Department of mathematics and statistics
Topology I
2. course exam 9.5.2012

Remark. The candidate is allowed to have a short abstract of size A4.

1. Define briefly completeness of a metric space.

2. Consider the subset $A = \{(s, s^2 - t, t) \in \mathbf{R}^3 \mid s, t \in \mathbf{R}\}$ of $\mathbf{R}^3$. Give a homeomorphism $f : \mathbf{R}^2 \to A$ and give also arguments that it really is as stated. When such a homeomorphism exists, indeed, is A compact or connected?

3. Let A be a subset of a space X such that $\partial A = \emptyset$, i.e., its boundary is empty.
(a) Show that then A is both open and closed in the space X.
(b) Can there then exist a path $\alpha : [0,1] \to X$ such that $\alpha(0) \in A$ and $\alpha(1) \in X \setminus A$? Arguments.

4. Let $A = [0,1] \times [0,1] = \{(x, y) \in \mathbf{R}^2 \mid x, y \in [0,1]\}$ and $f : A \to \mathbf{R}$ a continuous function. Show that the function $F : [0,1] \to \mathbf{R}$,

$$F(x) = \int_0^1 f(x,t)\,dt \quad \text{when } x \in [0,1],$$

is continuous. Is it uniformly continuous on $[0,1]$?
Remark. The integrals really exist by continuity of integrand.