Structure of tomographic X-ray data

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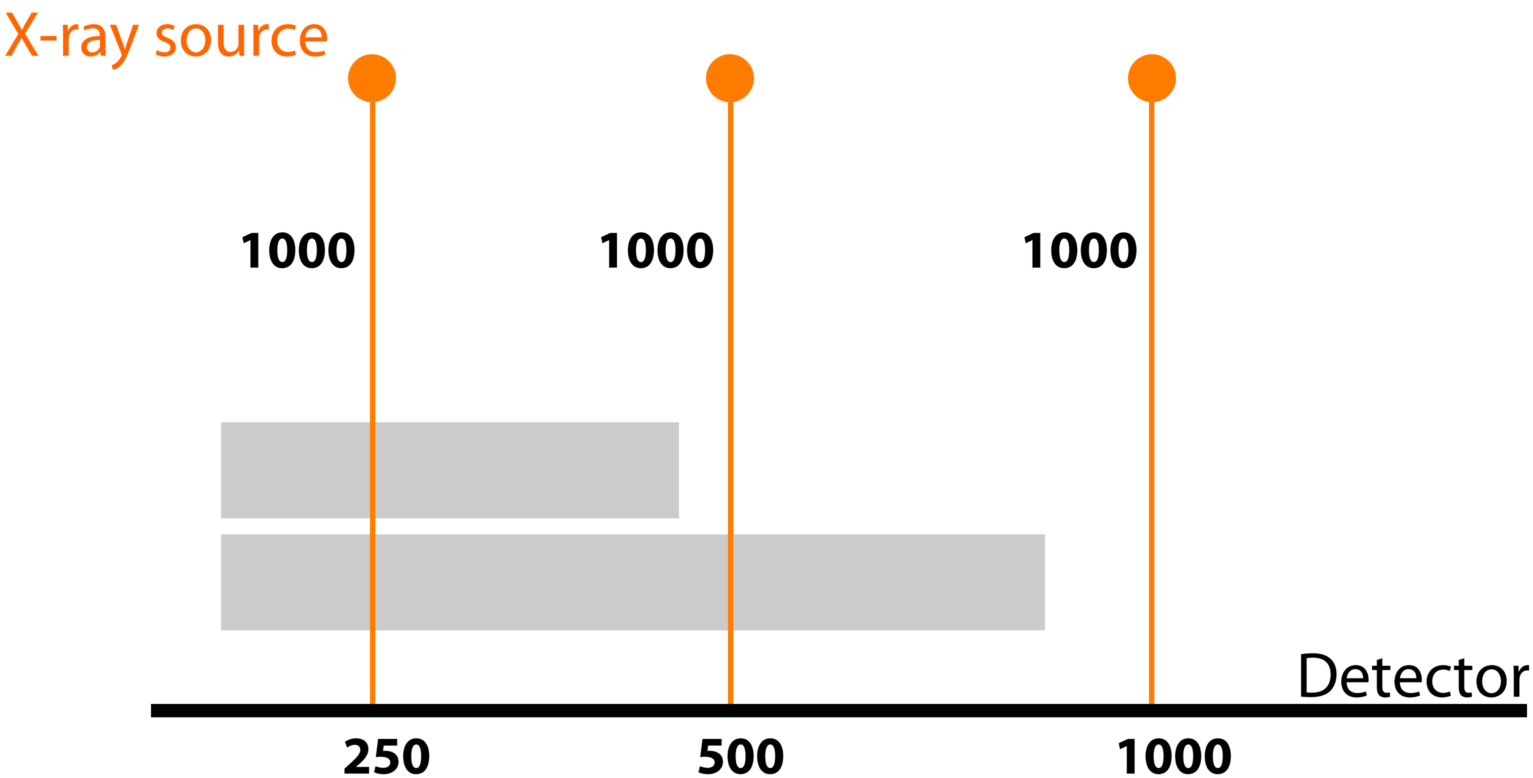
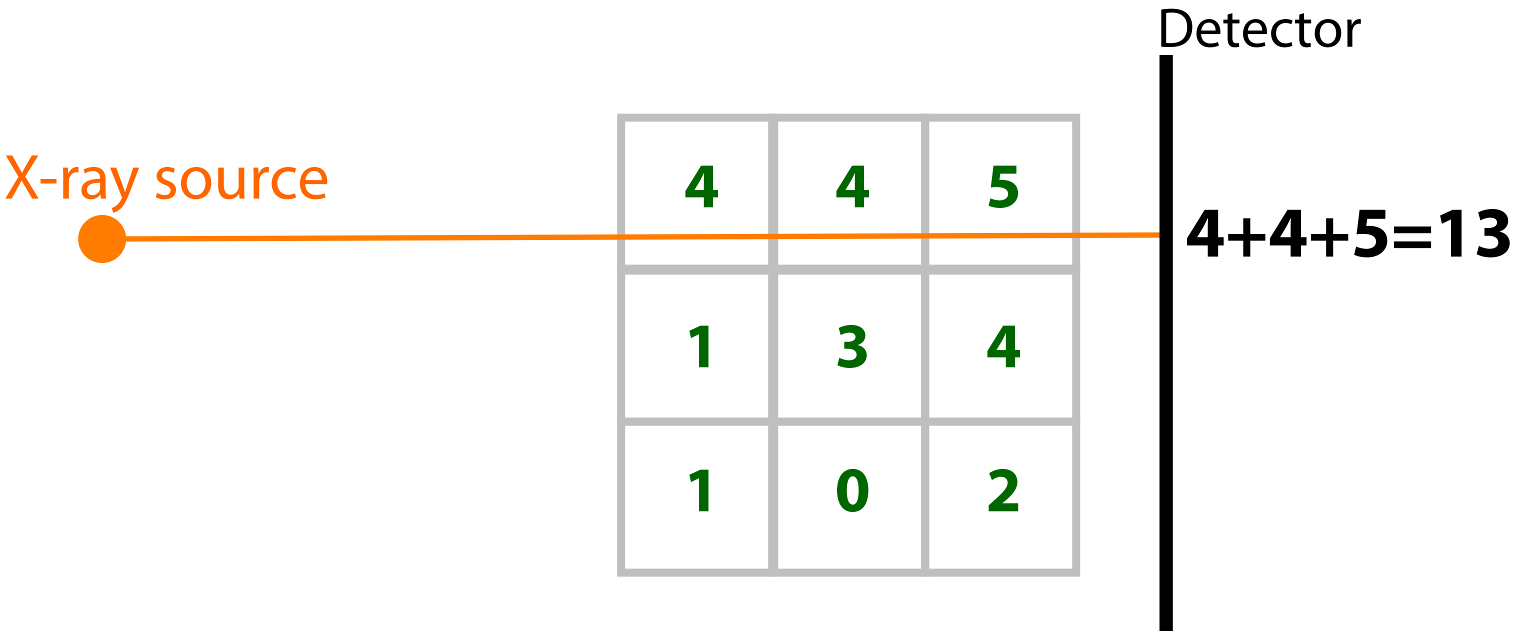
Course on computational inverse problems

X-ray images as measurements

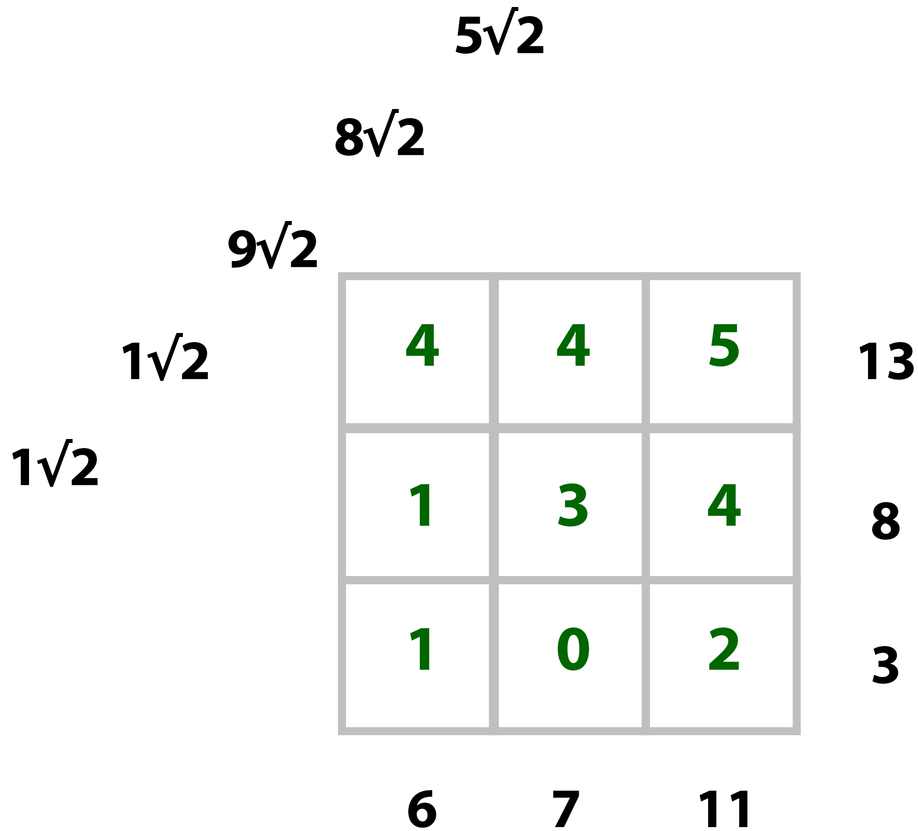


Logarithm	5,5	6,2	6,9
Density	1,4	0,7	0,0

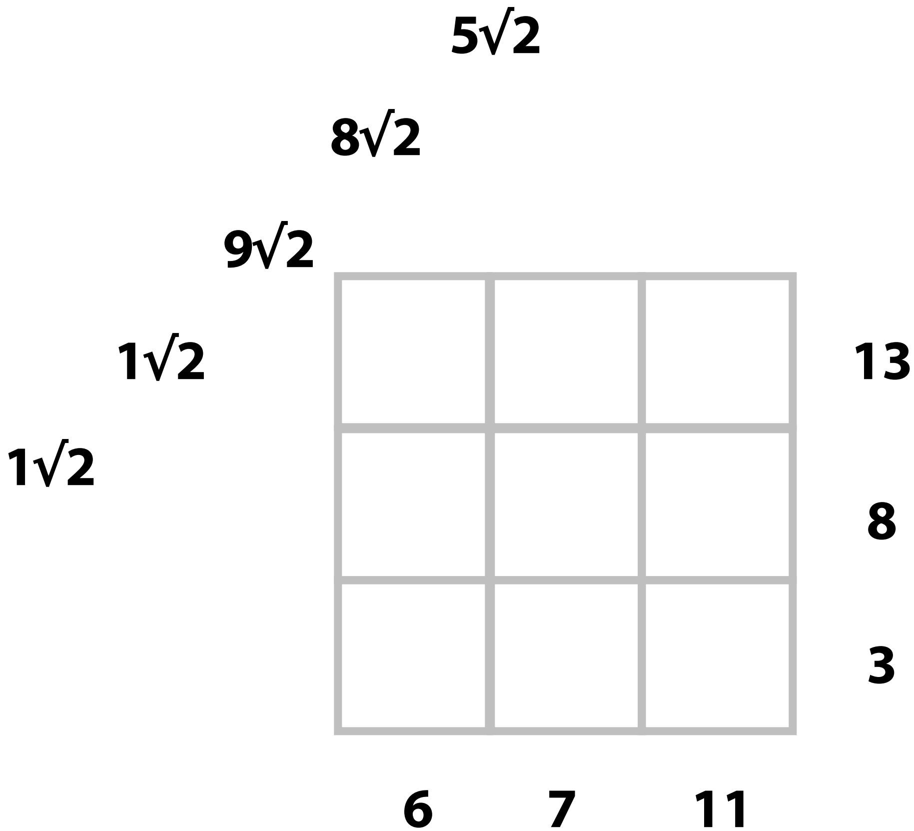
Every X-ray measures the sum of attenuation through tissue



Direct problem of tomography is to find the radiographs from given tissue

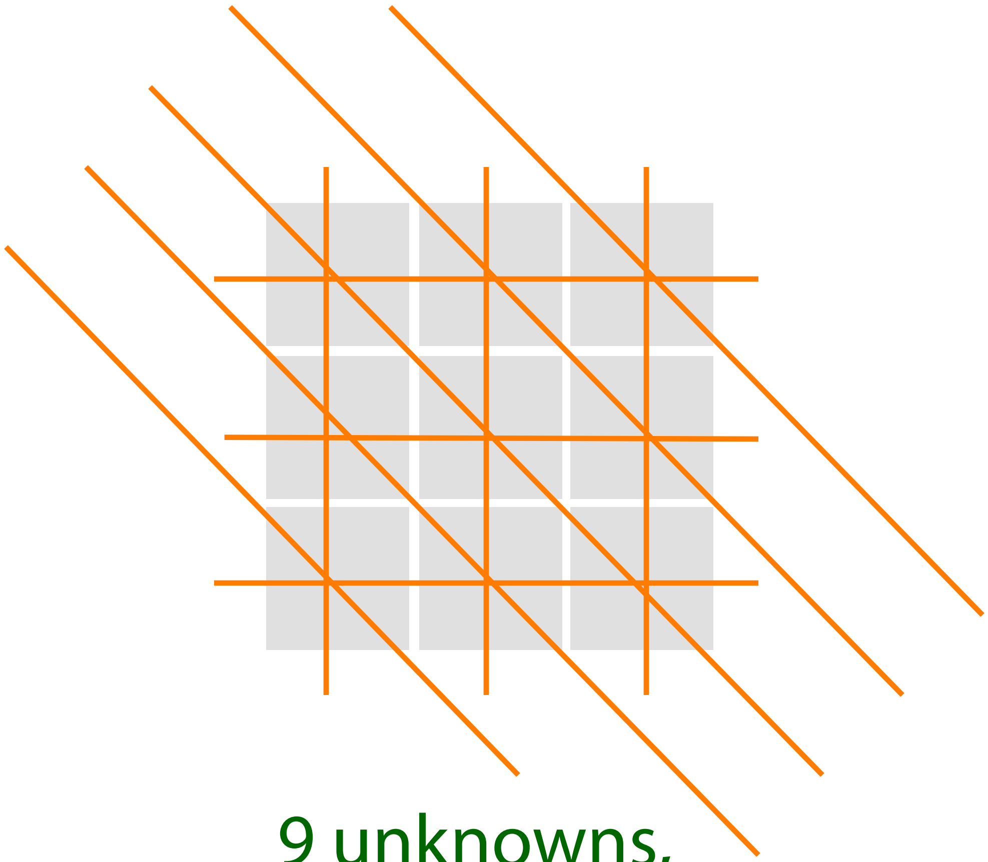


Inverse problem of tomography is to find the tissue from radiographs

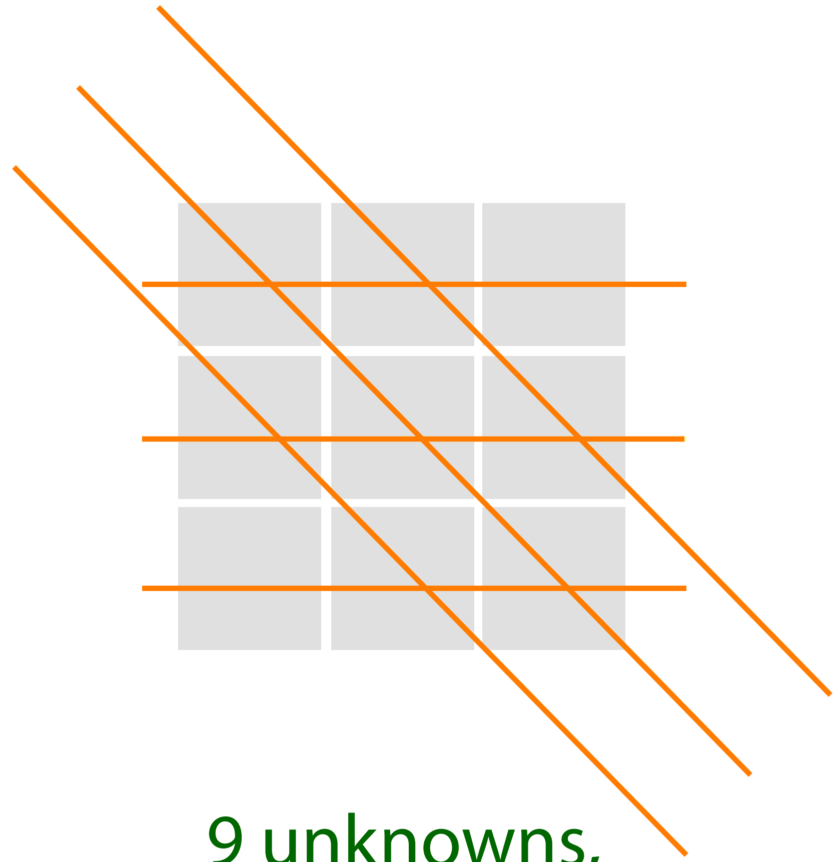


9 unknowns, 11 linear equations

The limited angle problem is harder than the full angle problem



9 unknowns,
11 linear equations



9 unknowns,
6 linear equations

In limited angle 3D imaging there are many tissues matching the radiographs

8√2

9√2

1√2

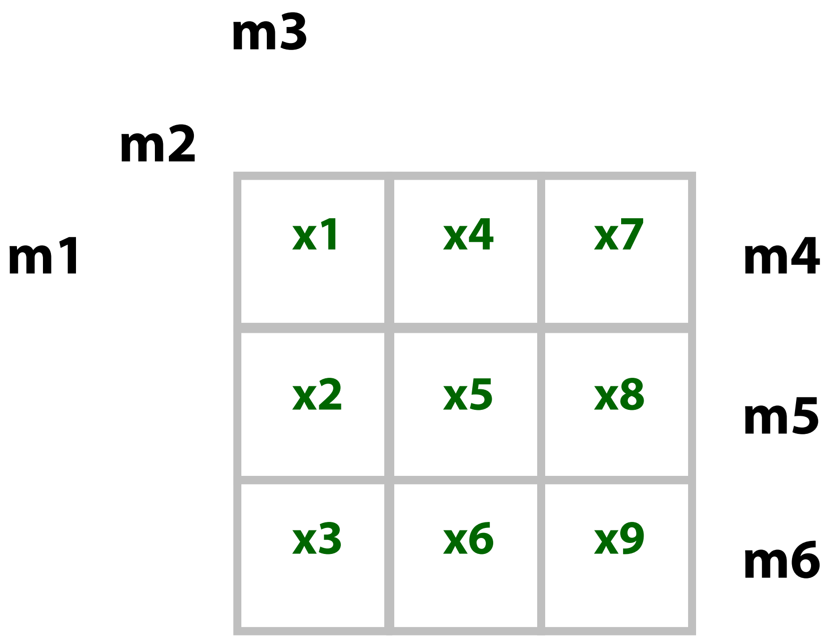
4	4	5	13
1	3	4	8
1	0	2	3

5	6	2
1	5	2
4	0	-1

9	1	3
1	0	7
3	0	0

a priori information is needed!

We write the reconstruction problem in matrix form and assume Gaussian noise

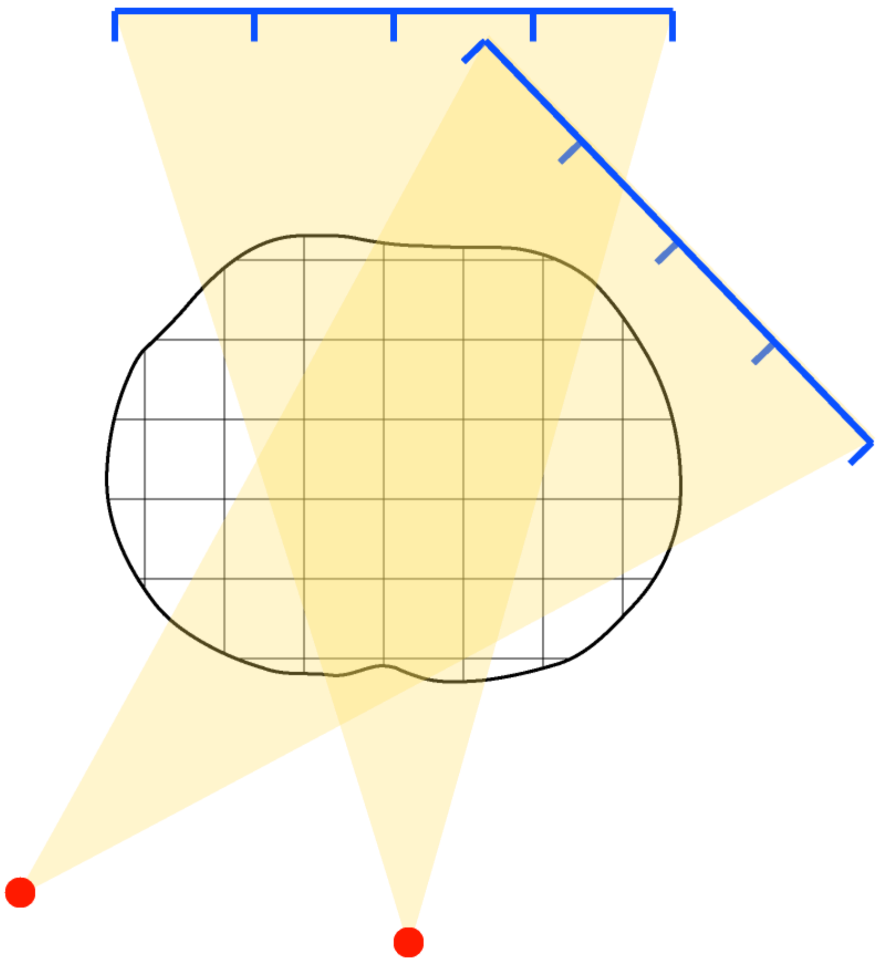


$$x = [x_1, x_2, \ldots, x_9]^T$$
$$m = [m_1, m_2, \ldots, m_6]^T$$

$$Ax = m$$

We consider the measurement model m=Ax+ε
with additive Gaussian noise ε of standard deviation σ.

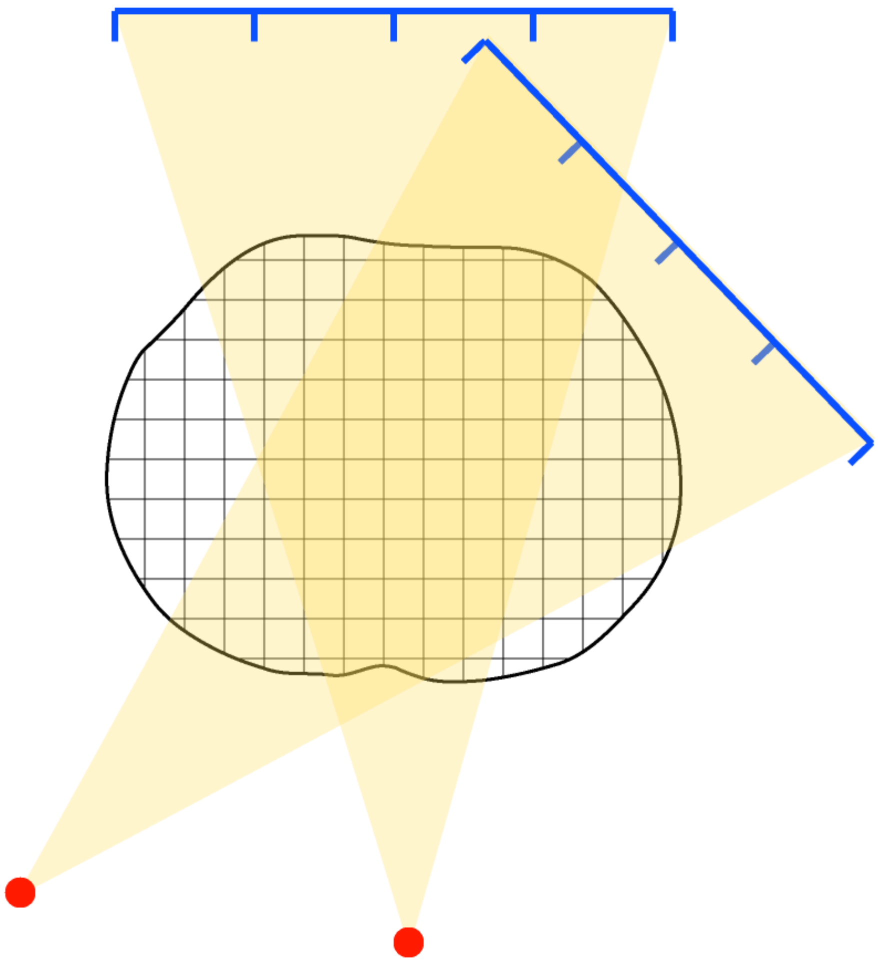
Number of data is k, number of unknowns is n



k=8

n=48

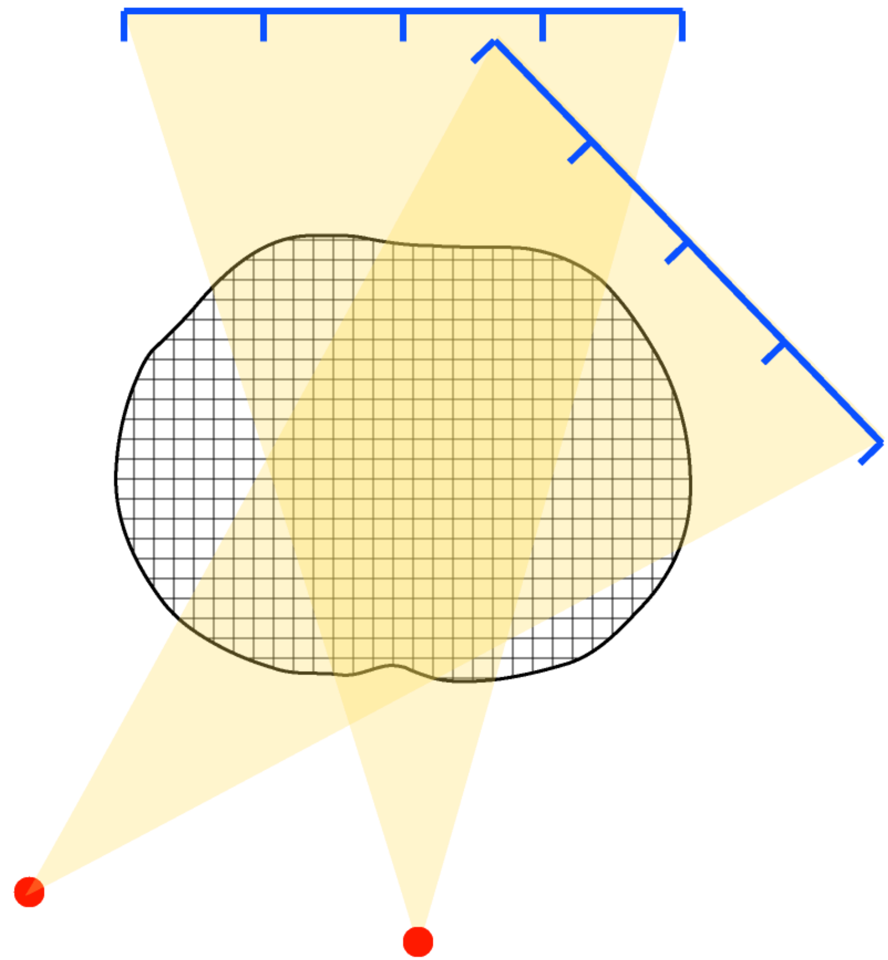
Number of data is k, number of unknowns is n



k=8

n=156

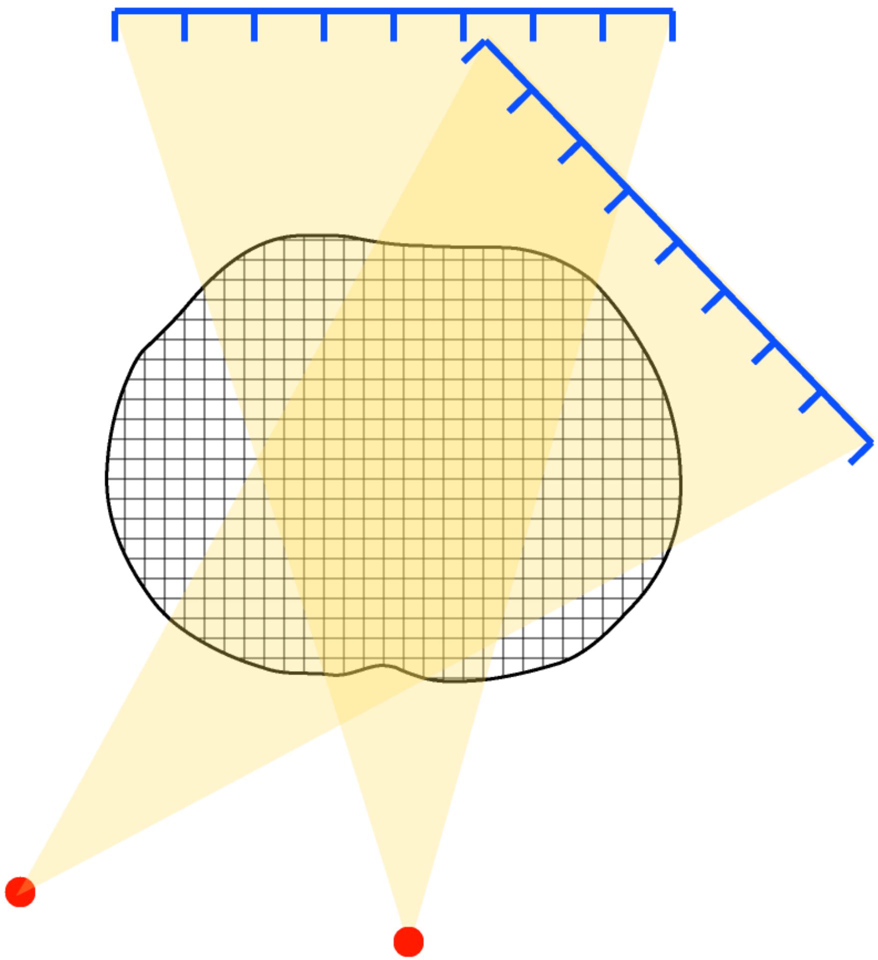
Number of data is k, number of unknowns is n



k=8

n=440

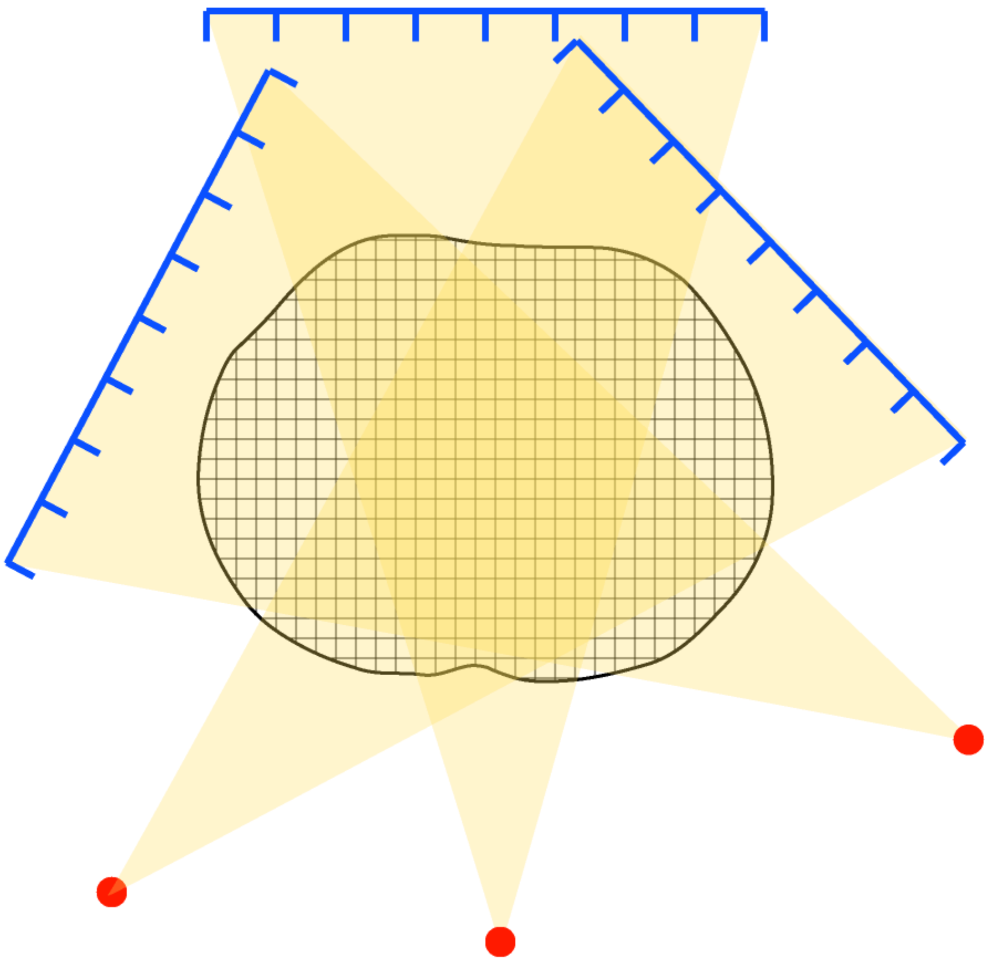
Number of data is k, number of unknowns is n



k=16

n=440

Number of data is k, number of unknowns is n

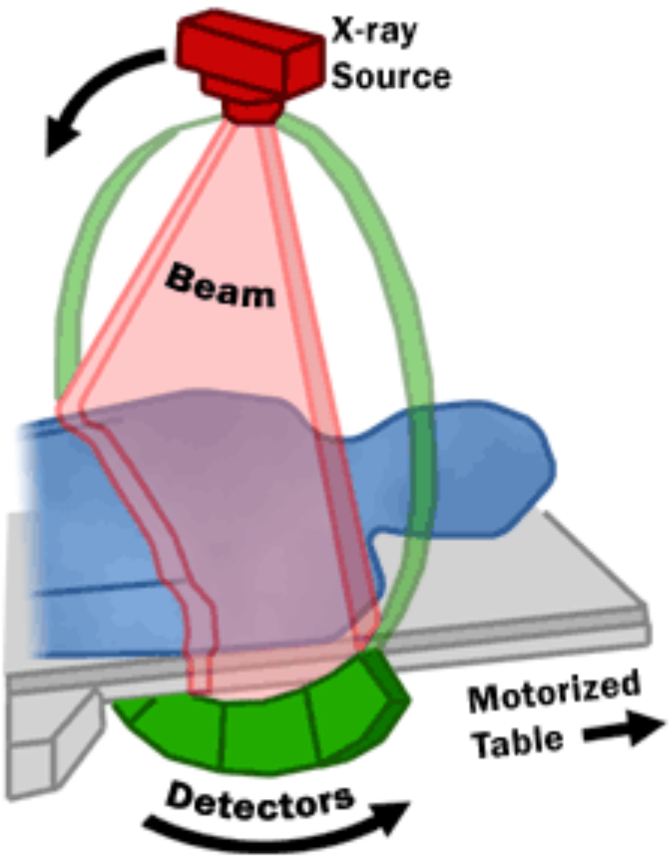


k=24

n=440

To explain low-dose 3D imaging, let's start by discussing traditional 3D imaging

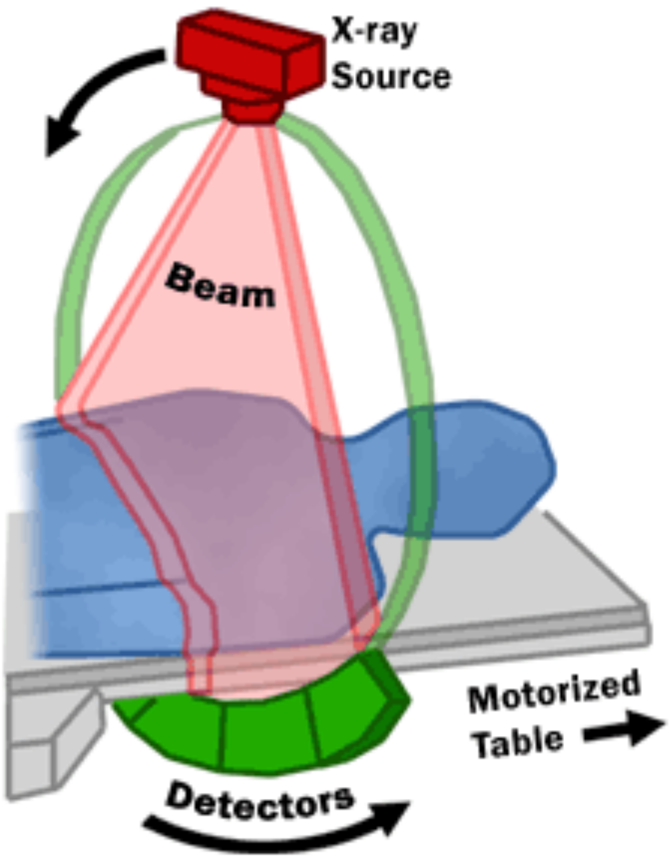
X-ray attenuation data is collected from 180 directions separately for each two-dimensional slice.



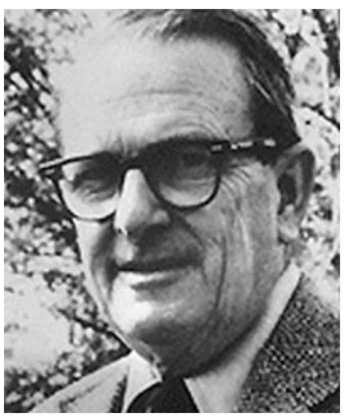
Images from http://www.fda.gov/cdrh/ct/what.html

Using a reconstruction algorithm, inner structure in the slice is revealed

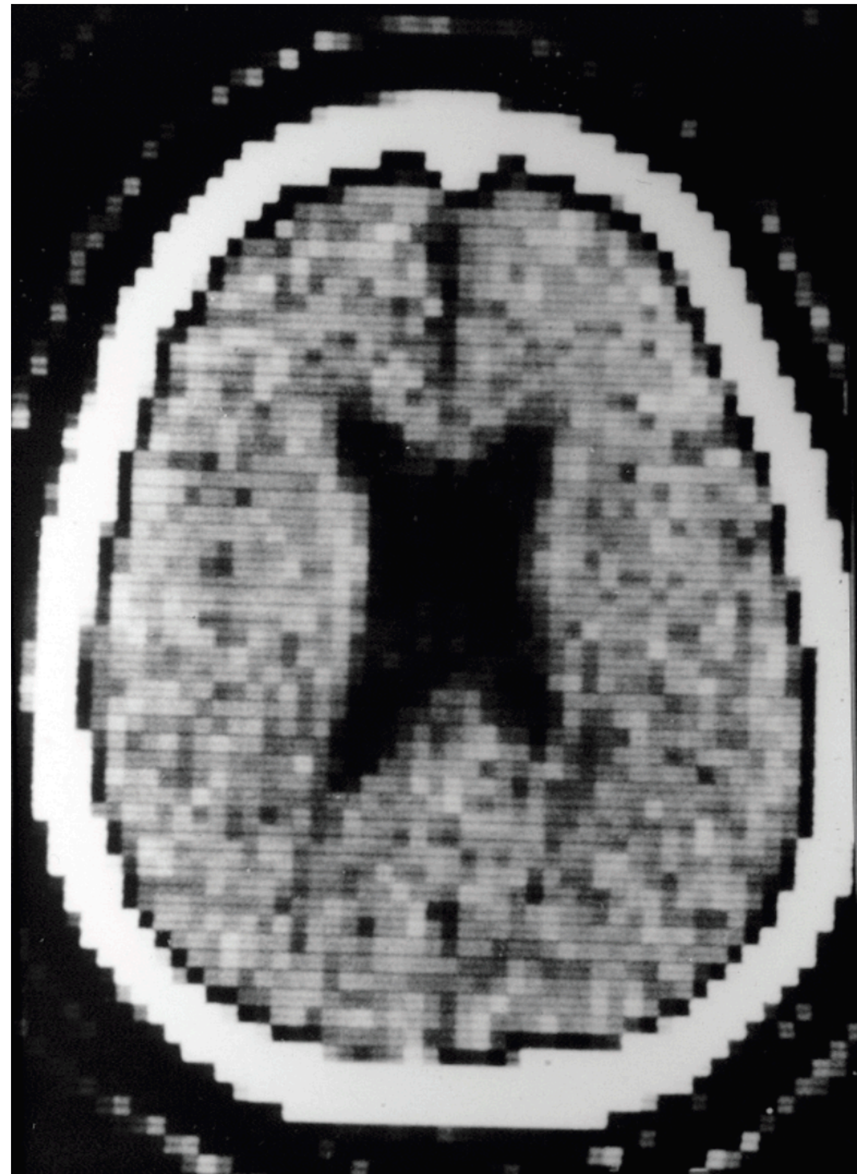
This is called computerized tomography (CT).



Godfrey Hounsfield and Allan McLeod Cormack were the first to develop X-ray tomography



Nobel prize was awarded to Hounsfield (top) and Cormack in 1979.



Reconstruction of a function from its line integrals was first invented by

Johann Radon (1887-1956).

This is the famous inversion formula from 1917
for the Radon transform Rf of a function f:

$$f(x) = \frac{1}{4\pi^2} \int_{S^1} \int_{\mathcal{R}} \frac{\frac{d}{ds}(Rf)(\theta, s)}{x \cdot \theta - s} ds d\theta$$

Filtered back-projection

$$f(x) = \frac{1}{4\pi^2}\int_{S^1}\int_{\mathcal{R}} \frac{\frac{d}{ds}(Rf)(\theta,s)}{x\cdot\theta - s}\,ds\,d\theta$$

Filtered back-projection (FBP) is mathematical technology used on a daily basis in hospitals around the world.

The quality of 3D reconstruction using FBP is excellent.

However, a comprehensive data set is mandatory for FBP.

Due to the radiation dose, a CT scan is only appropriate for seriously ill patients

In filtered backprojection, the mathematical reconstruction formula assumes dense angular sampling of full-angle data:
the chosen mathematics requires high radiation dose.

Think the opposite: take as few X-ray images as possible and use tailor-made mathematics to form a reconstruction that is good enough for the clinical task at hand. Then
the low level of radiation dose requires
new mathematics and more computational power.

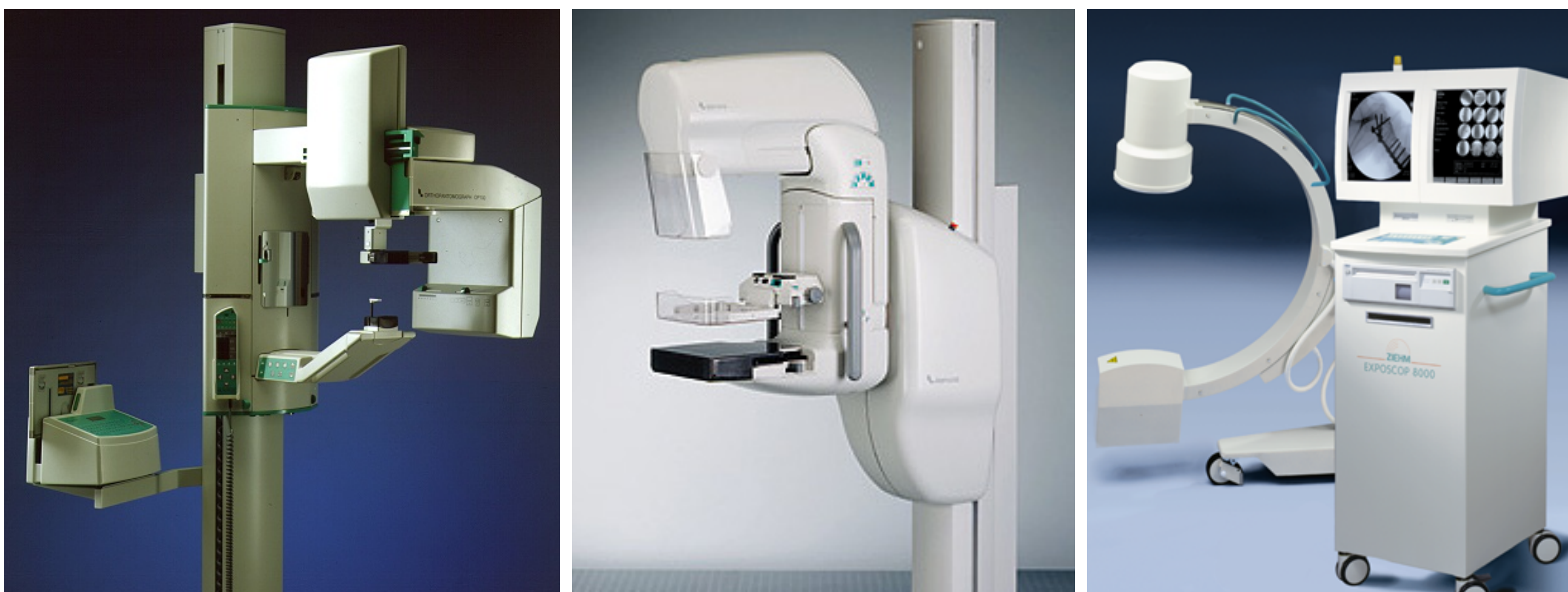
A series of projects started in 2001 aiming for a new type of low-dose 3D imaging

The goal was a mathematical algorithm with

Input: small number of digital X-ray images taken with any X-ray device

Output: three-dimensional reconstruction with high enough quality for the clinical task at hand

Products of Instrumentarium Imaging in 2001:



This work was done in 2001-2010 jointly with

Nuutti Hyvönen
Seppo Järvenpää
Jari Kaipio
Martti Kalke
Petri Koistinen
Ville Kolehmainen
Matti Lassas
Jan Moberg
Kati Niinimäki
Juha Pirttilä
Maaria Rantala
Eero Saksman
Henri Setälä
Erkki Somersalo
Antti Vanne
Simopekka Vänskä
Richard L Webber

INSTRUMENTARIUM imaging

GE Healthcare

PaloDEx Group

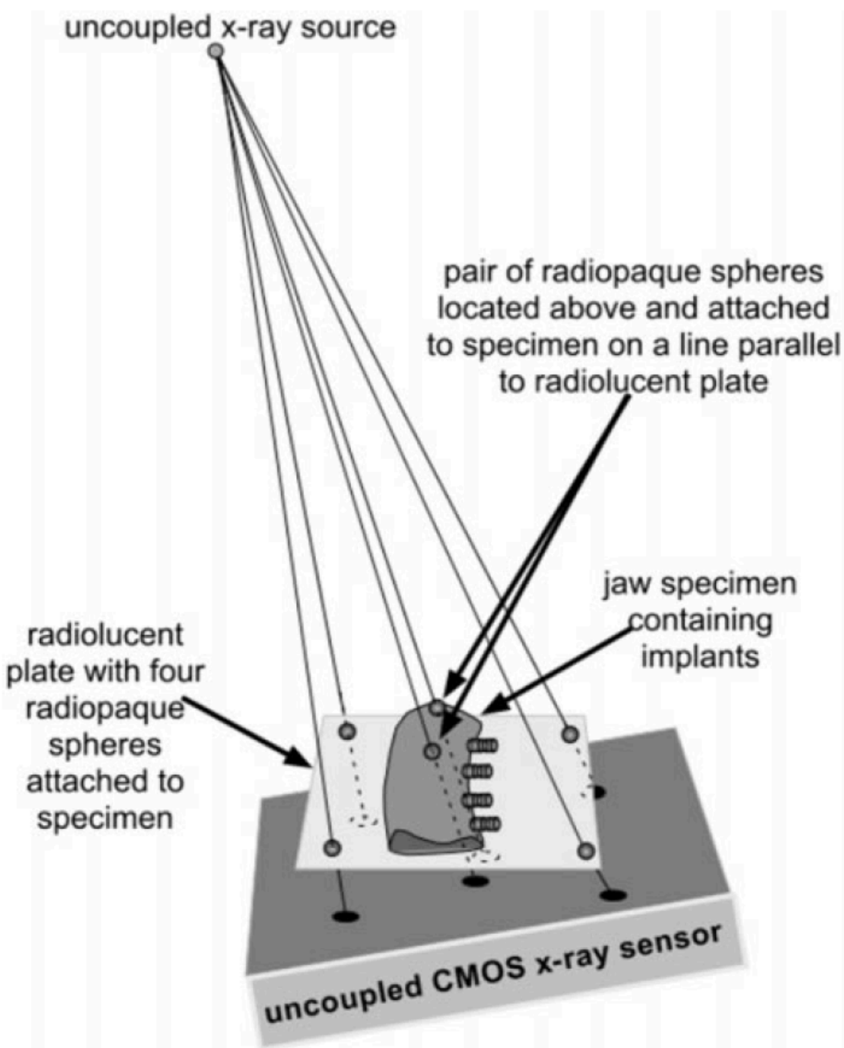
Essential history of the three projects:

Academic members: Inverse problems research groups in
University of Helsinki,
University of Eastern Finland,
Aalto University and
Tampere University of Technology
(all located in Finland)

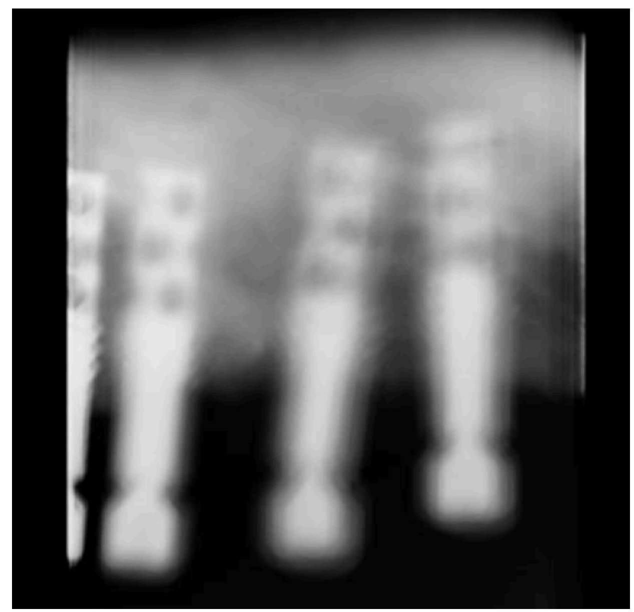
Industrial members:
2001-2002 Instrumentarium Imaging
2003-2004 GE Healthcare
2005-2007 PaloDEx Group

Funding by Finnish Technology Agency (TEKES).

Tuned Aperture Computed Tomography (TACT) was the starting point of our research

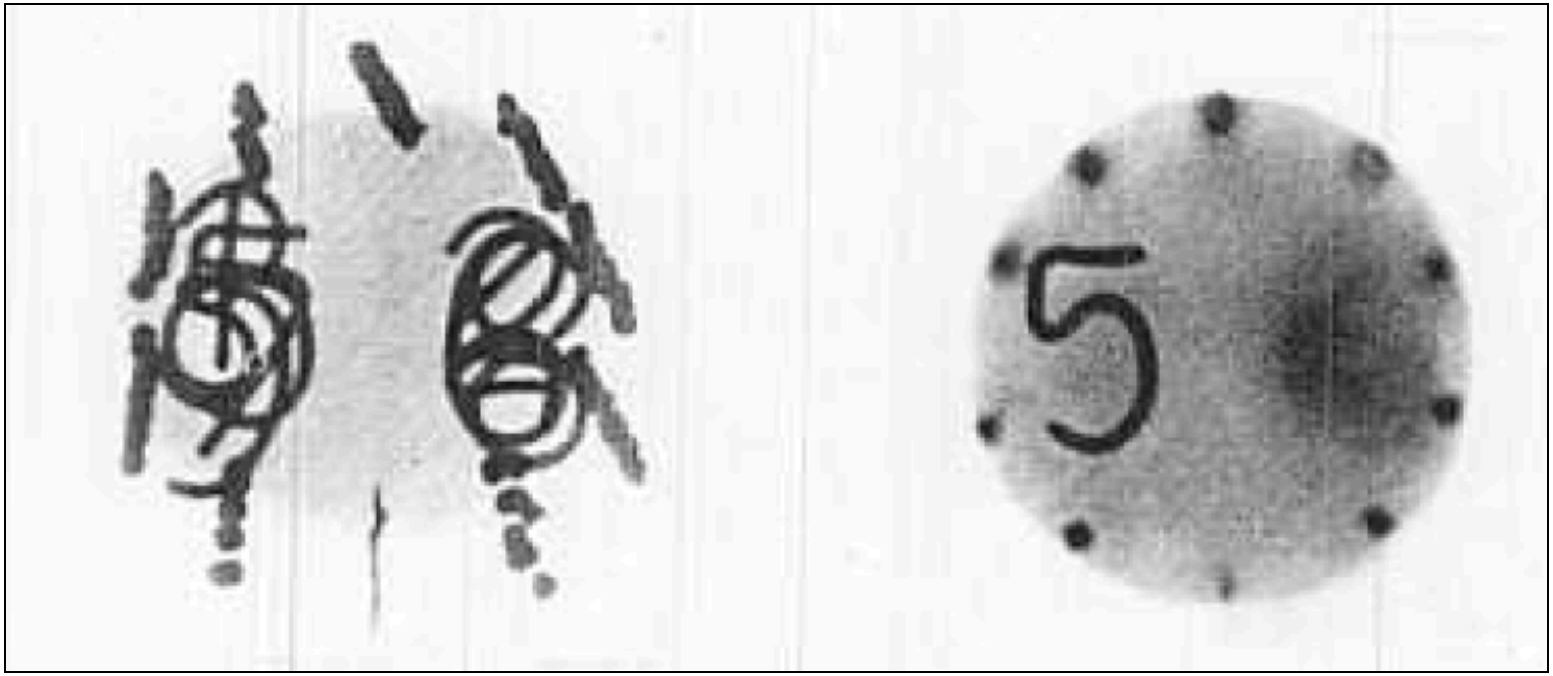


Instrumentarium Corp. licensed Richard Webber's tomosynthesis-based TACT patent in 1999.



Linnenbrugger, Webber and Lehmann 2002,
US Patent 6289235 (Webber & Horton 1998)

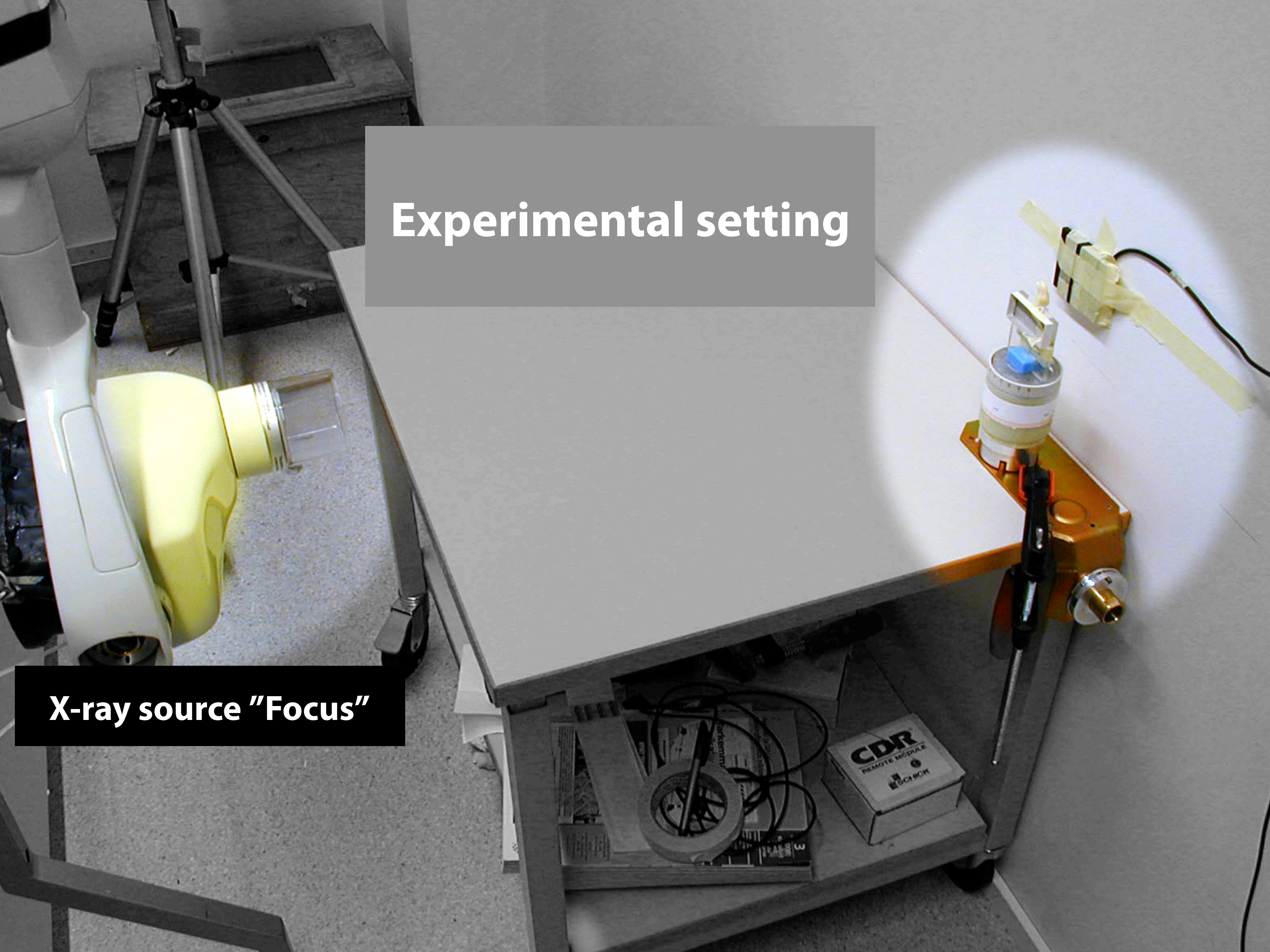
Tomosynthesis can be dated back to the work of Ziedses des Plantes in 1932

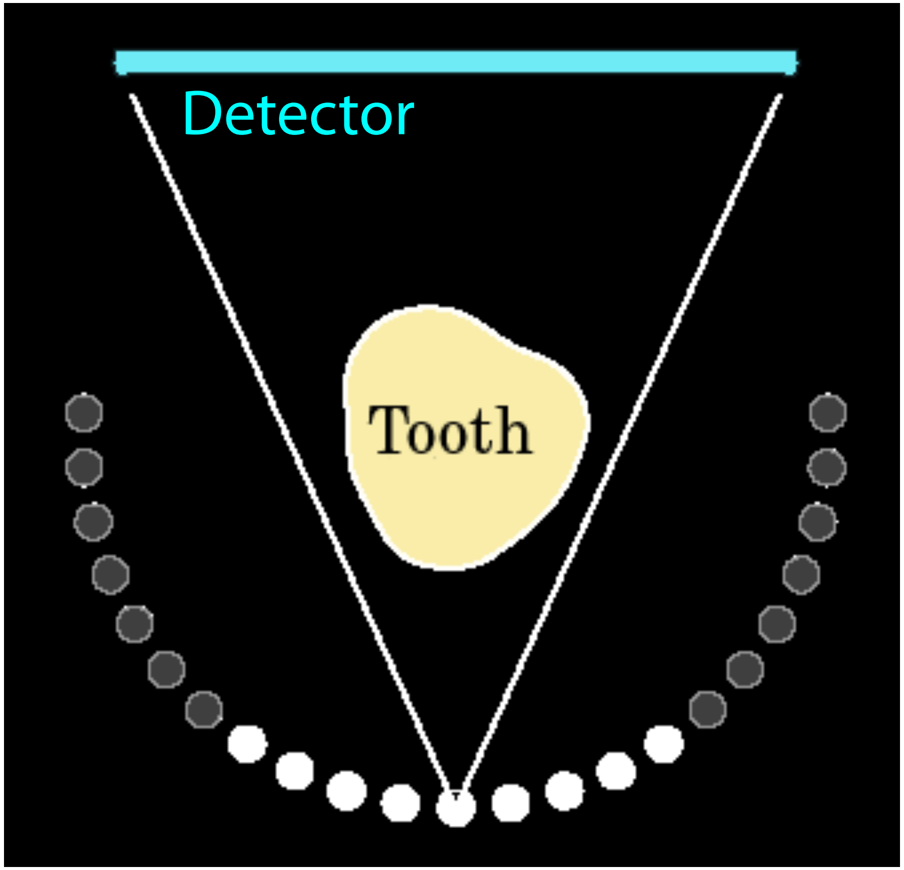


EINE NEUE METHODE ZUR DIFFERENZIERUNG IN DER RÖNTGENOGRAPHIE (PLANIGRAPHIE)[1]

von

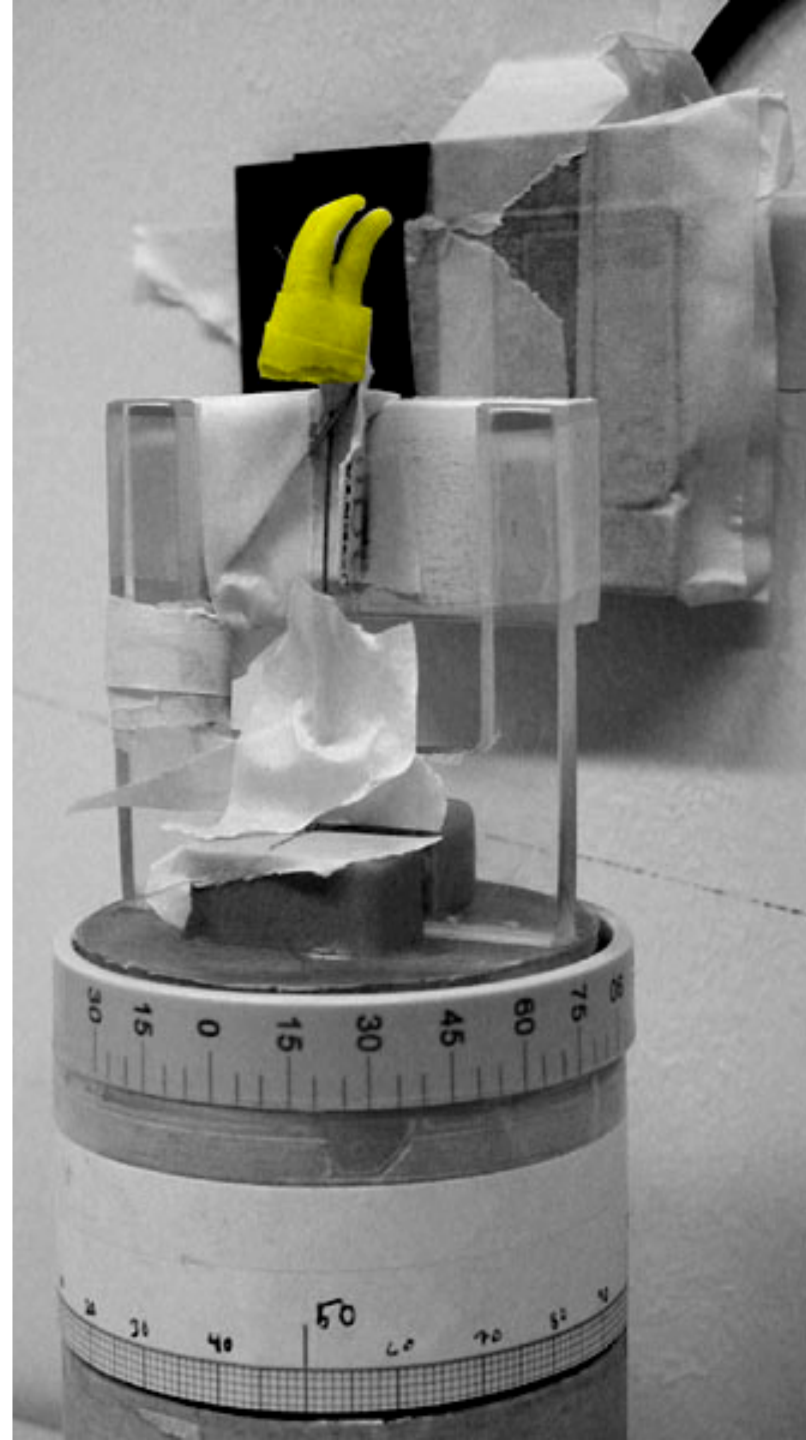
B. G. Ziedses des Plantes


Experimental setting
X-ray source "Focus"
CDR
REMOTE MODULE

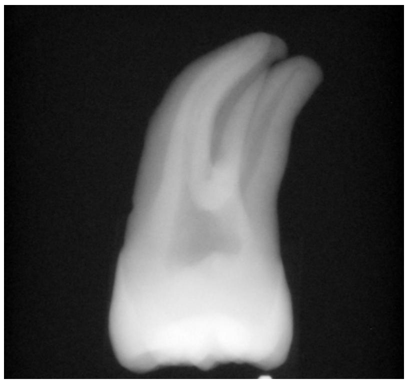


X-ray source positions
Full data: 23 angles, 187 degrees
Limited data: 9 angles, 68 degrees
Almost parallel beam geometry!

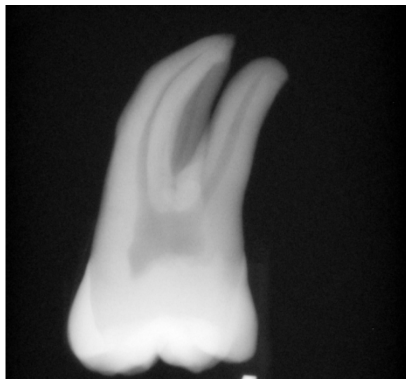
Tooth donated to science
by Helena Sarlin. Thanks!



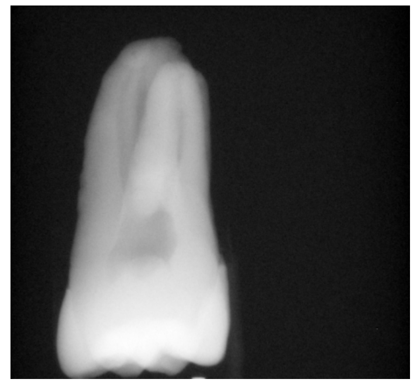
The projection images look like this



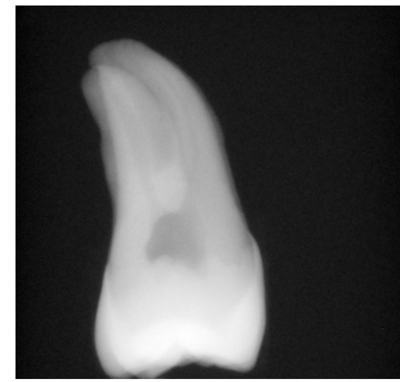
0



30



60



90

We stack 400 two-dimensional tomographic problems using the projection data line by line

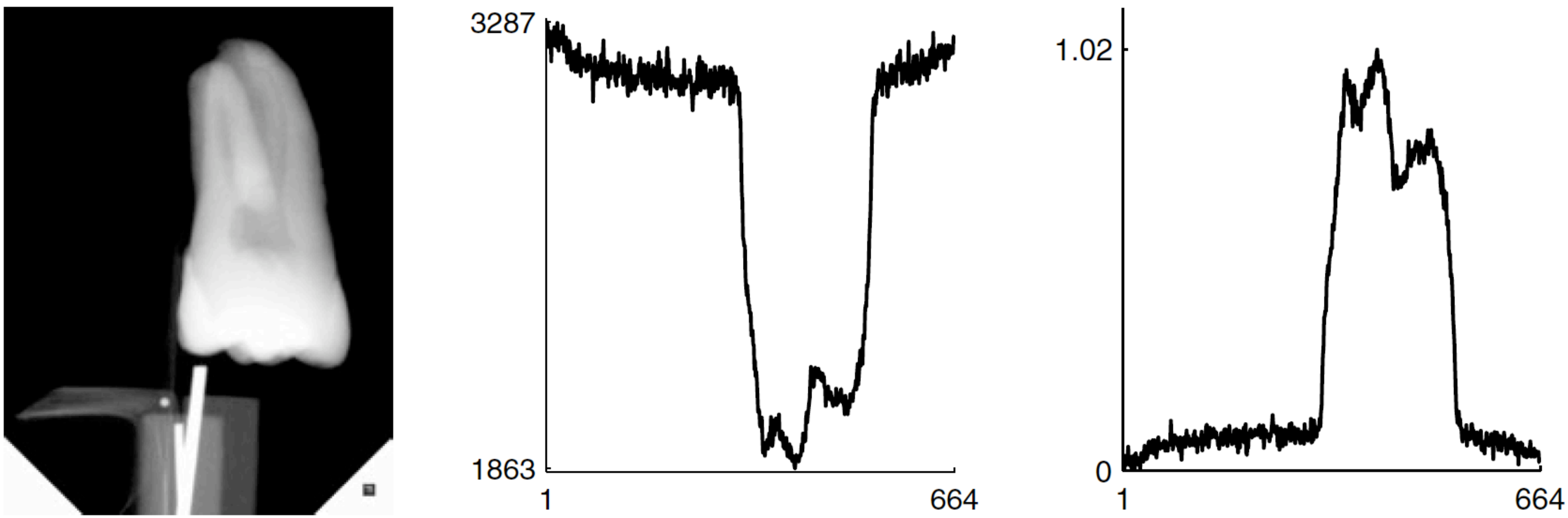


Figure 2. Left: 872 × 664 projection radiograph from the tooth phantom. Note that the image is shown with inverted colourmap (i.e., black corresponds to high photon counts). Middle: pixel values of the 200th row from the raw projection radiograph. Right: same row in the form of tomographic attenuation data.

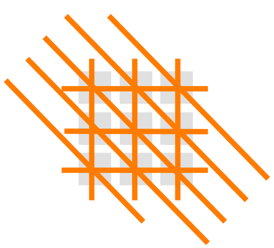
These are the dimensions of each of the 400 two-dimensional tomographic problems

Computational domain is a square with 26mm long sides.

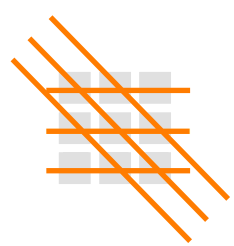
We divide the square into 166x166 = **27556** pixels.

Full angle data is collected from 23 directions, each having 664 data points. Thus the number of measurements is **15272.**

Limited angle data is collected from 9 directions, each having 664 data points. Thus the number of measurements is **5976.**

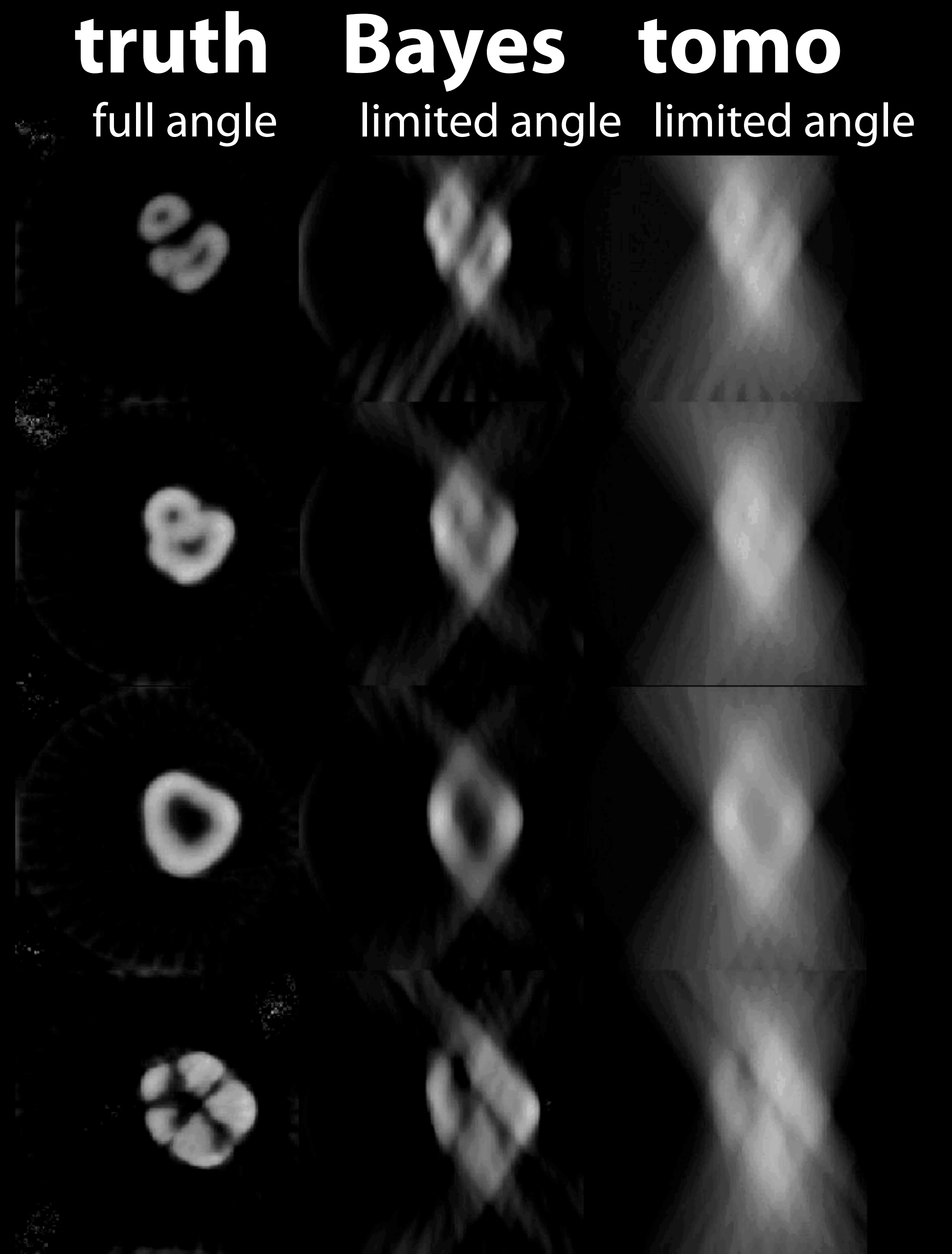


Compare:
11 measurements,
9 pixels

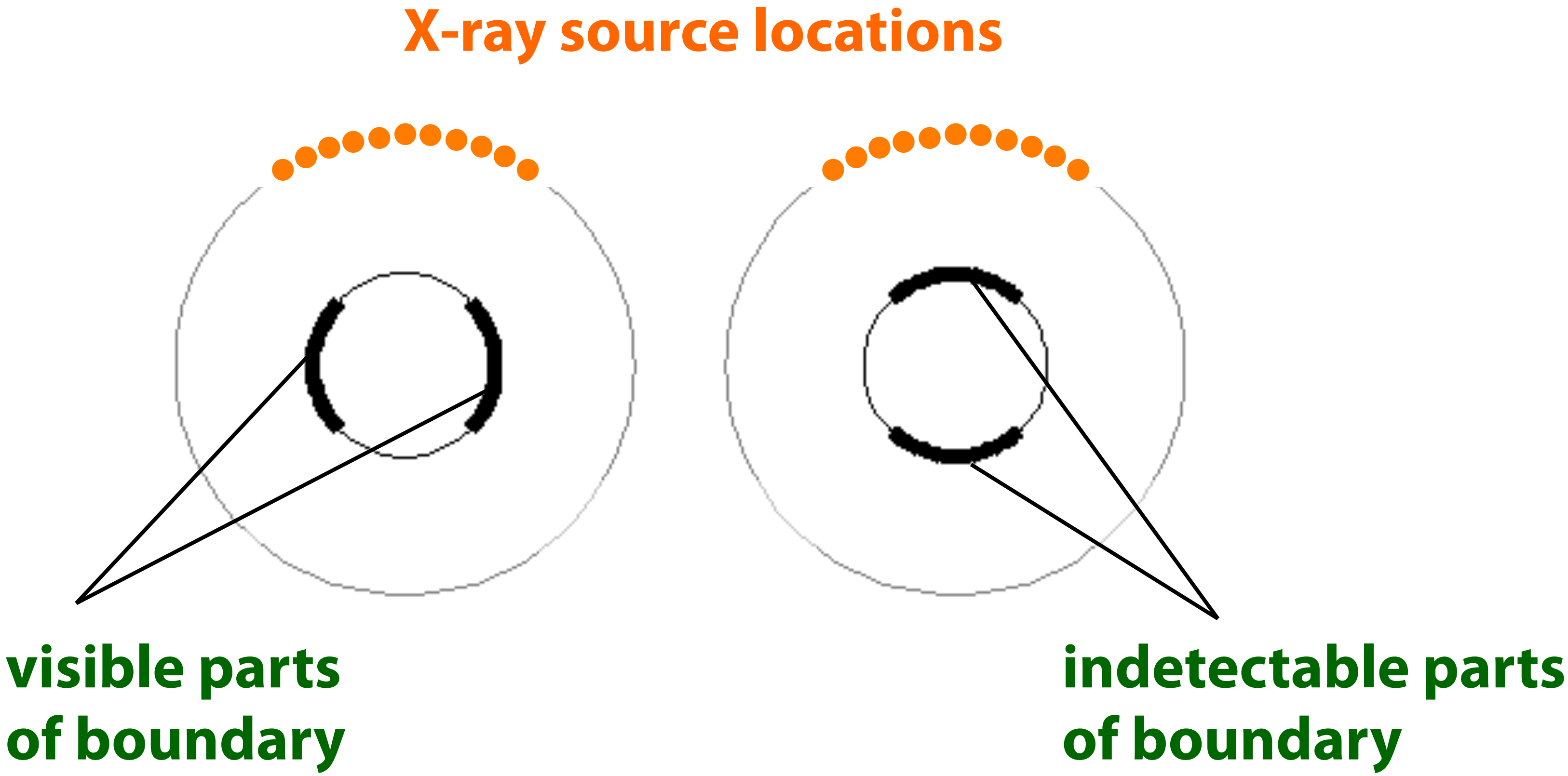


Compare:
6 measurements,
9 pixels

Horizontal slices:

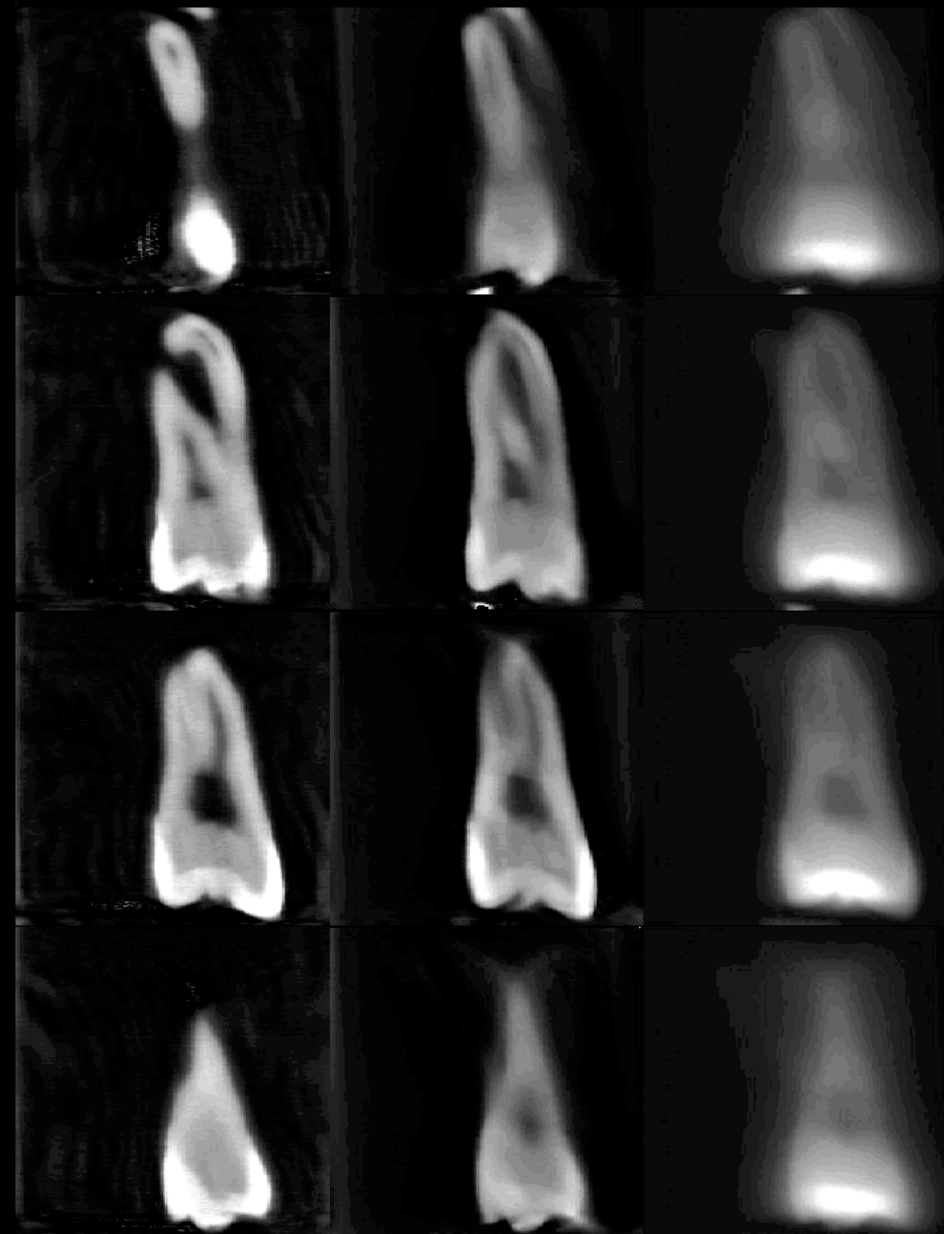


Some parts of the boundary are strongly visible in projection data



Microlocal analysis of recoverable singularities is available in **Greenleaf & Uhlmann** (1989), **Quinto** (1993) and **Ramm** & **Katsevich** (1996), based on earlier work by Guillemin.

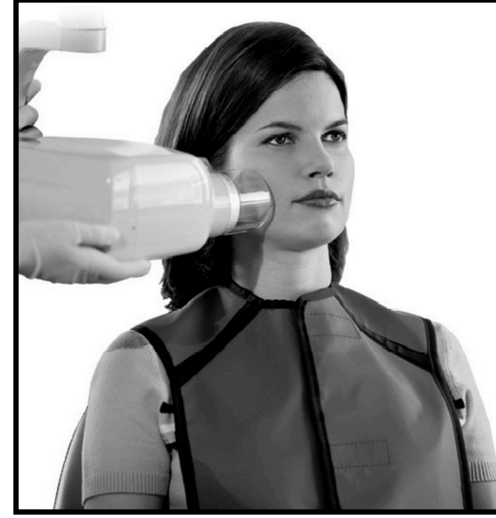
Vertical slices:

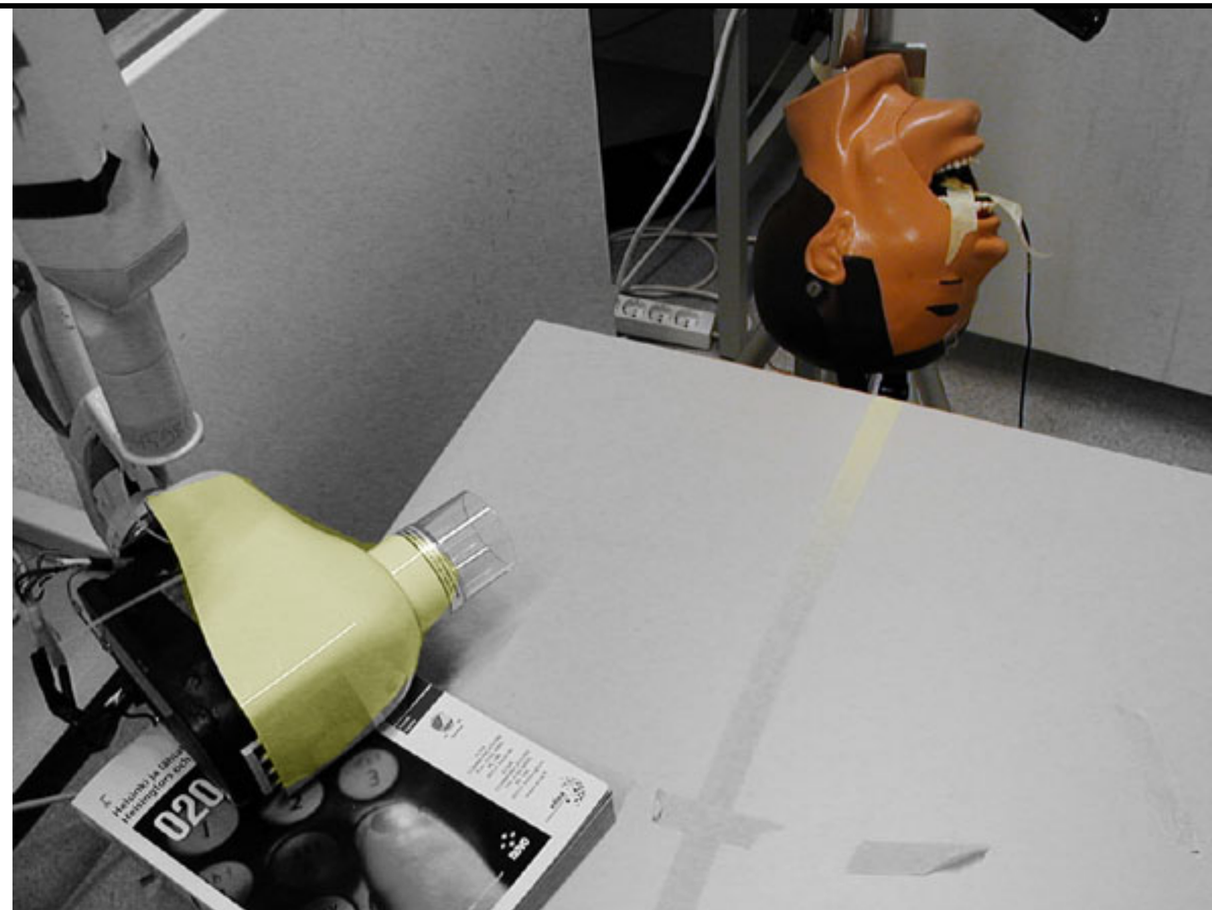


Kolehmainen, S, Järvenpää, Kaipio, Koistinen, Lassas, Pirttilä, Somersalo (2003)

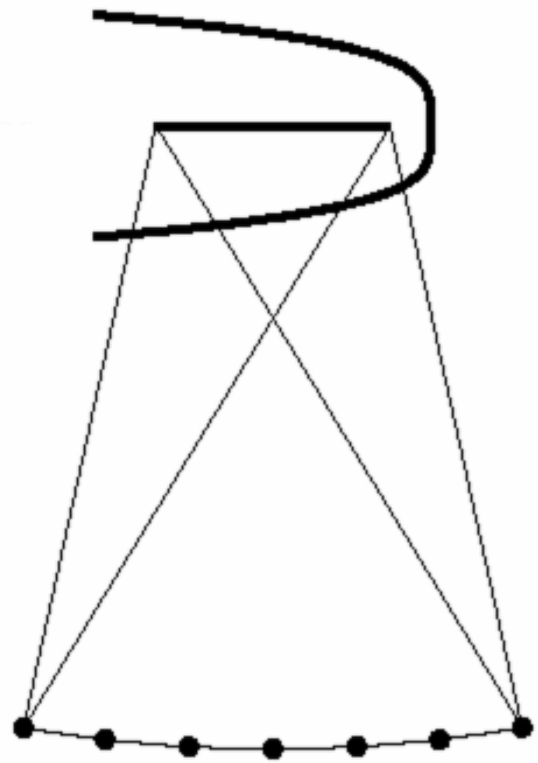
After 11 hours of laboratory work

Experimental setup for chairside 3D imaging models the clinical situation





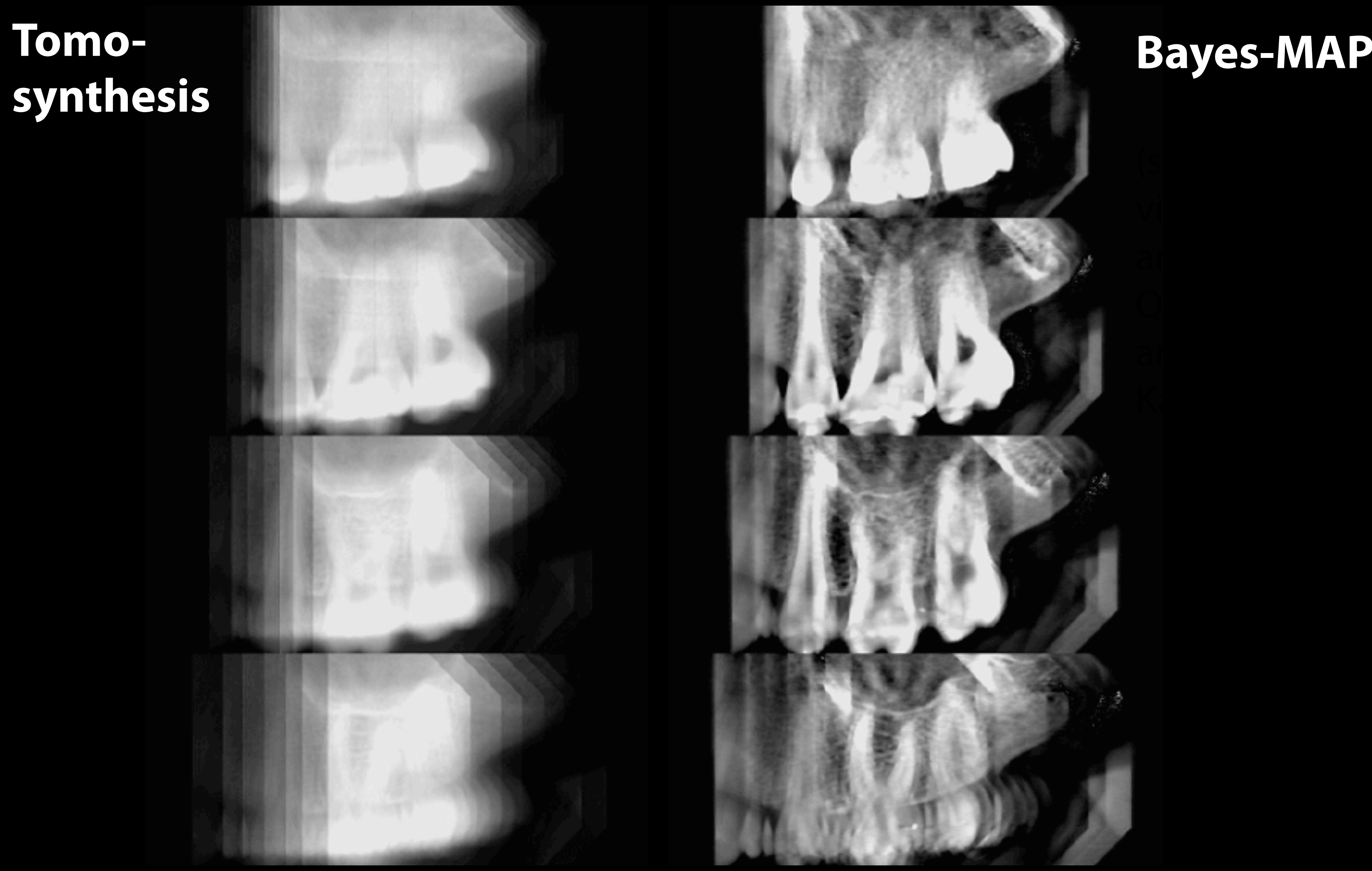
Details of this limited angle experiment



Opening angle 60 degrees

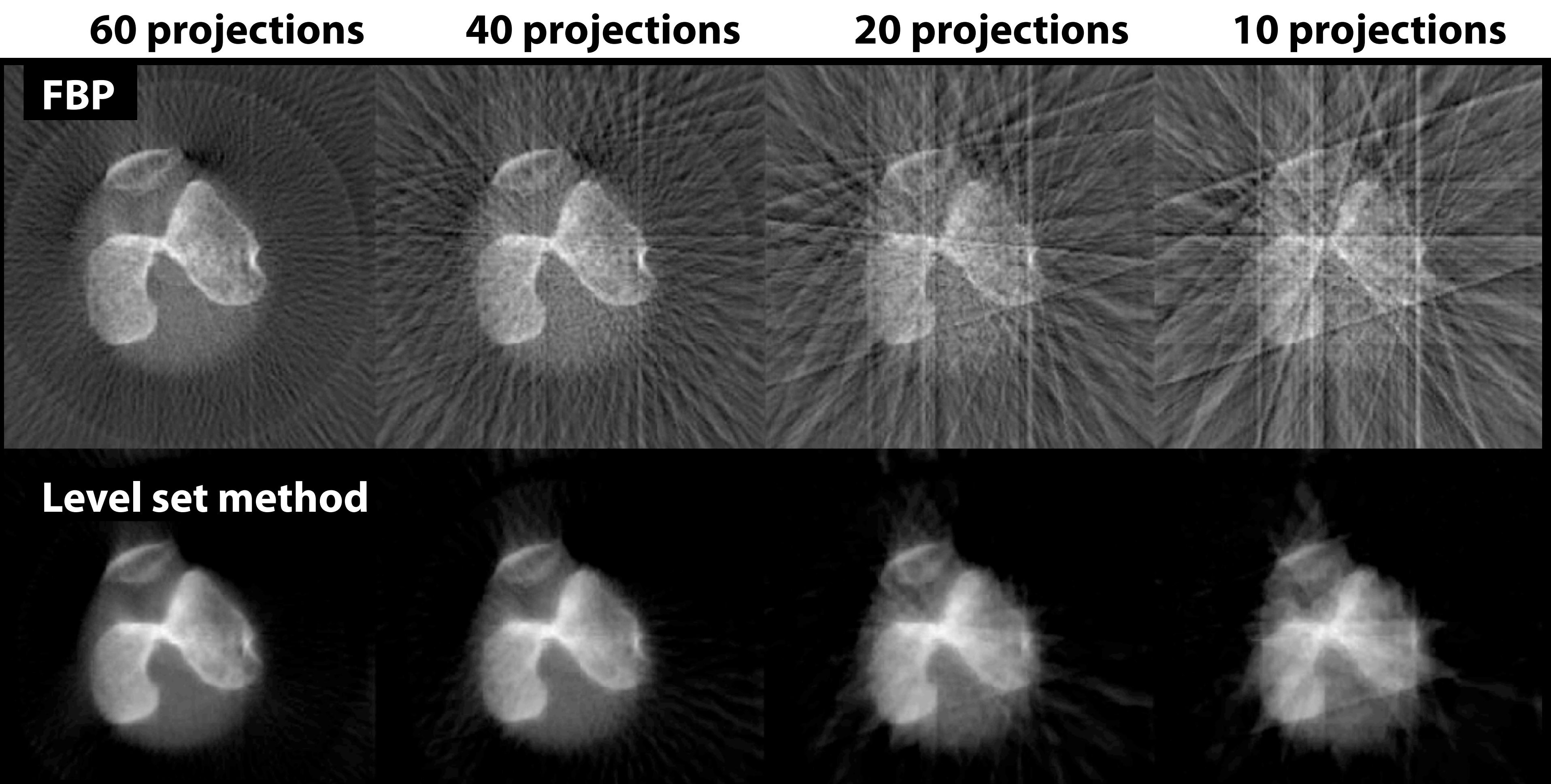
Seven digital intraoral radiographs
(664 x 872 pixels each)

There are 42 496 000 unknowns and 4 053 056 linear equations.
Computation is divided into 400 approximately 2D problems.



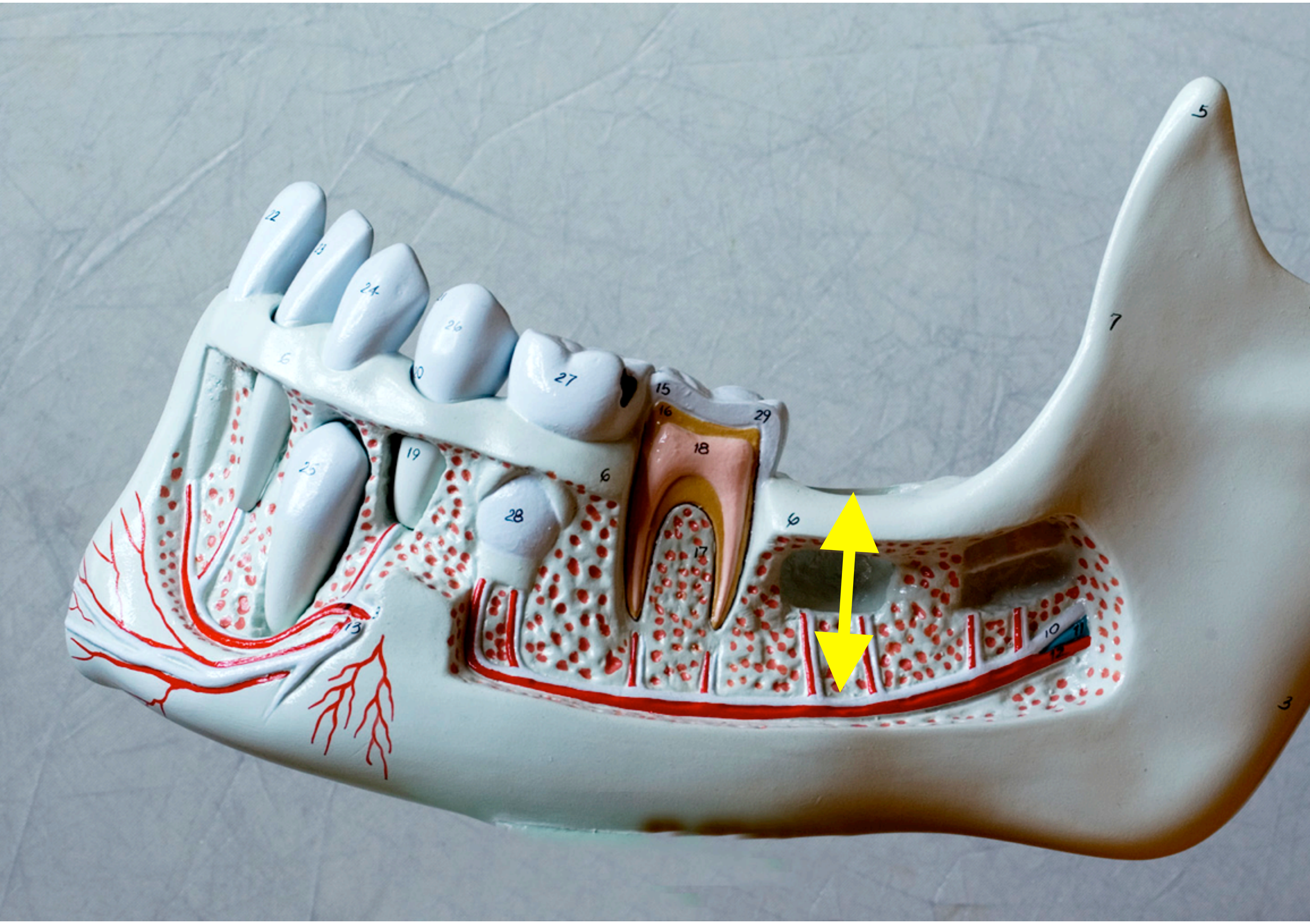
S, Kolehmainen, Järvenpää, Kaipio, Koistinen, Lassas, Pirttilä and Somersalo 2003

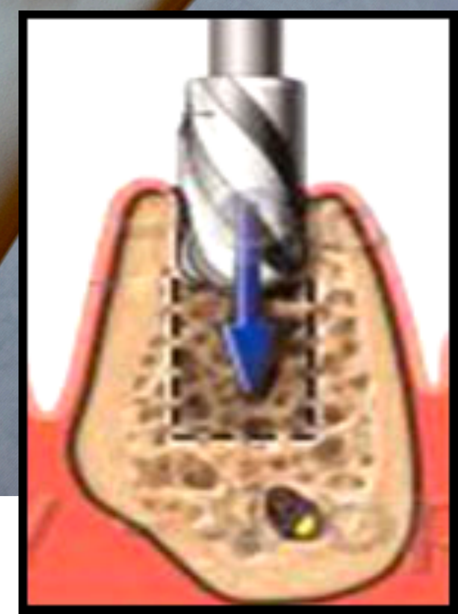
Data from a knee phantom using a surgical C-arm X-ray device, reconstruction by level set method



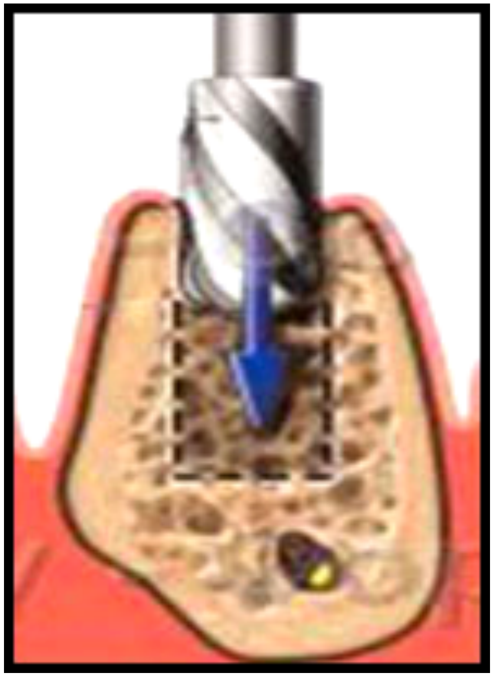
Kolehmainen, Lassas & S (2008), US patent 7274766

Application: dental implant planning, where a missing tooth is replaced by an implant





Three-dimensional information is crucial for dental implant planning



The hole must be drilled deep enough for sturdy attachment but not so deep that the mandibular nerve is damaged.

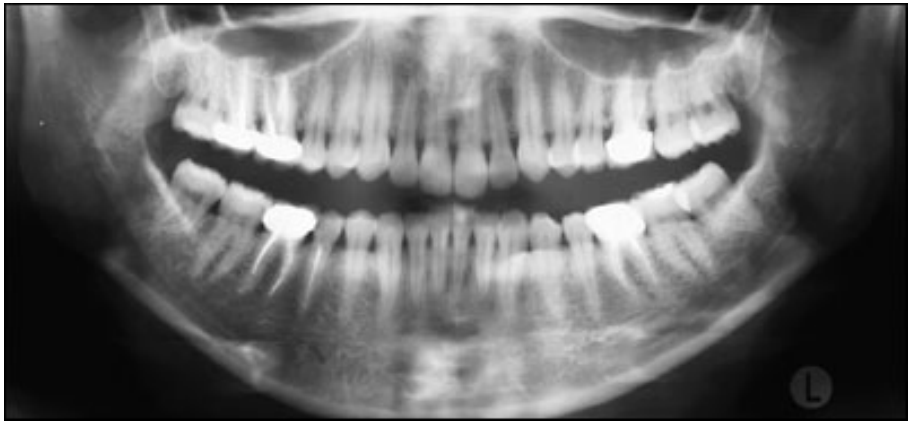
Two-dimensional X-ray projection images are not suitable for assessing the proper depth because of geometric distortion.

Three-dimensional reconstruction of the tissue is needed, but a traditional CT scan is not practical due to high cost, too low resolution and too high radiation dose.

Low-dose 3D X-ray imaging is an ideal solution: it is based on a small number of X-ray projection images recorded with a cost-effective device.

Panoramic X-ray device rotates around the head and produces a general picture of teeth

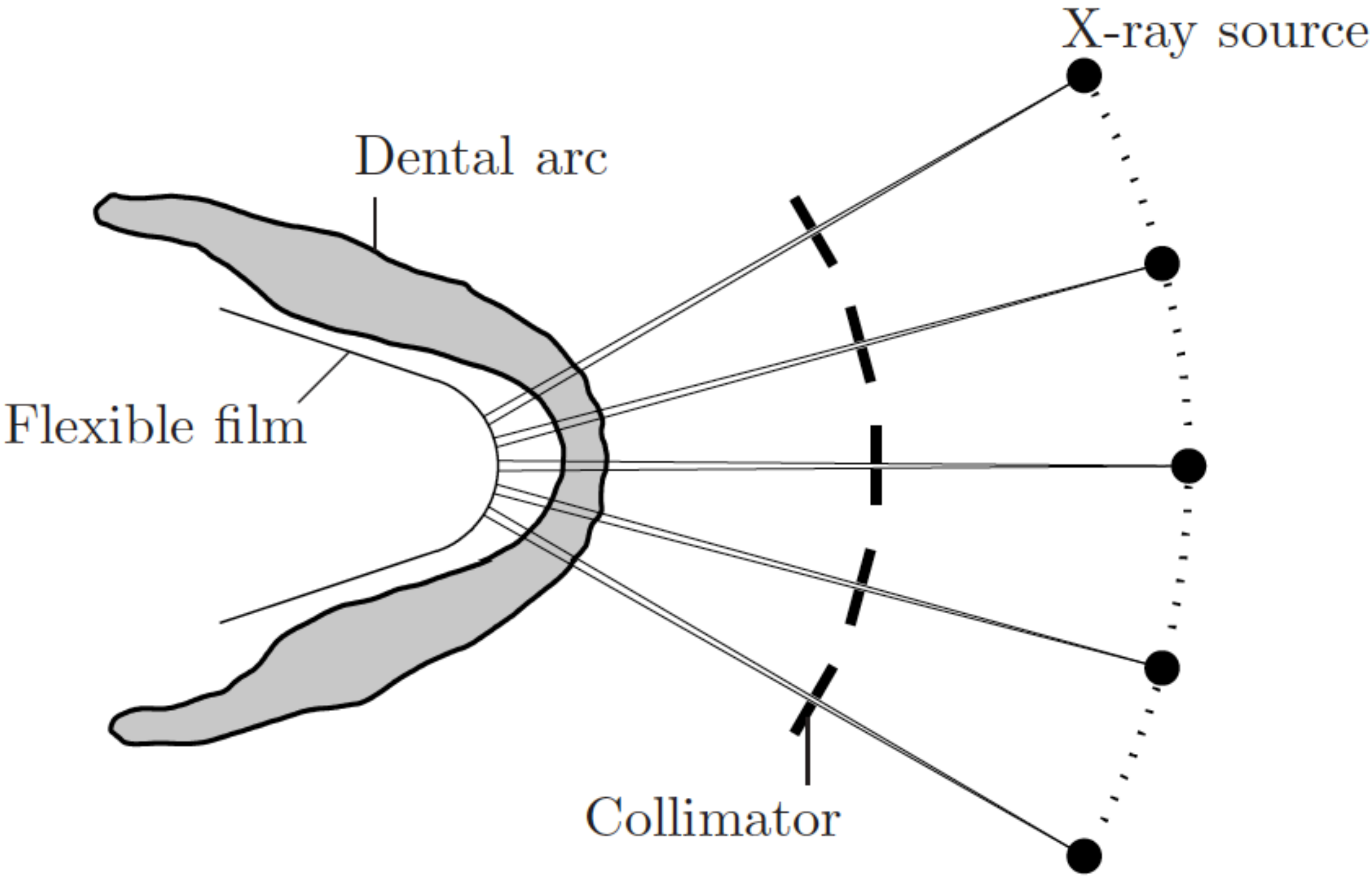
Panoramic imaging was invented by Yrjö Paatero in 1950's.



Nowadays a panoramic device is standard equipment at every dental clinic around the world.

In our project, we reprogrammed the device so that it collects limited-angle data.

Early approach for all-teeth X-ray imaging pioneered by Hisatugu Numata in 1933



This is the principle of panoramic X-ray imaging

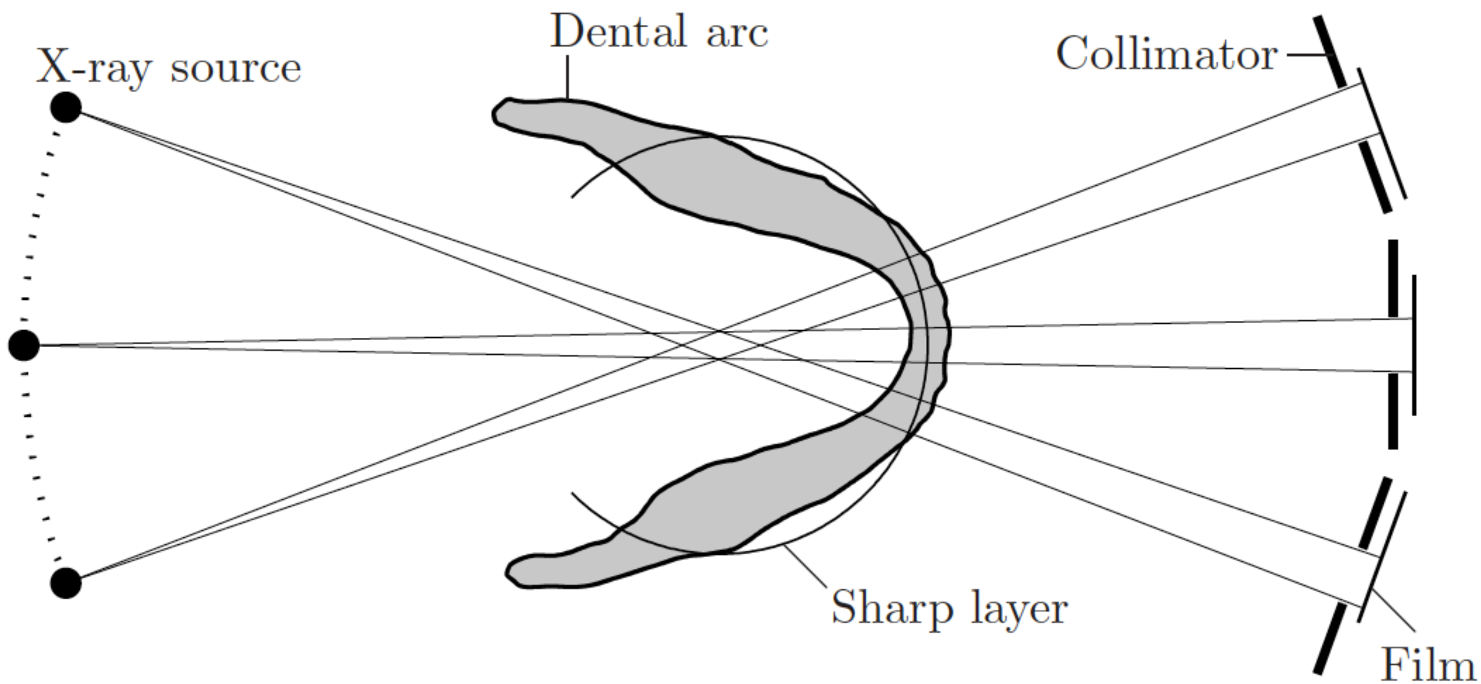


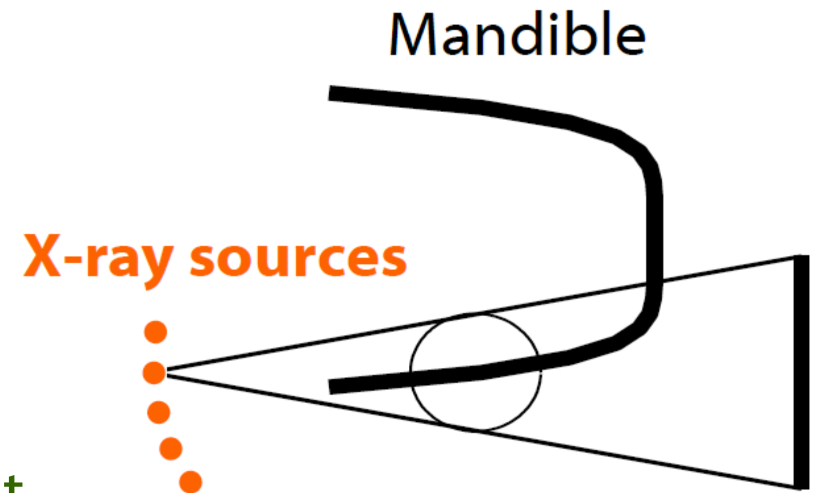
Figure 9.10: Principle of film-based extraoral (film outside the mouth) panoramic dental X-ray imaging, shown in a simplified cylindrical geometry and exaggerated angles for clarity. Note the crucial linear movement of the film with respect to the collimator slit. Due to the simplified geometry, the sharp layer does not follow the dental arc properly. In practice this problem is solved by varying the linear speed of the film and moving the center of rotation during the exposure.

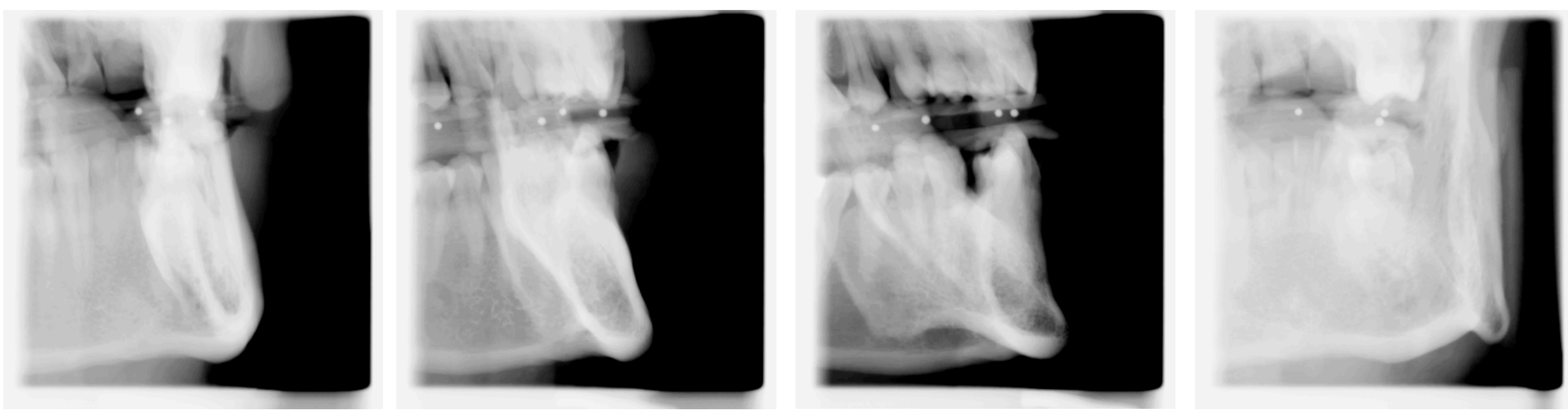
We reprogram the panoramic X-ray device so that it collects projection data by scanning

11 projection images of the mandibular area

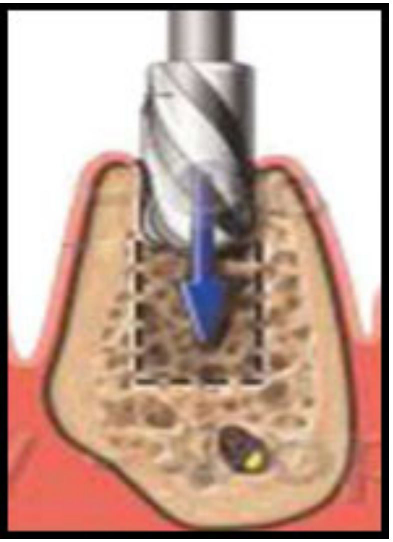
40 degrees angle of view

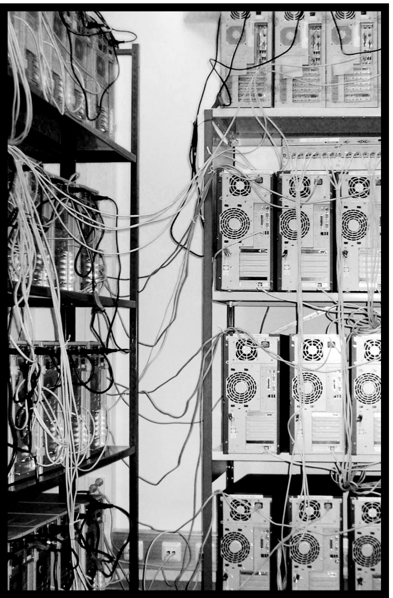
1000 x 1000 pixels per image, formed by a scanning movement



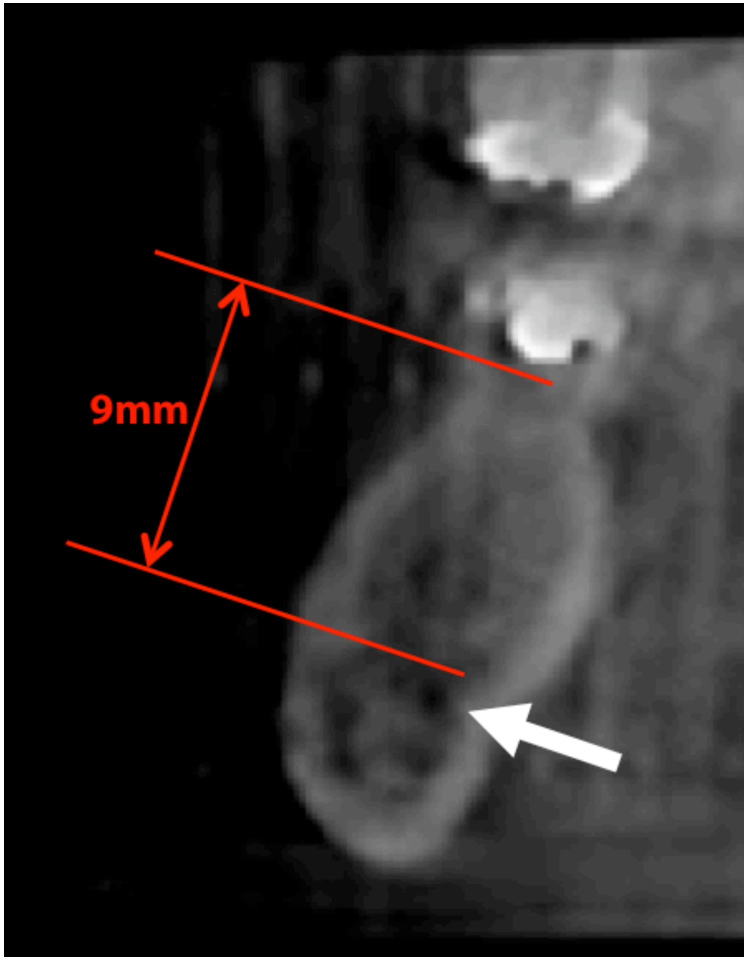


Limited angle Bayesian reconstruction can be used for locating the mandibular nerve





7 200 000 unknowns,
2 100 000 data points.
Parallel processing with a 13-node Beowulf cluster and GPGPU computing yields computation time less than 4 minutes.



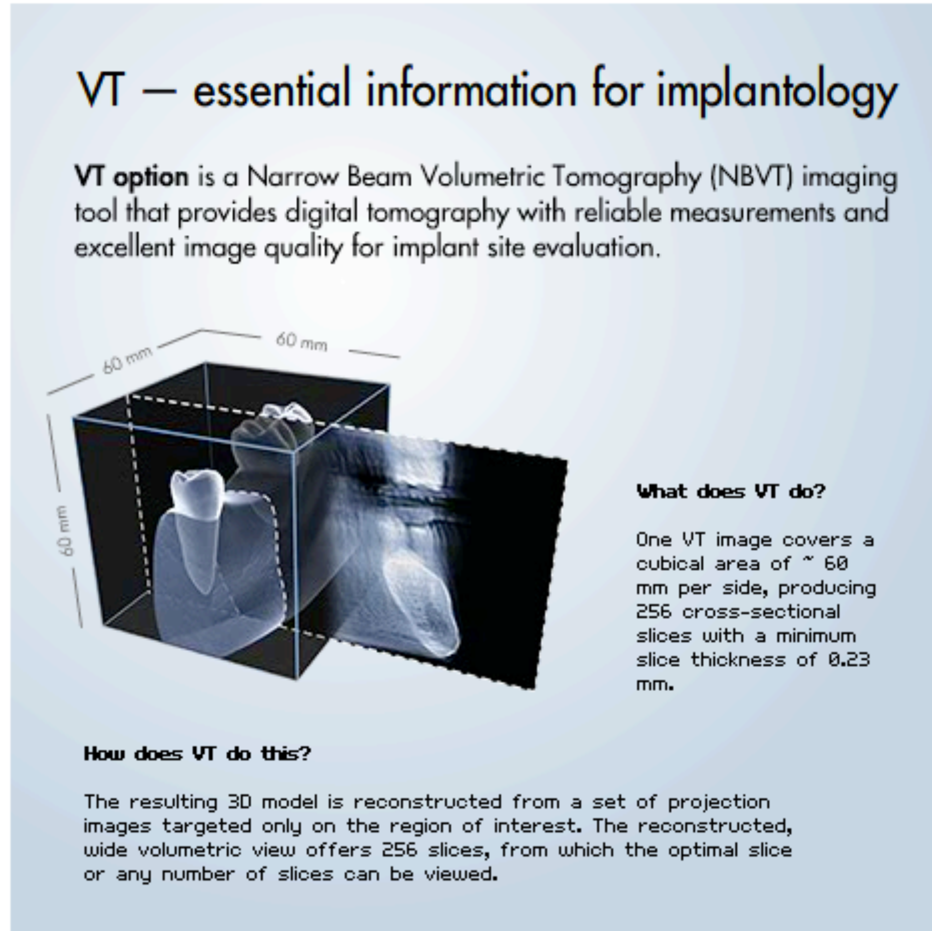
Cederlund, Kalke & Welander (2009)
Kolehmainen, Lassas & S (2008)
Kolehmainen, Vanne, S, Järvenpää, Kaipio, Lassas & Kalke (2006)
United States patent 7269241

The Bayesian low-dose imaging technique has been commercialized by Palodex Group

The **VT device** has been in the market from year 2007; thousands of units sold. See **www.vt-cube.com**.

It is remarkable that an existing 2D X-ray imaging product (panoramic device) becomes a 3D imaging product by a software update.

The core of that update is a mathematical inversion algorithm.

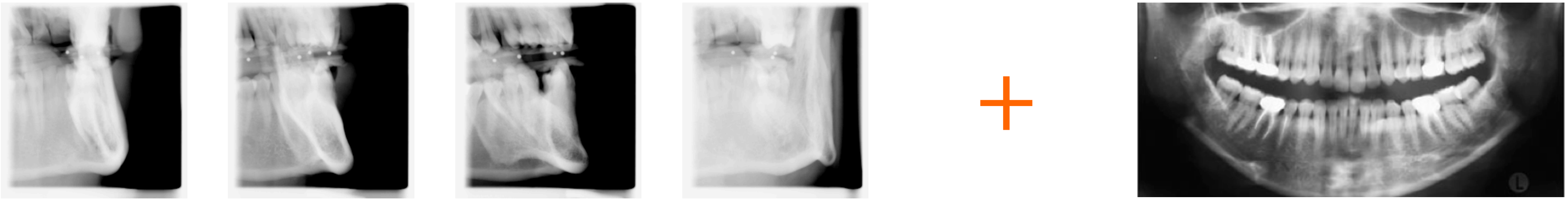


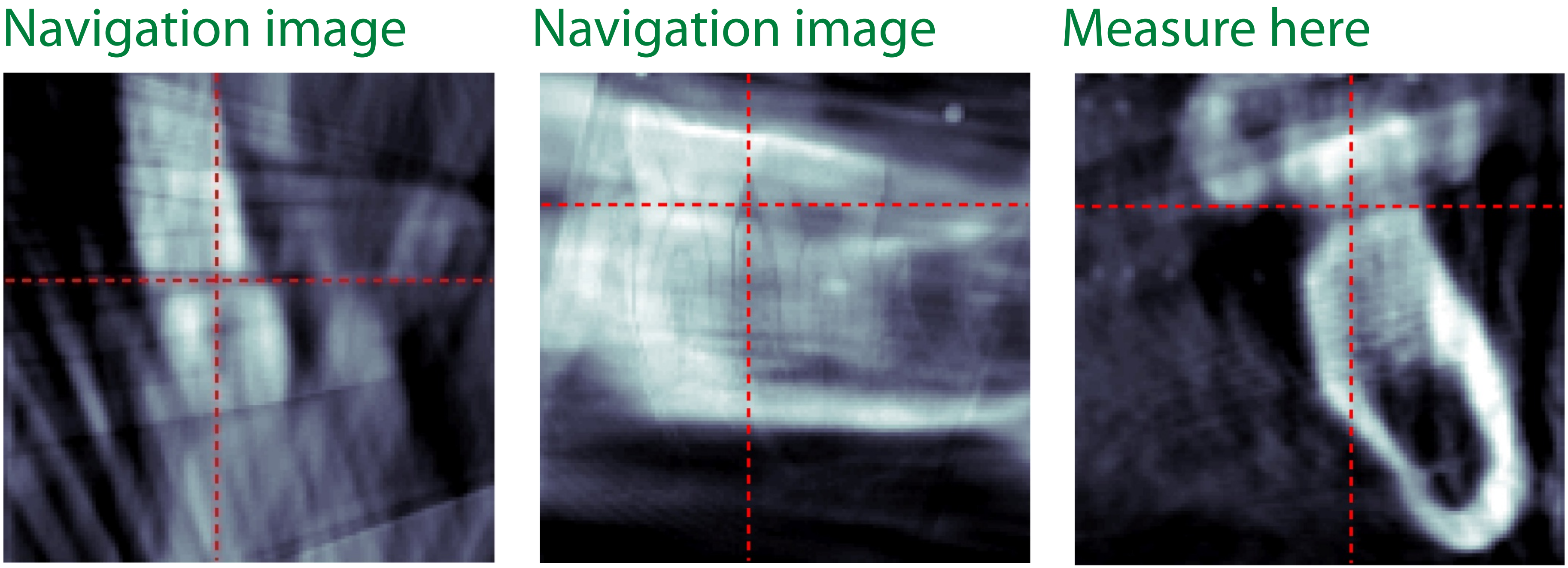
The radiation dose of the VT device is the lowest among 3D dental imaging modalities

Modality	**μSv**
Head CT	2100
CB Mercuray	558
i-Cat	193
NewTom 3G	59
VT device	**12**
Panoramic image (2D)	6

Ludlow et al, *Dentomaxillofacial Radiology* (2006) 35, 219–226

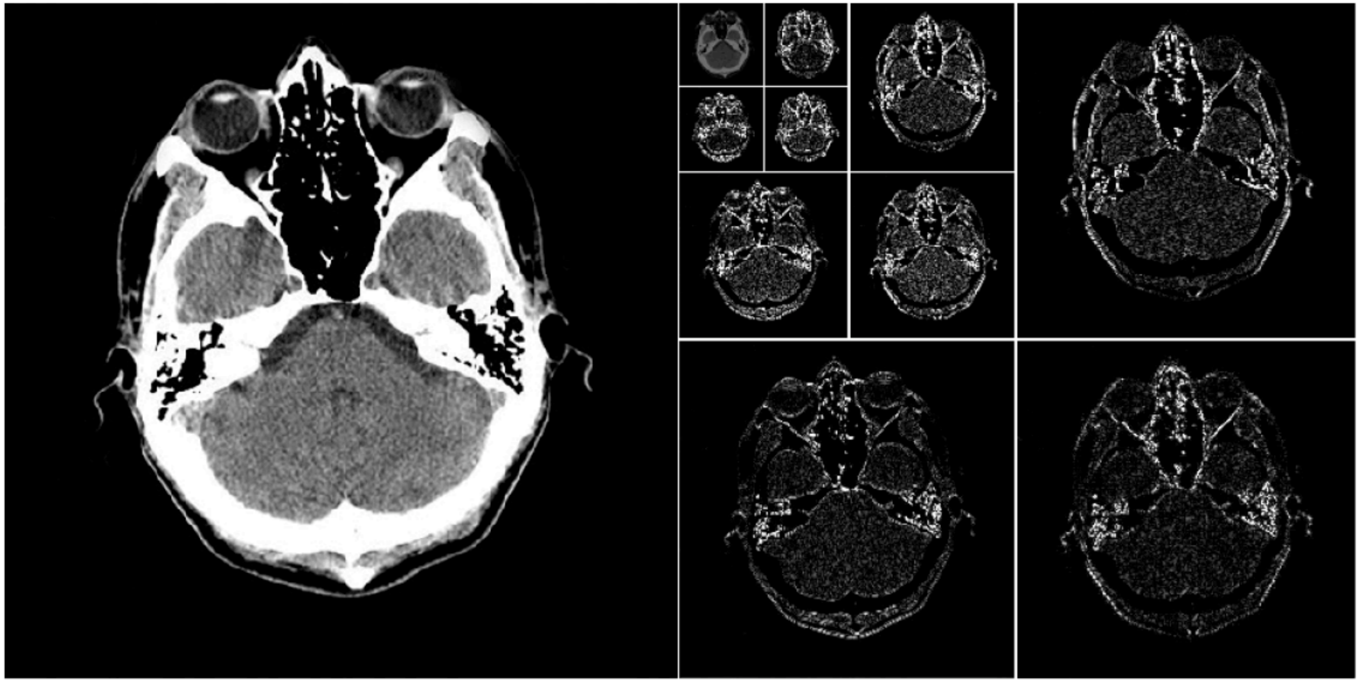
The use of a panoramic device for 3D imaging is enhanced by including panoramic data



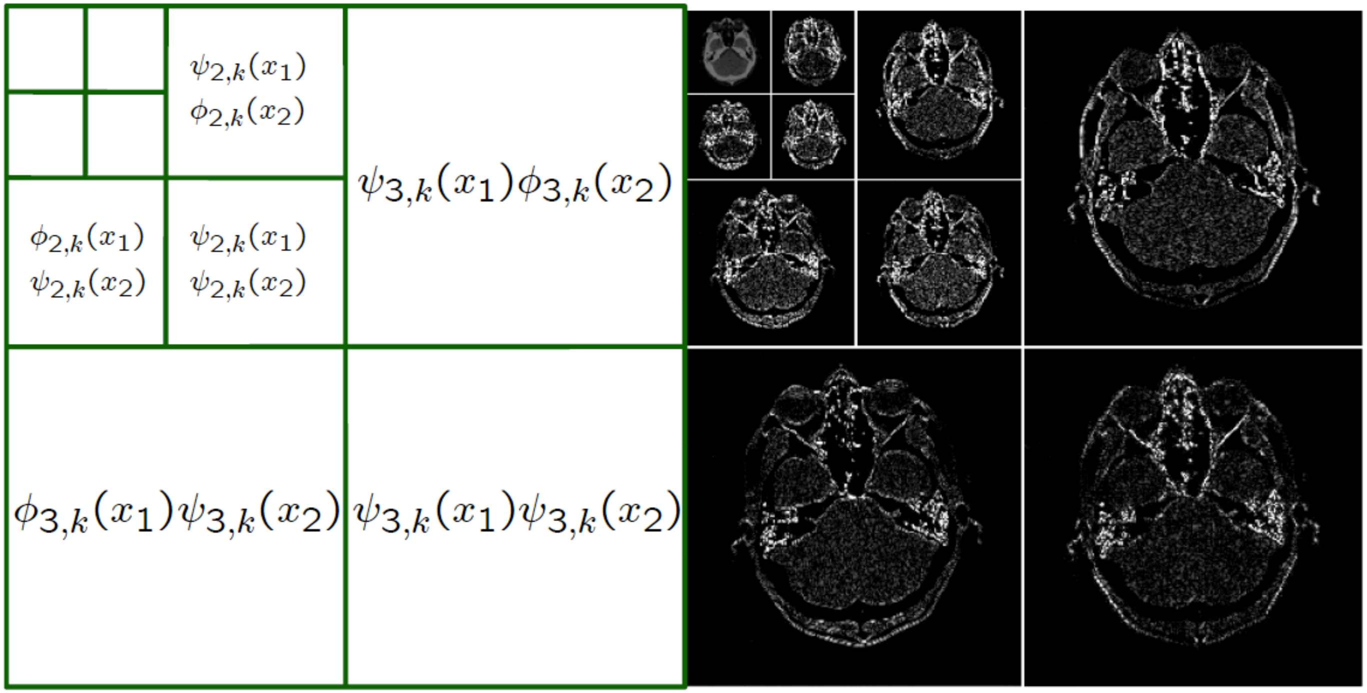


Hyvönen, Kalke, Lassas, Setälä and S (2010)

Wavelet transform divides a function into details at different scales



We introduce a convenient renumbering of the basis functions

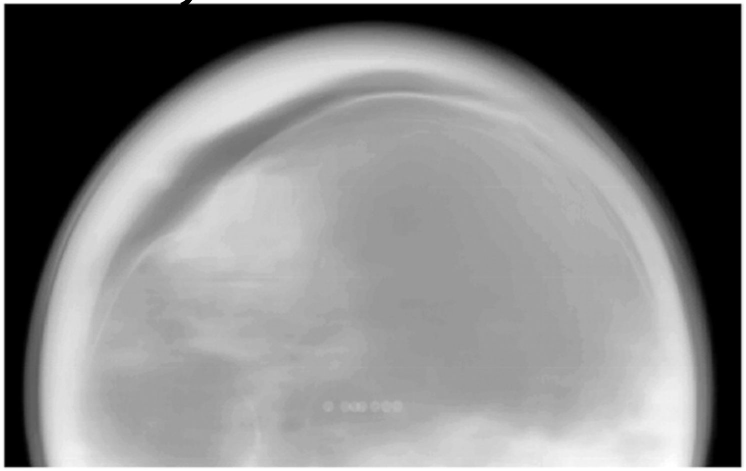


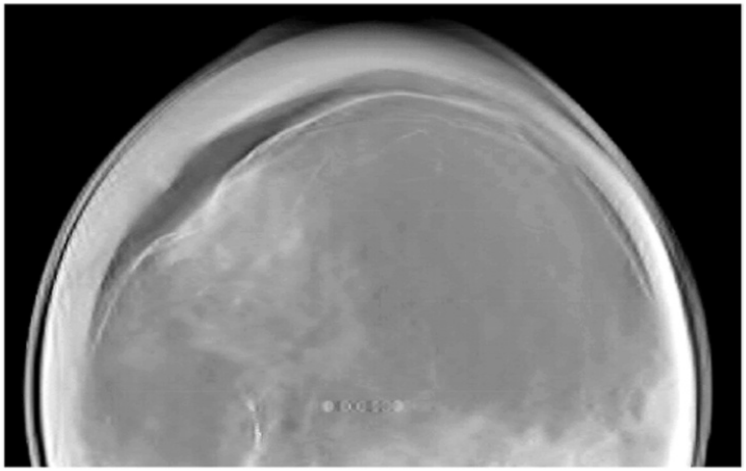
$$f(x) = \sum_{\ell=1}^{\infty} c_\ell \psi_\ell(x)$$

Limited angle tomography results for X-ray mammography

Rantala *et al.* (2006)
US patent 7215730

Tomosynthesis

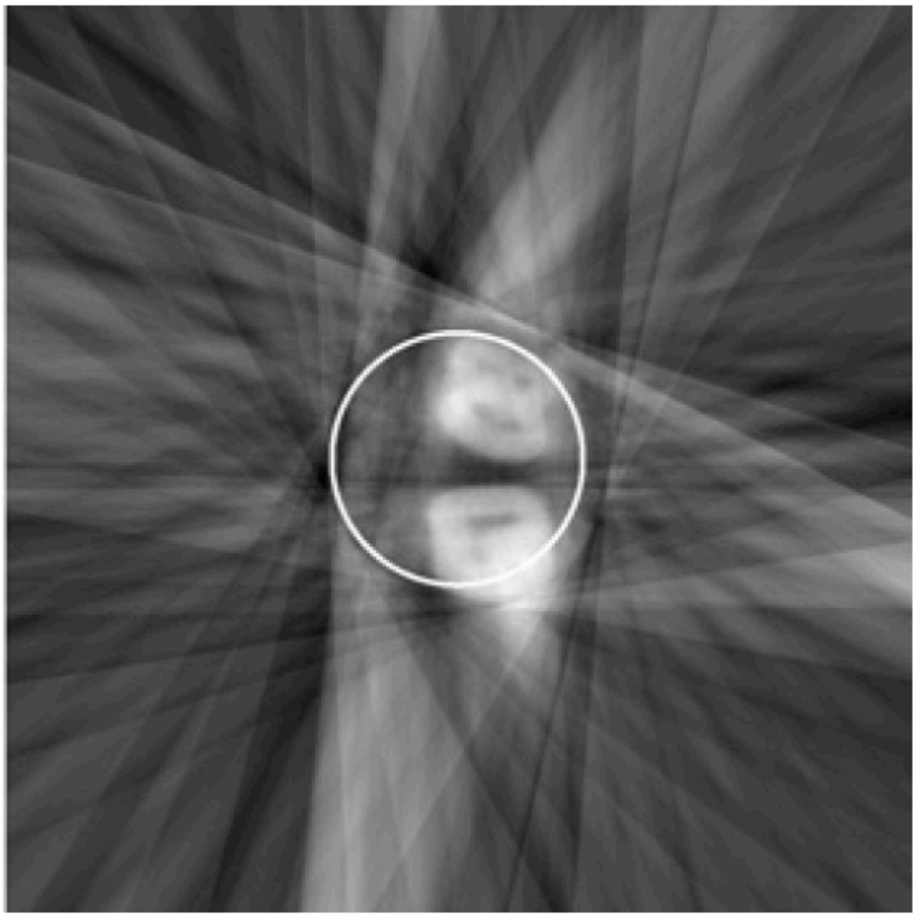




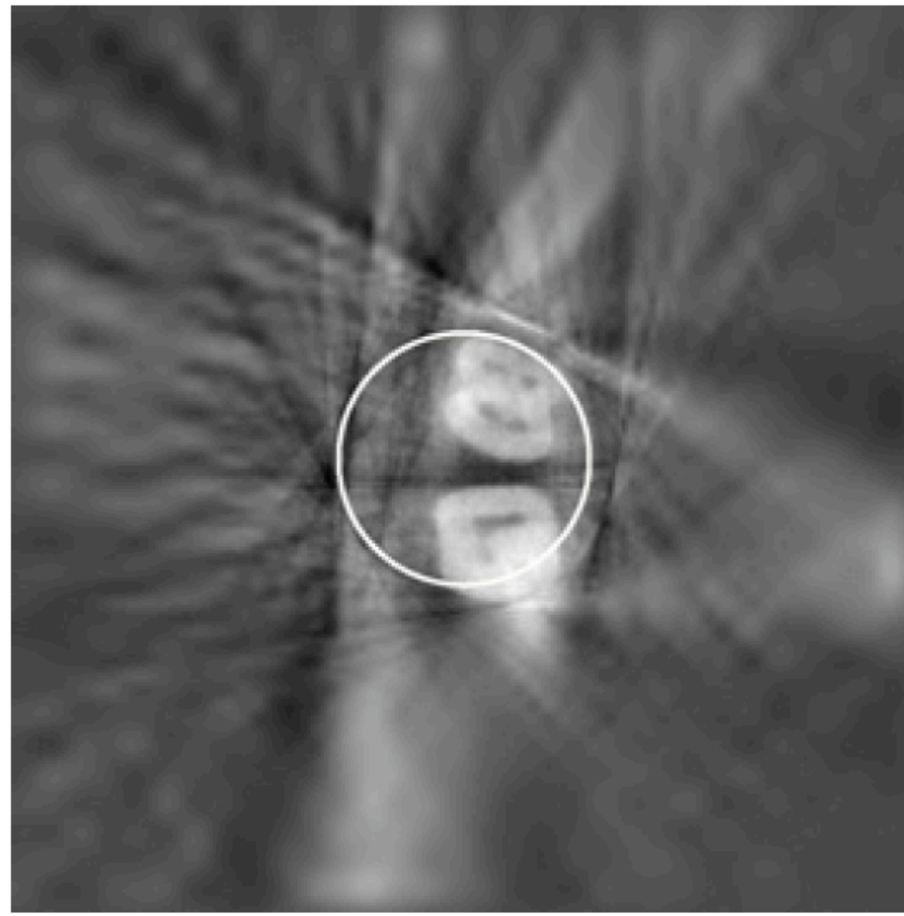
MAP estimate, Besov prior, p=1.5=q and s=0.5

Local tomography results for dental X-ray imaging; data from dry mandible (jawbone)

All 6 wavelet scales outside ROI



Only coarsest wavelets outside ROI

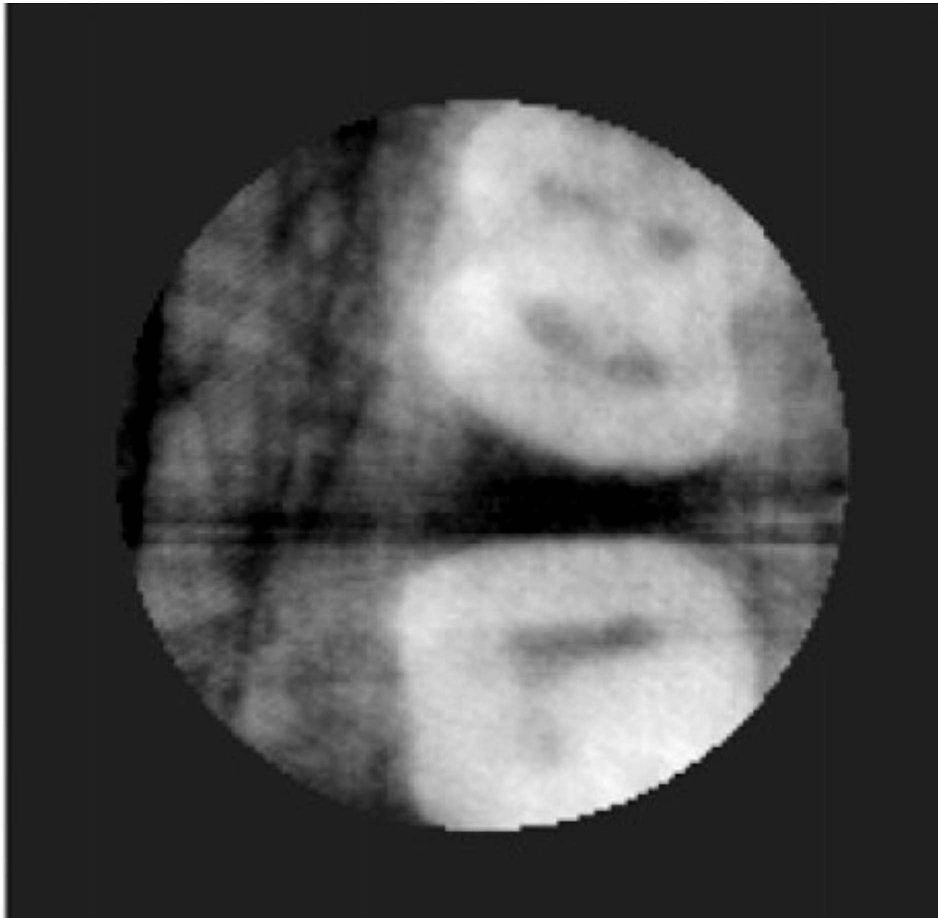


Niinimäki, S & Kolehmainen (2007); Vänskä, Lassas & S (2009); US patent 7215730

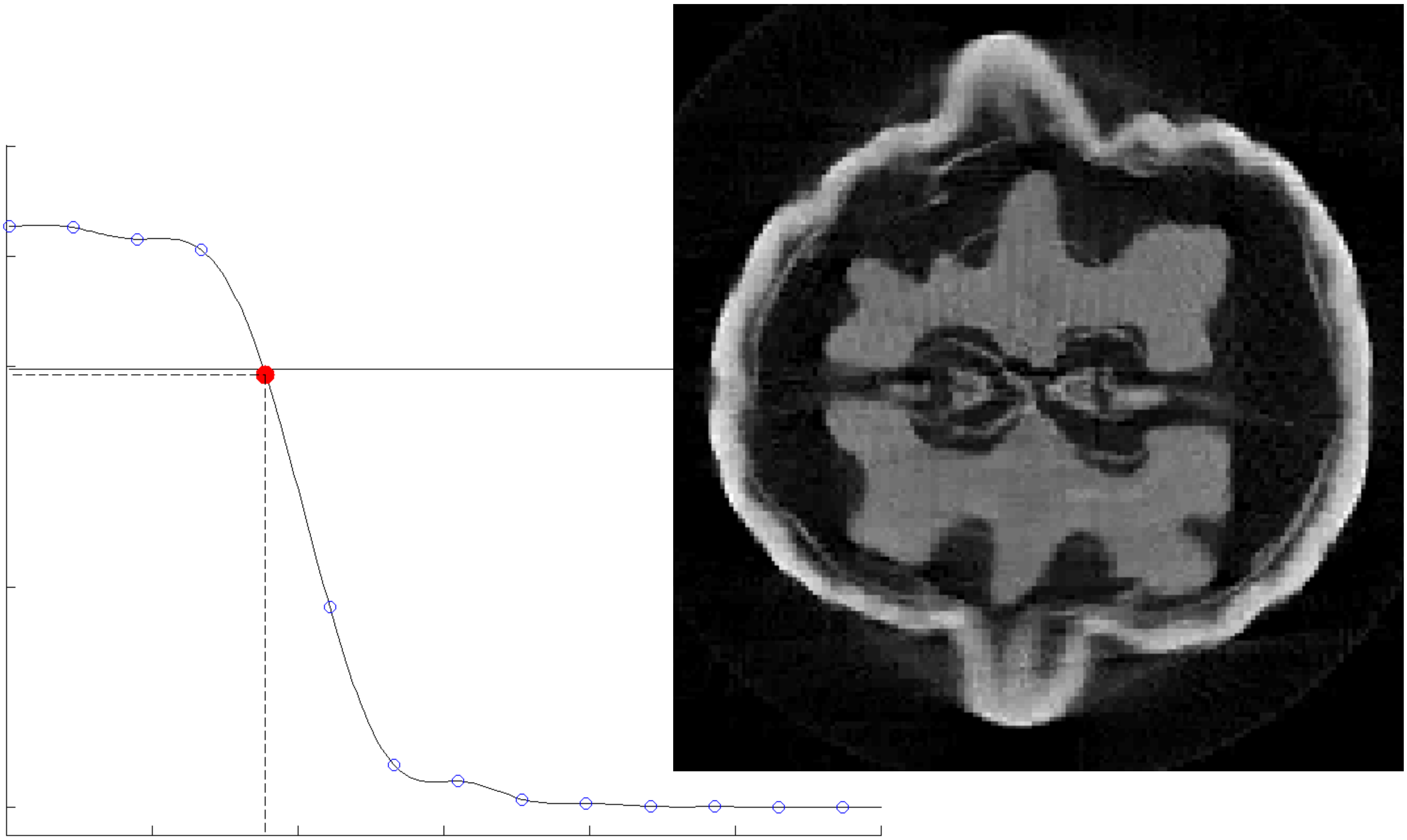
Comparison of our local tomography results with Lambda-tomography

Lambda-tomography

Besov prior, p=q=1.5 and s=0.5



This is a preliminary example of X-ray tomography using the B111 prior



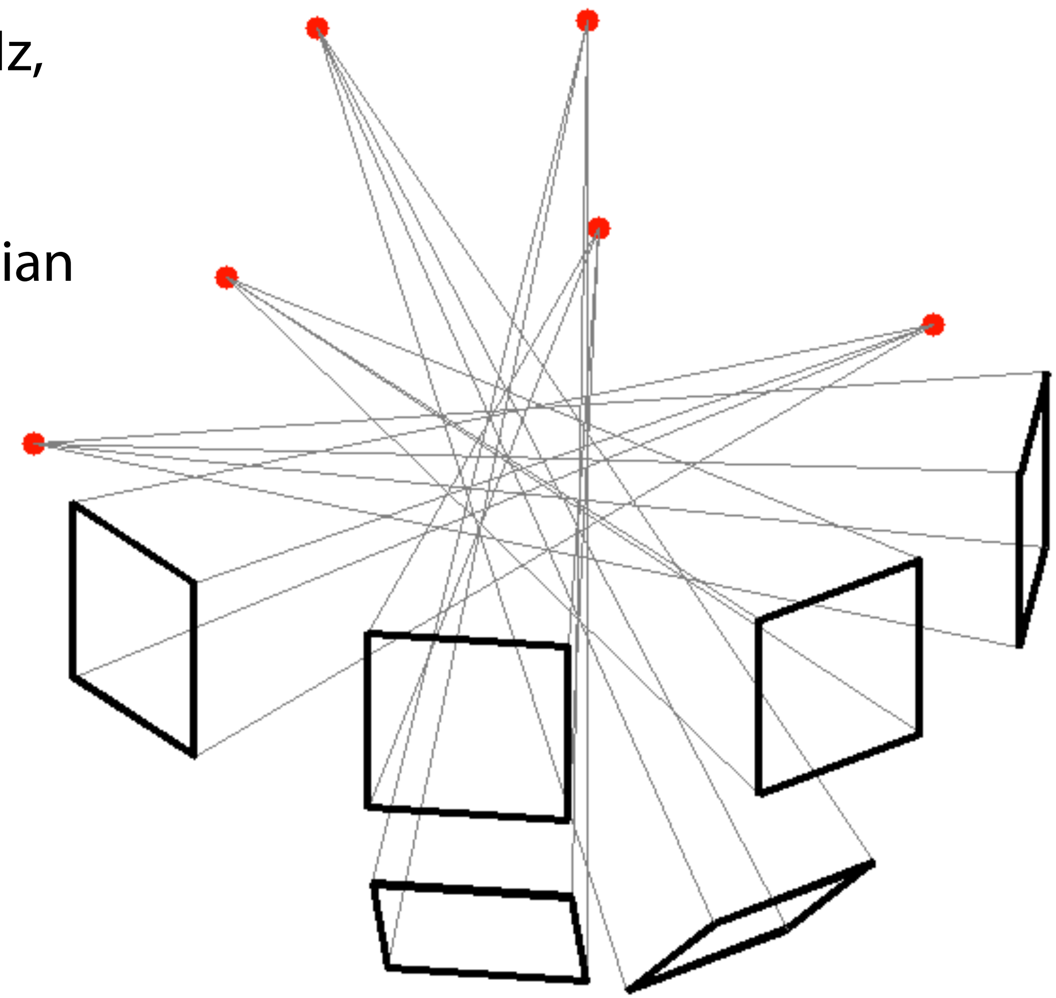
Combining several source-detector pairs enables 4-dimensional X-ray imaging

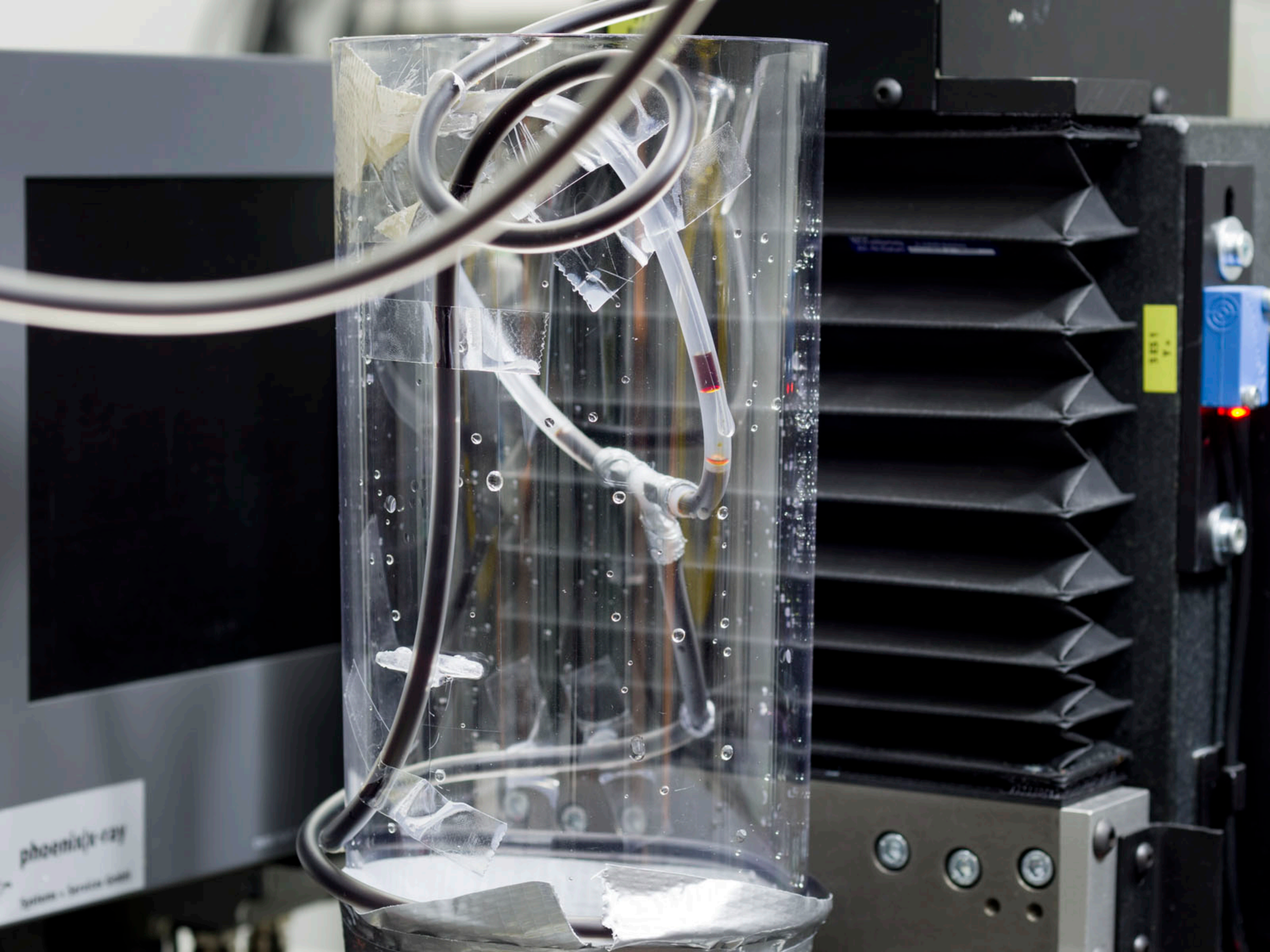
X-ray detectors are available with frame rates up to 200 Hz, providing dynamic data.

We can compute a 3D Bayesian estimate for each time; there are no moving parts.

Applications include
-cardiac imaging
-**angiography**
-dental cone-beam imaging
-veterinary medicine
-non-destructive testing

Patent pending

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