Inverse Problems, Problems session 1

1. Suppose $f,\ h \in S(\mathbb{R}^n),\ a > 0$ and $b \in \mathbb{R}^n$. Show that

a) $\widehat{(f * h)}(\xi) = \widehat{f}(\xi)\widehat{h}(\xi)$

b) $\int_{\mathbb{R}^n} \widehat{f}(z)g(z)dz = \int_{\mathbb{R}^n} \widehat{g}(z)f(z)dz$

c) $\mathcal{F}_{x\to\xi}(f(ax)) = a^{-n}\widehat{f}(\frac{\xi}{a}), \quad \mathcal{F}_{x\to\xi}(e^{ib\cdot x}f(x)) = \widehat{f}(\xi - b)$

2. Calculate the Radon transform of function

$$\chi_{B(0,1)}(\omega, s) = \begin{cases} 1, & \text{when } (\omega, s) \in B(0,1) \subset \mathbb{R}^2, \\ 0, & \text{otherwise.} \end{cases}$$

Do the functions $\chi_{B(0,1)},\ R_\omega\chi_{B(0,1)}$ belong to C^α with some α?

3. Calculate the Fourier transform $\mathcal{F}\delta_p$, where $\delta_p \in S'(\mathbb{R}^n)$.

4. Let $A_n \in S'(\mathbb{R}^n)$. We define

$$A_n \to A \qquad \text{in } S'(\mathbb{R}^n),\ \text{when } n \to \infty \tag{1}$$

if for all $f \in S(\mathbb{R}^n)$

$$\lim_{n\to\infty} \langle A_n, f\rangle = \langle A, f\rangle.$$

Define the open sets of the space $S'(\mathbb{R}^n)$ so that (1) is limit in the sense of topology of $S'(\mathbb{R}^n)$.