Lineaariset mallit, Spring 2012, Exercise 6, week 18

1. Continuation of Exercises 4.3 and 5.1. Derive an F-test for the hypothesis H : $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2$. (*Hint*: Express the F-test statistic in terms of residual sum of squares.)

2. Consider the linear regression model $Y_i = \beta_1 + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \varepsilon_i$, $i = 1, ..., n$, where $\varepsilon_1, ..., \varepsilon_n$ are independent and $\varepsilon_i \sim \mathsf{N}\left(0, \sigma^2\right)$, and the purpose is to test the hypothesis $H : \beta_2 = \cdots = \beta_p = 0$. First show that $\text{SSE} = \left(1 - R^2\right) \sum_{i=1}^n \left(y_i - \bar{y}\right)^2$, where R^2 is the coefficient of determination (see. Exercises 3.3), and using this show further that the F-test statistic for the preceding hypothesis can be written as

$$F = \frac{(n-p)\, R^2}{(p-1)\,(1-R^2)}.$$

3. Continuation of Exercises 4.4 and 5.2. Suppose that the hypothesis $\mu_1 = \mu_2$ has been rejected. Construct a $100(1-\alpha)\%$ confidence interval for the difference $\mu_1 - \mu_2$.

4. Continuation of Exercises 4.3 and 6.1. (i) Construct a $100(1-\alpha)\%$ confidence interval for the linear combination $\mathbf{a}'\boldsymbol{\beta}_1$ ($\mathbf{a} = [a_1 \ \cdots \ a_p]' \neq 0$). (ii) Do the same by assuming the constraint $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2$.

5. Suppose that in the linear model $\mathbf{Y} \sim \mathsf{N}\left(\mathbf{X}\boldsymbol{\beta}, \sigma^2 \mathbf{I}_n\right)$ ($\boldsymbol{\beta} \in \mathbb{R}^p$, $\sigma^2 > \mathbf{0}$, $\mathsf{r}(\mathbf{X}) = p$) the regressors are orthogonal or that $\mathbf{X}'\mathbf{X}$ is a diagonal matrix. Derive the least squares estimate of the parameter β_j, the t-test statistic for the hypothesis $H : \beta_j = 0$ and a $100(1-\alpha)\%$ confidence interval for the parameter β_j (= the jth component of $\boldsymbol{\beta}$). How do these change if one of the regressors x_k ($k \neq j$) is removed from the model?