Matematiikan ja tilastotieteen laitos
Topics in geometric Fourier analysis
Exercise 5

Let $d(n)$ be the infimum of the Hausdorff dimension of Besicovitch sets in $\mathbb{R}^n$.

1. Show that $d(n) \leq d(n+1)$. Hint: Consider projections.

2. Show that $d(m+n) \leq d(m) + n$.

3. Prove that there exists a Borel subset of $\mathbb{R}^2$ with positive Lebesgue measure whose orthogonal projections on all lines have empty interior. Hint: Consider Besicovitch sets.

4. Peres and Schlag have proved that if $B \subset \mathbb{R}^n$ is a Borel set with Hausdorff dimension $\dim B > 2k$, then almost all projections $P_V(B), V \in G(n,k)$, have non-empty interior. Use this to get another proof for Falconer's theorem 8.3.

5. Prove that m_δ is not an L^p-multiplier if $p \leq \frac{2n}{n+1+2\delta}$ or $p \geq \frac{2n}{n-1-2\delta}$. You can of course use the kernel K_δ and its asymptotic properties.