STATISTICAL MECHANICS - EXERCISE 2

1. Show that in the high temperature case (i.e. small enough β), for any $A \subset \mathbb{Z}^d$, the correlation function $\langle \sigma_A \rangle_\Lambda^{\bar{\sigma}}$ converges as we take $\Lambda \to \mathbb{Z}^d$ (take the limit along cubes for simplicity: let $\Lambda = \{-L..., L\}^d$ and $L \to \infty$).

Hint: Proceed as when proving uniqueness of the limit: estimate $|\langle \sigma_A \rangle_\Lambda^{\bar{\sigma}} - \langle \sigma_A \rangle_{\Lambda'}^{\bar{\sigma}}|$ by duplicating the summation variable and show that we are dealing with a Cauchy sequence.

2. Prove clustering in the high temperature case: for small enough β and for any $X, Y \subset Z^d$

$$|\langle \sigma_X \sigma_Y \rangle_\Lambda^{\bar{\sigma}} - \langle \sigma_X \rangle_\Lambda^{\bar{\sigma}} \langle \sigma_Y \rangle_\Lambda^{\bar{\sigma}}| \leq C e^{-\alpha \mathrm{dist}(X,Y)} \tag{1}$$

for some $\alpha, C > 0$ independent of Λ and $\bar{\sigma}$.

3. Let $\Lambda \subset \mathbb{Z}^d$ and $x \in \Lambda$. Let us assume that we have some configuration of contours in Λ. For a path P from x to Λ^C, define $N(P)$ to be the number of times the path crosses a contour. Show that for any two such paths P and P', $N(P) - N(P')$ is an even number.

4. What is the appropriate way to define a contour when $d \geq 3$? Prove Lemma 4.4 and relations (4.7) and (4.8) from the lecture notes in the case that $d \geq 3$.