Department of mathematics and statistic
Topology I
1. course exam 28.2.2012

Remark. The candidate is allowed to have a short abstract of size A4.

1. Let $f : \mathbf{R} \to \mathbf{R}$ be a function (arbitrary). Show that its graph

$$G(f) = \{(x, f(x)) \in \mathbf{R}^2 \,|\, x \in \mathbf{R}\}$$

is not open in the Euclidean plane $\mathbf{R}^2$.

2. Suppose (X, d) is a metric space, and A and B are nonempty sets of it such that $d(A, B) = \inf\{d(x, y) \,|\, x \in A, y \in B\} > 0$. Show that

$$d(x, A) + d(x, B) > 0 \quad \text{for all } x \in X.$$

3. Consider $X = \{(x, y) \in \mathbf{R}^2 \,|\, x + y > 1\}$ (a half plane), equipped with the Euclidean metric, and a subset $A = \{(x, y) \in \mathbf{R}^2 \,|\, x > 0 \text{ and } y - x/2 > 1\}$ of it. Determine the closure $cl_X A$ of A in the space X. Arguments.

4. Consider the vector space $E = raj([0, 1], \mathbf{R})$ of all the bounded functions $f : [0, 1] \to \mathbf{R}$, equipped with the supnorm $\|f\|_\infty = \sup\{|f(x)| : x \in [0, 1]\}$, and its subset

$$A = \{f \in E \,|\, f(x) \geq x \text{ for all } x \in [0, 1]\}.$$

(a) Fix $x \in [0, 1]$. Show that the corresponding mapping

$$\psi_x : E \to \mathbf{R}, \quad f \mapsto f(x) - x,$$

is continuous.

(b) Show that the set A is closed in the space E.

(c) Show that it is not open in E.