Department of mathematics and statistics
Topology I
Compensating 2. course exam 15.5.2012

Remark. The candidate is allowed to have a short abstract of size A4.

1. Let $A = \{(x, y) \in \mathbf{R}^2 \mid 0 \leq y < x^2\}$ and $B = \mathbf{R}^2 \setminus A$ (draw a figure for yourself). Denote briefly $z = (x, y)$. Define a function $f : \mathbf{R}^2 \to \mathbf{R}$ by setting

$$f(z) = f(x, y) = \begin{cases} 1, & \text{when } z = (x, y) \in A, \\ 0, & \text{when } z = (x, y) \in B. \end{cases}$$

(a) Give a sequence (z_n) in $\mathbf{R}^2$ such that $z_n \to \mathbf{0}$, but the corresponding sequence $\big(f(z_n)\big)$ of values of f does not converge in $\mathbf{R}$. Arguments not necessary. Here $\mathbf{0} = (0, 0)$.

(b) Is the function f continuous at $\mathbf{0}$? Argue briefly.

2. Let $X =]0, 1]$ and $Y = [0, \infty[$ (with the Euclidean metric). Consider the function $f : X \to Y$, $f(x) = 1/x - 1$ for $x \in X$ (suppose to be known that it maps a point $x \in X$ into Y). Show that f is a homeomorphism.

3. Suppose the sets $A_k \subset X$ are compact, $k \in \mathbf{N}$.

(a) Show that a finite union $\cup_{k=1}^{n} A_k$ of them is always compact.

(b) Give an example such that $X = \mathbf{R}$ and the union $\cup_{k=1}^{\infty} A_k$ is not compact, though the sets A_k are.

4. Show that the subset $A = \{(x, y, z) \in \mathbf{R}^3 \mid z = x^2 - y\}$ of $\mathbf{R}^3$ is

(a) complete, (b) connected.

A tip (b). Represent A as an image set of an appropriate mapping.