Department of mathematics and statistic
Topology I
Compensating 1. course exam 12.3.2012

Remark. The candidate is allowed to use a brief abstract of size A4.

1. Show that the mapping $d : X \times X \to \mathbf{R}_+$, where

$$d(x, y) = |\ln(x + 1) - \ln(y + 1)|,$$

is a metric in the set $X = \mathbf{R}_+ = [0, \infty[$. The elementary properties of the function $x \to \ln x$ are supposed to be known.

2. Consider the function $f : \mathbf{R}^2 \to \mathbf{R}^2$, where

$$f(x, y) = (3y - 1,\ -2x + 1).$$

(a) Show that it is continuous.
(b) Is it even a Lipschitz (in the whole plane $\mathbf{R}^2$)?

3. Consider the function $f : \mathbf{R}^2 \to \mathbf{R}$,

$$f(x, y) = \begin{cases} -1, & \text{when } x < 0, \\ x - 1, & \text{when } 0 \le x < 1, \\ (x - 1)^2 y^2, & \text{when } x \ge 1. \end{cases}$$

Show that it is continuous.

4. Let $f : \mathbf{R} \to \mathbf{R}$ be a continuous function. Show that the graph

$$G(f) = \{(x, y) \in \mathbf{R}^2 \mid x \in \mathbf{R},\, y = f(x)\}$$

of it is a closed set in the (Euclidean) plane $\mathbf{R}^2$.