MATHEMATICAL MODELING 2012
EXERCISES 1-3

1. Model the following i-level processes as unimolecular or bimolecular reactions: (a) migration from one habitat to another; (b) asexual reproduction; (c) territory owner starts a fight with an intruder; (d) hatching of an egg; (e) predator captures prey; (f) death; (g) sexual reproduction; (h) predator discovers prey and starts stalking the prey; (i) two competitors meet and one eliminates the other.

2. Consider the reactions $A \xrightarrow{\alpha} B$ and $2A \xrightarrow{\alpha} A + B$. (a) What i-level processes could they represent? (b) For each reaction, solve the corresponding differential equations for the concentration of A. (c) Let T be the time when a particular A-particle undergoes a reaction counting from $t = 0$. For each reaction, give the probability density of T and calculate its expected value. (d) For each reaction, we measure in the lab how the concentration of A changes over time. How can we derive from this date the value of the reaction constants α? (*Hint: plot* $\log a(t)$ *and* $a(t)^{-1}$ *as a function of time* t.)

3. Let E denote an egg, A an adult and R an adult recovering from egg laying, and consider the following reaction network:

$$\begin{aligned}
\text{egg hatching:} &\quad E \xrightarrow{\alpha} A \\
\text{reproduction:} &\quad A \xrightarrow{\beta} R + E \\
\text{recovery:} &\quad R \xrightarrow{\gamma} A \\
\text{cannibalism:} &\quad E + A \xrightarrow{\delta} A \\
\text{death of egg:} &\quad E \xrightarrow{\lambda} \dagger \\
\text{death of A-adult:} &\quad A \xrightarrow{\mu} \dagger \\
\text{death of R-adult:} &\quad R \xrightarrow{\nu} \dagger
\end{aligned}$$

Give the corresponding differential equations for the population densities of eggs, adults and adults in the recovery phase. (*N.B., egg cannibalism is very common among both fish and insects.*)