MATHEMATICAL MODELING 2012
EXERCISES 4-6

4. Assuming mass-action, give a possible interpretation of the following population model. That is, what i-level processes could underly the equations:

$$\begin{cases} \frac{dx}{dt} &= \varphi - \alpha x - \beta xy \\ \frac{dy}{dt} &= \gamma xy - \delta y \end{cases}$$

Give a phase-plane analysis for the system.

5. The life cycle of many butterfly species is linked to that of a so-called parasitoid, which is another insect species (usually a wasp) that lies its eggs inside the larvae of the butterfly so that the larvae do not develop into a new butterfly but into one or more individuals of the parasitoid instead.

Suppose that each adult butterfly produces new larvae at a constant rate. If a larva is found by a parasitoid, then the parasitoid deposits a single egg inside the larva, independently of the number of eggs that may already be inside. A larva containing zero eggs of the parasitoid develops into an adult butterfly, but a larva containing a single egg develops into an adult parasitoid. Larvae with two or more eggs inside do not develop into anything at all but just die (presumably because there is not enough food present for the parasitoids to complete their development). Larvae, butterflies and parasitoids are also subject to death from other (unspecified) causes.

Formulate a model of the above system, i.e., (a) specify the various i-states, (b) model the i-level processes with a network of mono- and bimolecular reactions, and (c) assuming mass-action, give the corresponding population model as a system of differential equations for the population densities.

This exercise potentially has many good answers, but try to keep it as simple as you can.

6. Consider the model on page 29 of the lecture notes of 02-02-2012:

$$\begin{cases} \frac{dx}{dt} &= +\beta(n_0 - x)y - \mu x & \text{(site owner)} \\ \frac{dy}{dt} &= -\beta(n_0 - x)y + \alpha x - \nu y & \text{(free indiv.)} \end{cases}$$

Use time-scale separation to split this system into two one-dimensional equations assuming that sites (and hence site owners as well) are very rare compared to free individuals, i.e., $x, n_0 \ll y$.