Matematiikan ja tilastotieteen laitos
Topics in geometric Fourier analysis
Exercise 4

1. Prove that the $L^2 - L^2$-operator norm of the multiplier operator T_m is $\|m\|_\infty$.

2. Prove that the multiplier operator T_m is bounded $L^p \to L^p$ for $1 < p < \infty$ if $m \in \mathcal{S}$.

3. Prove that if P is a polyhedral domain, then $T_{\chi_P} : L^p \to L^p$ is bounded for $1 < p < \infty$.

4. Prove that the operator norms of $T_{B(0,1)}$ and $T_{B(x,r)}$ for any $x \in \mathbb{R}^n, r > 0$, are equal.

5. Prove that if the characteristic function of the unit ball is an L^p- multiplier for some p in some $\mathbb{R}^n$, then

$$f(x) = \lim_{R\to\infty} \int_{B(0,R)} e^{2\pi i x\xi} \hat{f}(\xi) d\xi \text{ as } R \to \infty \text{ in } L^p(\mathbb{R}^n) \text{ sense?}$$

6. Verify the formula

$$T_j^r f(x) = e^{2\pi i r v_j \cdot x} T_r(e^{-2\pi i r v_j \cdot \xi} f)(x),$$

in the proof of Lemma 7.8.