(29-03-2012)

Non-random motion : advection

Advection is a form of non-random motion, e.g., due to particles suspended in a moving medium, or particles falling under the influence of gravity.

$$\boxed{J(t,x) = v(t,x)\, n(t,x)}$$

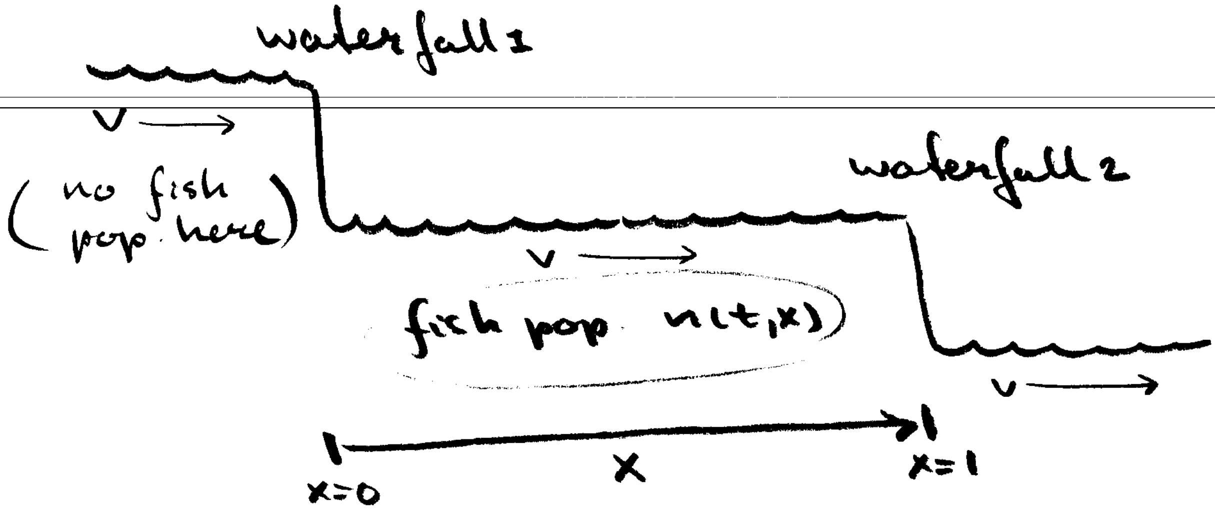
(velocity of medium, or terminal velocity of falling.)

$$\Rightarrow \boxed{\partial_t n(t,x) = -\partial_x \big(v(t,x)\, n(t,x)\big)}$$

Combining advection with diffusion gives the Fokker-Planck equation

$$\boxed{\partial_t n = \partial_x(\alpha \partial_x n) - \partial_x(v n)}$$

Example I



① $$\boxed{\partial_t n = D\,\partial_x^2 n - v\,\partial_x n + a n}$$

diffusion term — advection term (const. velocity) — reaction term (birth − death).

Boundary conditions.

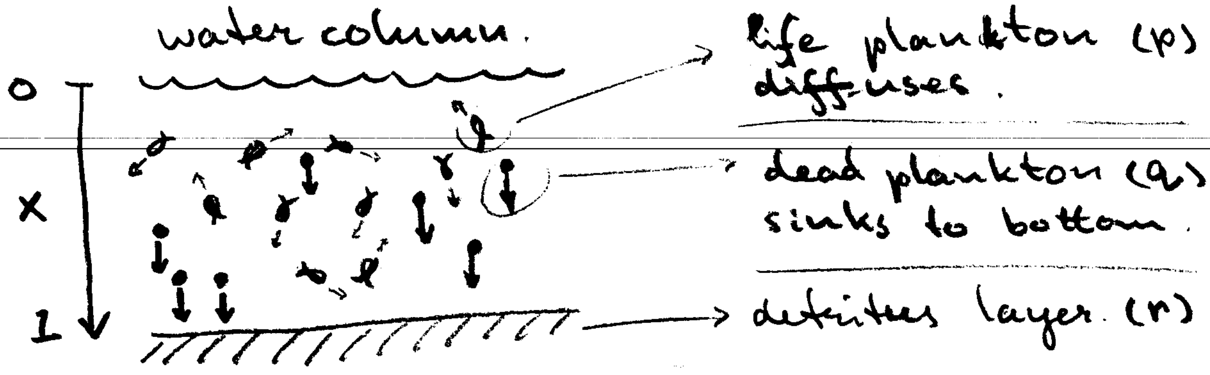
Waterfall 1 is reflecting bnd: Flux is $-D\partial_x n + vn$ and hence

② $$\boxed{-D\partial_x n(t,0) + v\,n(t,0) = 0}$$

Waterfall 2 is absorbing bnd:

③ $$\boxed{n(t,1) = 0}$$

Example II



$$\text{④}\quad \boxed{\partial_t p = \underbrace{\lambda(x)\,p}_{\text{depth-dependent birth}} - \overbrace{\mu p}^{\text{death}} + \underbrace{D\,\partial_x^2 p}_{\text{diffusion}}} \quad \left(\begin{array}{c}\text{life}\\ \text{plankton}\end{array}\right)$$

$$\text{⑤}\quad \boxed{\partial_t q = \underbrace{\mu p}_{\text{death of life plankton}} - \underbrace{v\,\partial_x q}_{\text{advection}}} \quad \left(\begin{array}{c}\text{dead}\\ \text{plankton}\end{array}\right)$$

Boundary conditions:

$$\text{⑥}\quad \boxed{-D\,\partial_x p = 0 \quad \text{for } x=0 \text{ and } x=1}$$

(refl. bnd. for life plankton)

$$\text{⑦}\quad \boxed{q = 0 \quad \text{for } x=0}$$

(zero conc. bnd for dead plnk.)

(Note: ⑤ needs only <u>one</u> bnd cond.)

The detritus layer grows as a consequence of the influx of dead plankton at $x=1$ (bottom):

(8) $$\boxed{\frac{dr(t)}{dt} = \underbrace{v\,q(t,1)}_{\text{influx of detritus}} - \underbrace{f(r(t))\,c(t)}_{\text{removal of detritus by a consumer } \boxed{c} \text{ (worm, bacteria?) with functional response } \boxed{f}}}$$

- Notice that the dynamics of the detritus is given by an ODE.
- We can continue by giving equations for the consumer, e.g.

(9) $$\boxed{\frac{dc}{dt} = \gamma f(r) c - \delta c}$$