STATISTICAL MECHANICS - EXERCISE 6

Note: Exercise session is on Friday the 16th at 10.15 in C129.

1. Prove Wick's theorem.

2. Prove that for $d \geq 3$ and $r = 0$, there is some constant c_d (find it) so that

$$(-\Delta)^{-1}_{x,y} = \frac{c_d}{|x-y|^{d-2}}(1 + o(1)), \tag{1}$$

as $|x - y| \to \infty$, where Δ is the discrete Laplacian.

3. Consider a graph with vertices $x, y, x_1, ..., x_n$, simple edges $\{x, x_1\}$, $\{x_1, x_n\}$ and $\{x_n, y\}$ and double edges from x_i to x_{i+1} (so there's a line with a chain of bubbles attached to it). Calculate the value of this graph and calculate the sum of all such graphs.

4. Show that for a general graph in momentum space (fourier space), the number of integrals we are left after getting rid of the δ-functions is the number of independent cycles in the graph.

5. Show that

$$\int \frac{1}{(q-p)^2 q^2} dq \sim \begin{cases} \mathcal{O}(1), d \geq 5 \\ \log |p|, d = 4, \\ |p|^{d-4}, d \leq 3 \end{cases} \tag{2}$$

as $p \to 0$.

6. Show that the $\lambda, r \to \infty$ with $\frac{\lambda}{r}$ fixed limit of the Ginzburg-Landau model is the Ising model. More precisely, consider the generating function for the correlation functions of the GL-model in finite volume:

$$Z(h) = \int \prod_{x \in \Lambda_L} d\phi_x e^{-H_{GL}(\phi)} e^{-\sum_x \phi_x h_x} \tag{3}$$

and show that after suitably rescaling Z (and h and ϕ), one gets the generating function for the correlation functions of the Ising model.

Hint: You might want to make use of a Gaussian approximation to the Dirac δ function.