MATHEMATICAL MODELING 2012
EXERCISES 17-18

17.

(a) Consider the delay logistic equation on page 5 of the lecture notes of 15-03-2012, i.e.,

$$\frac{dn}{dt} = \frac{\lambda\beta F(T)}{\gamma}(S_0 - n)n_T - \delta n$$

where $n_T(t) := n(t-T)$, and do a local stability analysis of the zero equilibrium and of the positive equilibrium (whenever the latter exists).

(b) Compare the above results with a local stability analysis of the delay logistic equation of Hutchinton and May, i.e.,

$$\frac{dn}{dt} = rn\left(1 - \frac{n_T}{K}\right).$$

18.

Consider the delay differential equation

$$\frac{dn}{dt} = f(n, n_\varphi)$$

where $f : \mathbb{R}^2_+ \to \mathbb{R}$ is a continuously differential function with $f(\bar{n}, \bar{n}) = 0$ for some given $\bar{n} > 0$, and where $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ is a probability density on the positive real numbers and

$$n_\varphi(t) := \int_0^\infty \varphi(\tau)n(t-\tau)d\tau.$$

Do a local stability analysis of the equilibrium $\bar{n}$ if φ is the probability density of the uniform distribution on the positive interval $[T_1, T_2]$, i.e.,

$$\varphi(\tau) = \begin{cases} (T_2 - T_1)^{-1} & \text{if } T_1 \le \tau \le T_2 \\ 0 & \text{elsewhere} \end{cases}$$