**STATISTICAL MECHANICS - EXERCISE 5**

**This exercise session will be on THURSDAY the 15th in place of the lectures.**

**1.** Calculate $\langle \phi_x \phi_y \rangle_\lambda$ explicitly for the Ginzburg-Landau model in terms of a graphical expansion in connected graphs up to order $\lambda^2$ (so calculate everything and show that only connected graphs contribute and calculate the symmetry factors).

**2.** For $V(\phi) = \sum_x \phi_x^4$, show that

$$\frac{d^n}{d\lambda^n}\bigg|_{\lambda=0} \log \int e^{-\lambda V(\phi)} d\mu_G(\phi) = \langle V; ...; V \rangle \tag{1}$$

and that this correlation function only contains connected graphs in its expansion.

**3.** Prove that

$$G_4^c(x_1,x_2,x_3,x_4) := G_4(x_1,x_2,x_3,x_4) - G_2(x_1,x_2)G_2(x_3,x_4) - G_2(x_1,x_3)G_2(x_2,x_4) - G_2(x_1,x_4)G_2(x_2,x_3) \tag{2}$$

has only connected graphs in its perturbation expansion.

**4.** Let

$$F(\lambda) = \int_{-\infty}^{\infty} e^{-\lambda\phi^4 - \phi^2} d\phi. \tag{3}$$

Show that this defines an analytic function in $\mathbb{C} \setminus (-\infty, 0]$ and that the function has an essential singularity at $\lambda = 0$, but it is still $C^\infty$ at the origin. Moreover, estimate the error made when approximating the function by its Taylor polynomial around zero. Try to use this to motivate why in some cases, physicists get extraordinarily good results acting as though the graphical expansion was a convergent one.

**5.** Derive graphical rules for a model where the interaction $\phi^4$ is replaced by $\phi^n$ for non-negative integer $n$.