, ∂A and $\bar{A}$ in the space $Y = \{(x, y) \in \mathbf{R}^2 \,|\, xy \geq 0\}$. Quite detailed arguments again.
What a simple relation does hold between the boundaries $\partial_X A$ and $\partial_Y A$? Coincident?

4. (7:7) Let $A, B \subset X$ and $\bar{A} \cap B = A \cap \bar{B} = \emptyset$. Show that A and B are both open and closed sets in the space $A \cup B$, this naturally equipped with the metric coming from X.
Are A and B necessarily open or closed sets in the space X?

5. Let $A \subset X$, and suppose $f : X \to Y$ is a function such that its restriction $f|A$ is continuous at an inner point $a \in int(A)$. Show that also f, as a function $f : X \to Y$, is continuous at a.
On the basis of the first part design a continuity result concerning the function $f : X \to Y$ and open sets U_i, $i \in I$, in X such that $\cup_{i \in I} U_i = X$.

6. Show that $[0, \infty[\approx] - \infty, a]$, where $a \in \mathbf{R}$ is a constant and the ordinary Euclidean metric is used.

A tip. Construct a homeomorphism needed and show that your function indeed is a homeomorphism.