where

$$\mathcal{W}(s_V^i) = \prod_J \lambda^{2|J|} \prod_{i=1,2} W(s_-^i)W(s_+^i) \min_{s_-^i, s_+^i} (1 + g_J(s_-^i, s_+^i)). \tag{37}$$

The product over J runs over all the time intervals of $\Lambda \backslash V$ (so that the variables $s_\pm^1$, $s_\pm^2$ are indexed by sites in V).

Let us insert this representation of $Z^0(\Lambda\backslash V|s_V^i)$ in (35) and then expand the product over J of $(1 + \tilde{g}_J(s_-^i, s_+^i))$. The result is:

$$(35) = Z(\Lambda)^{-2} \sum_{\mathcal{X},\mathcal{J}} \sum_{s_V^i, i=1,2} \tilde{F}\tilde{G} \prod_{X \in \mathcal{X}} f_X \prod_{J \in \mathcal{J}} \tilde{g}_J \exp(-\mathcal{H}_V^0)\mathcal{W}(s_V^i) \tag{38}$$

where the sum over $\mathcal{J}$ runs over families of intervals $J \subset \Lambda\backslash V$.

From now on, we can proceed as we did previously: the main observation is again that if, for each term in (38), we decompose $V \cup (\cup_{\mathcal{J}} J)$ into connected components (where connected is defined in an obvious way: any two sets can be joined by a "connected path" $P = (Z_i)_{i=1}^n$ where each Z_i is either an $\underline{X}$ or a J and the distance between Z_{i+1} and Z_i is less than 1, $\forall i = 1, \cdots, n-1$), and if A and B belong to different components, then that term vanishes. Let us check this: as before, we interchange s_α^1 and s_α^2 for each α in the connected component containing A. $\tilde{F}$ is odd under such an interchange, while $\tilde{G}$ is even (if A and B belong to different components), and f_X, $\tilde{g}_J$ are obviously even. Next, observe that $\mathcal{W}(s_V^i)$ can be factorized into a product of functions, each of which depends only of s_α^i for α belonging to a connected component of V. Observe also that $\mathcal{H}_V^0$ does not contain terms where X intersects different connected components of V (since Φ_0 has range R and V is defined as a union of R-intervals). So, the two last factors in (38) factorize over connected components of V, and are therefore also even under our interchange of s_α^1 and s_α^2.

Hence, for each non-zero term in (38), we can choose a connected path, as defined above, $P = (Z_i)_{i=1}^n$ where $Z_1 = \underline{A}, Z_n = \underline{B}$. Then, using the positivity of f_X, $\tilde{g}_J$, we bound the sum in (38) by a sum over such paths, and control that sum essentially as in (9). The exponential decay comes from combining (34), when Z_i is an $\underline{X}$ and (36) when Z_i is a J. The uniqueness of the Gibbs state is proven as in (18); for details, see [4]. □

3 SRB measures for expanding circle maps.

We start by recalling the standard theory of invariant measures for smooth expanding circle maps, in a formulation that will be used later. To describe the dynamics, we first fix a map $F : S^1 \to S^1$. We take F to be an expanding, orientation preserving $C^{1+\delta}$ map with $\delta > 0$ (i.e. F is differentiable and its derivative is Hölder continuous of exponent δ). We describe F in terms of its lift to $\mathbf{R}$, denoted by f and chosen, say, with $f(0) \in [0, 1[$. We assume that

$$f'(x) > \lambda^{-1} \tag{1}$$

where $\lambda < 1$. Note that there exists an integer $k > 1$ such that

$$f(x+1) = f(x) + k \quad \forall x \in \mathbf{R}. \tag{2}$$

A probability measure μ on S^1 is called an SRB measure if it is F-invariant and absolutely continuous with respect to the Lebesgue measure. The following results are well-known for maps F as above (see [28, 7, 20]):

(a) There is a unique SRB measure μ.

(b) For any absolutely continuous probability measure ν, and any continuous function G,

$$\int G \circ F^N d\nu \to \int G d\mu \tag{3}$$

as $N \to \infty$.

(c) There exists $C < \infty$, $m > 0$, such that $\forall G \in L^\infty(S^1), \forall H \in \mathcal{C}^\delta(S^1)$,

$$|\int G \circ F^n H d\mu - \int G d\mu \int H d\mu| \leq C\|G\|_\infty \|H\|_\delta e^{-mn}, \tag{4}$$

where $\mathcal{C}^\delta(S^1)$ denotes the space of Hölder continuous functions, with the norm

$$\|H\|_\delta = \|H\|_\infty + \sup_{x,y} \frac{|H(x) - H(y)|}{|x-y|^\delta}.$$

Remark. There are different ways to define an SRB measure. In [11], they are introduced as measures whose restriction on the expanding directions is absolutely continuous with respect to the Lebesgue measure. Since here the whole phase space S^1 is expanding, our definition is natural (besides, with this definition, the SRB measure is unique). But, as we mentioned in the introduction, one of the most interesting properties of the SRB measure is that it describes the statistics of the orbits of almost every point, which means that

$$\frac{1}{N} \sum_{i=0}^{N-1} G \circ F^i(x) \to \int G d\mu, \tag{5}$$

for almost every x, and every continuous function G. Note that, if we integrate (5) with an absolutely continuous probability measure ν, we obtain the Cesaro average of (3). So, both properties are related to each other.

Let us sketch now the construction of μ. In doing so, we shall establish the connection with the statistical mechanics of one-dimensional spin systems. This way of constructing μ may not be the simplest one in the present context, but the connection to statistical mechanics will be essential in the analysis of coupled maps (see [28, 7, 20] for different approaches, although the one below is close to [28]).

The Perron-Frobenius operator P for F is defined by

$$\int G \circ F H dm = \int G P H dm \tag{6}$$

for $G \in L^\infty(S^1)$, $H \in L^1(S^1)$, and dm being the Lebesgue measure. Let us work in the covering space $\mathbf{R}$ and replace G, H by periodic functions denoted $g, h : g(x+n) = g(x)$, $\forall n \in \mathbf{Z}$. We get

$$\int_{[0,1]} g \circ f h dx = \int_{[0,1]} g(x) Ph(x) dx. \tag{7}$$

More explicitely,

$$Ph(x) = \sum_s \frac{h(f^{-1}(x+s))}{f'(f^{-1}(x+s))} \tag{8}$$

where $s \in \{0, \cdots, k-1\}$ (and k was introduced in (2)). Note that P maps periodic functions into periodic functions because the sum is periodic even if the summands are not: indeed, (2) implies that f' is periodic and that $f^{-1}(x+1+k-1) = f^{-1}(x+k) = f^{-1}(x)+1$ (so that, if we add 1 to x, it amounts to a cyclic permutation of s).

By (7), the density $h_\mu(x)$ of the absolutely continuous invariant measure $d\mu = h_\mu(x)dx$ satisfies $Ph_\mu = h_\mu$. We shall construct h_μ as the limit, as $N \to \infty$, of $P^N 1$. $P^N 1$ has a direct statistical mechanical interpretation which we now derive.

First, iterating (8), we get

$$(P^N 1)(x) = \sum_{s_1, \cdots, s_N} \prod_{t=1}^N [f'(f_{s_t}^{-1} \circ \cdots \circ f_{s_1}^{-1}(x))]^{-1} \tag{9}$$

where $f_s^{-1}(x) \equiv f^{-1}(x+s)$.

From now on, we shall consider $x \in [0,1]$. We introduce a convenient notation: $x \in [0,1]$ and $s_1, \cdots, s_N$ in (9) collectively define a configuration on a lattice $\{0, \cdots, N\}$. To any subset $X \subset \mathbf{Z}_+$ associate the configuration space $\Omega_X = \times_{t \in X} \Omega_t$ where Ω_t equals [0,1] if $t = 0$, and equals $\{0, \cdots, k-1\}$ if $t > 0$. We could use the existence of a Markov partition for F to write x as a symbol sequence, as is usually done, e.g. in [6], but we shall not use explicitely this representation.

Let $\mathbf{s} = (x, s_1, \cdots, s_N) \in \Omega_N$. Then (9) reads

$$(P^N 1)(x) = \sum_{s_1 \cdots s_N} e^{-\mathcal{H}_N(\mathbf{s})} \tag{10}$$

with $e^{-\mathcal{H}_N(\mathbf{s})}$ being the summand in (9). And we want to construct the limit:

$$\int_{[0,1]} g(x) d\mu = \lim_{N \to \infty} \int_{[0,1]} g(x)(P^N 1)(x) dx = \lim_{N \to \infty} \sum_{s_1 \cdots s_N} \int_{[0,1]} g(x) e^{-\mathcal{H}_N(\mathbf{s})} dx \tag{11}$$

for any continuous function g.

This is the statistical mechanical representation we want to use. In that language, $d\mu$ is the restriction to the "time zero" phase space of the Gibbs state determined by $\mathcal{H}$. One can also rewrite the time correlation functions (3) as follows: let $d\nu = h_\nu(x)dx$; then, replacing again G by a periodic function g and F by its lift,

$$\int_{[0,1]} g \circ f^N h_\nu dx = \int_{[0,1]} g P^N h_\nu dx = \sum_{(s_i)_{i=1}^N} \int_{[0,1]} g(x) \exp(-\mathcal{H}_N(\mathbf{s})) h_\nu(\mathbf{s}) dx \tag{12}$$

where

$$h_\nu(\mathbf{s}) = h_\nu(f_{s_N}^{-1} \circ \cdots \circ f_{s_1}^{-1}(x)),$$

and the last equality in (12) follows by iterating (8).

On the other hand, since $\int Phdx = \int hdx$, by definition of P (use (7) with $g = 1$), one has

$$\sum_{(s_i)_{i=1}^N} \int_{[0,1]} \exp(-\mathcal{H}_N(\mathbf{s}))h_\nu(\mathbf{s})dx = \int_{[0,1]} P^N h_\nu dx = \int_{[0,1]} h_\nu dx = 1, \tag{13}$$

since $d\nu$ is a probability measure. So, using (11, 12), one sees that (3) is translated, in the statistical mechanics language, into

$$\begin{aligned}
& \lim_{N\to\infty} (\int G \circ F^N d\nu - \int G d\mu) \\
= \ & \lim_{N\to\infty} (\int_{[0,1]} gP^N h_\nu dx - \int_{[0,1]} gP^N 1 dx) \\
= \ & \lim_{N\to\infty} (\sum_{(s_i)_{i=1}^N} \int_{[0,1]} g(x) e^{-\mathcal{H}_N(\mathbf{s})} h_\nu(\mathbf{s}) dx \\
- \ & (\sum_{(s_i)_{i=1}^N} \int_{[0,1]} g(x) e^{-\mathcal{H}_N(\mathbf{s})} dx)(\sum_{(s_i)_{i=1}^N} \int_{[0,1]} e^{-\mathcal{H}_N(\mathbf{s})} h_\nu(\mathbf{s}) dx)) \\
= \ & 0
\end{aligned} \tag{14}$$

where we used (13) to insert the last factor (which equals one). We shall see below that the last equality expresses the decay of correlation functions for the Gibbs state determined by $\mathcal{H}$. A similar observation holds for (4).

One advantage of these representations is that one may use the statistical mechanics formalism to control the limit. To make the connection with statistical mechanics more explicit, it is convenient to write $\mathcal{H}_N$ in terms of many-body interactions. First of all, from (9, 10), we get

$$\mathcal{H}_N = \sum_{t=1}^N V_t \tag{15}$$

with

$$V_t(\mathbf{s}) = \log(f'(f_{s_t}^{-1} \circ \cdots \circ f_{s_1}^{-1}(x))). \tag{16}$$

We may localize V_t by writing it as a telescopic sum:

$$V_t(\mathbf{s}) = \sum_{l=0}^t \Phi_{[l,t]}(\mathbf{s}) + V_t(\mathbf{0}_{[0,t]}) \tag{17}$$

where, for $l \neq 0, t$,

$$\Phi_{[l,t]}(\mathbf{s}) = V_t(\mathbf{s}_{[l,t]} \vee \mathbf{0}_{[0,l-1]}) - V_t(\mathbf{s}_{[l+1,t]} \vee \mathbf{0}_{[0,l]}) \tag{18}$$

and $\mathbf{0}_{[0,l]}$ denotes the configuration equal to 0 for all $i \in [0, l]$ (note that 0 belongs to the phase space for all i's). For $l = 0, t$, $\Phi_{[l,t]}(\mathbf{s})$ is given by a similar formula, where the intervals that would appear in (18) as $[0, -1]$ and $[t+1, t]$ are replaced by the empty set.

Combining (15, 17), we may write the Hamiltonian as a sum of many-body interactions:

$$\mathcal{H}_N(\mathbf{s}) = \sum_{t=1}^{N} \sum_{l=0}^{t} \Phi_{[l,t]}(\mathbf{s}) + C \tag{19}$$

where the constant $C = \sum_{t=1}^{N} V_t(\mathbf{0}_{[0,t]})$.

The main point of (19) is that $\Phi_{[l,t]}(\mathbf{s})$ depends on $\mathbf{s}$ only through $\mathbf{s}_{[l,t]}$. In the statistical mechanics language, these are many-body interactions coupling all the variables in the interval $[l, t]$. The next Proposition shows that these interactions decay exponentially with the size of the interval $[l, t]$.

Proposition 2 *There exists $C < \infty$, such that*

$$|\Phi_{[l,t]}(\mathbf{s})| \leq C\lambda^{\delta(t-l)}. \tag{20}$$

Proof. This combines two bounds: First, since F is $C^{1+\delta}$, one has

$$|\log f'(x) - \log f'(y)| \leq C|x - y|^{\delta} \tag{21}$$

and, by (1),

$$|f_s^{-1}(x) - f_s^{-1}(y)| \leq \lambda |x - y|. \tag{22}$$

Then, iterating (22), one gets:

$$|f_{s_t}^{-1} \circ \cdots \circ f_{s_{l+1}}^{-1}(x) - f_{s_t}^{-1} \circ \cdots \circ f_{s_{l+1}}^{-1}(y)| \leq \lambda^{t-l}$$

since $|x - y| \leq 1$. Then (20) follows from this and (21), since the s variables in both terms of (18) (see (16)) coincide in the first $t - l$ places. □

We can formulate the system here in the language of Section 2 as follows: we write the interaction as the sum of an interaction Φ^0 of finite range R, which does not necessarily have a small norm plus a long range "tail" Φ^1 whose norm can be made as small as we wish by choosing R large enough. Concretely, choose now R to be the smallest integer such that

$$\lambda^{\frac{\delta R}{2}} < \epsilon. \tag{23}$$

Then, we define Φ^0 as grouping all the $\Phi_{[l,t]}$'s with $t - l \leq R$ and Φ^1 to collect all the longer range $\Phi_{[l,t]}$'s. Since, for an interval $X = [l, t]$, $d(X) = t - l$, we easily have (for all $s \in \mathbf{Z}_+$) the bound:

$$\sum_{X \ni s} e^{\gamma d(X)} \|\Phi_X^1\| \leq C\epsilon \tag{24}$$

for γ small enough (e.g. so that $e^{\gamma} \leq \lambda^{\delta/2}$) , and where C depends on λ^{δ}. Note also that, here, $d(X) = |X| - 1$, so that, in this one-dimensional situation, the norms (2.24) and (2.25) are equivalent. However, we cannot use Theorem 1 directly, because the way this Theorem is stated, ϵ depends on Φ^0, i.e. on R, and, here, we choose R in (23) in an ϵ-dependent way. It turns out that all we would need in the proof of Theorem 1 is that $R^{d+1}\epsilon$ is small enough (with $d = 0$ here), and that is compatible with (23), for R large (see [4] for details). Of course, in this example, one can also apply directly the transfer matrix formalism to infinite-range interactions decaying as in (20), see [25].

In order to prove the decay of the correlation functions (3,4) one proceeds as follows. First, note that we can approximate the L^1 function h_ν, in the L^1 norm, by a smoother function, $\tilde{h}_\nu$, e.g. by a Hölder continuous function of exponent δ. Since G in (3,14) is bounded, this means that we have the following approximation, uniformly in N:

$$|\int G \circ F^N h_\nu dx - \int G \circ F^N \tilde{h}_\nu dx| \leq \|G\|_\infty \|h_\nu - \tilde{h}_\nu\|_1.$$

So, it is enough to prove (14) when h_ν is Hölder continuous.

Thus, we may write a telescopic sum, as in (17):

$$h_\nu(\mathbf{s}) = \sum_{l=0}^{N} h_{\nu,[l,N]}(\mathbf{s}) + h_\nu(\mathbf{0}_{[0,N]}) \tag{25}$$

and one has an exponential decay of the form:

$$|h_{\nu,[l,N]}(\mathbf{s})| \leq C\lambda^{\delta(N-l)} \tag{26}$$

as in Proposition 2, since h_ν is Hölder continuous. Now insert (25) in (14), observe that $g(x)$ depends only on the "time zero" variable x, while $h_{\nu,[l,N]}(\mathbf{s})$ depends only on $\mathbf{s}_{[l,N]}$. Since (14) has the form of correlation function, we can use the exponential decay of the Gibbs state determined by $\mathcal{H}$, i.e. (2.26) with $A = 0$ and $B = [l, N]$, hence $d(A, B) = l$. Combining this exponential decay with the exponential decay of $h_{\nu,[l,N]}(\mathbf{s})$ and with

$$\sum_{l=0}^{N} e^{-ml}\lambda^{\delta(N-l)} \leq Ce^{-m'N} \tag{27}$$

for $m' < \min(m, \delta|\log\lambda|)$, one proves (3).

If h_ν is not Hölder continuous, the limit in (3) is still reached (via our approximation argument), but not necessarily exponentially. The proof of (4) is similar. One sees also why, in (4), one requires H to be Hölder continuous, while G is only bounded: in order to prove exponential decay, we had to use (25, 26).

4 Coupled map lattices.

We consider now a lattice of coupled expanding circle maps. The phase space $\mathcal{M} = (S^1)^{\mathbf{Z}^d}$ i.e. $\mathcal{M}$ is the set of maps $\mathbf{z} = (z_j)_{j\in\mathbf{Z}^d}$ from $\mathbf{Z}^d$ to the circle.

To describe the dynamics, we first consider a map $F : S^1 \to S^1$ as in Section 3. We let $\mathcal{F} : \mathcal{M} \to \mathcal{M}$ denote the Cartesian product $\mathcal{F} = \mathrm{X}_{i\in\mathbf{Z}^d} F_i$ where F_i is a copy of F. $\mathcal{F}$ is called the uncoupled map.

The second ingredient in the dynamics is given by the coupling map $A : \mathcal{M} \to \mathcal{M}$. This is taken to be a small perturbation of the identity in the following sense. Let A_j

be the projection of A on the j$^{\text{th}}$ factor and let a_j denote the lift of A_j : $A_j = e^{2\pi i a_j}$. We take, for example,

$$a_j(\mathbf{x}) = x_j + \epsilon \sum_k g_{|j-k|}(x_j, x_k)$$

where g is a periodic $C^{1+\delta}$ function in both variables, with exponential falloff in $|j-k|$. We shall come back later on the reasons for considering this somewhat unusual model (see Remark 3 below). More general examples of such $A's$ can be found in [4, 3] (note, however, that in [3], we restricted ourselves to analytic maps).

The coupled map $T : \mathcal{M} \to \mathcal{M}$ is now defined by

$$T = A \circ \mathcal{F}.$$

We are looking for "natural" T-invariant measures on $\mathcal{M}$. For this, write, for $\Lambda \subset \mathbf{Z}^d$, $\mathcal{M}_\Lambda = (S^1)^\Lambda$, and let m_Λ be the product of Lebesgue measures.

Definition 1 *A Borel probability measure μ on $\mathcal{B}$ is a SRB measure if*

- *(a) μ is T-invariant*
- *(b) The restriction μ_Λ of μ to $\mathcal{B}_\Lambda$ is absolutely continuous with respect to m_Λ for all $\Lambda \subset \mathbf{Z}^d$ finite.*

Remark 1. This is a natural extension to infinite dimensions of the notion of SRB measure, given in section 3, since each S^1 factor can be regarded as an expanding direction. However, unlike the situation for single maps, we do not show that the SRB measure is unique (although we expect it to be so). In [4], we prove a weaker result, namely that there is a unique "regular" SRB measure. We also show that (3.3) holds for ν being a "regular" measure, but we have not extended (3.5). The extension of (3.4) is given below.

Our main result is:

Theorem 2 *Let F and A satisfy the assumptions given above. Then there exists $\epsilon_0 > 0$ such that, for $\epsilon < \epsilon_0$, T has an SRB measure μ. Furthermore, μ is invariant and exponentially mixing under the space-time translations: there exists $m > 0$, $C < \infty$, such that, $\forall B, D \subset \mathbf{Z}^d, |B|, |D| < \infty$ and $\forall G \in L^\infty(\mathcal{M}_B), \forall H \in \mathcal{C}^\delta(\mathcal{M}_D)$,*

$$|\int G \circ T^n H d\mu - \int G d\mu \int H d\mu| \leq C \|G\|_\infty \|H\|_\delta e^{-m(n+d(B,D))}, \tag{1}$$

where $d(B, D)$ is the distance between B and D and C depends on $d(B), d(D)$.

Remark 2. The proof combines the ingredients from the previous two sections. We first derive a formula for the Perron-Frobenius operator of T which is similar to (3.9, 3.10). And we express the Hamiltonian in terms of potentials as in (3.19), using a telescopic sum. The decay of the potentials is proven again using the Hölder continuity of the differential of T and the expansivity of F. We may write the potential Φ as a sum of two terms, as in Theorem 1, $\Phi^0 + \Phi^1$, with Φ^0 one-dimensional and of finite range and $\|\Phi^1\|_2$ small, but on a "space-time" $\mathbf{Z}_+^{d+1}$ lattice. And, using Theorem 1 (which can trivially be extended to this lattice), we construct the SRB measure and prove the exponential decay of correlations. For a discussion of previous work on this problem, see [4].

Remark 3. One would like to extend this Theorem to coupled maps of the interval $[0, 1]$ into itself, where the uncoupled map is not smooth, but, say, of bounded variation. Indeed, all examples were phase transitions are expected to occur are of this form (see e.g. [22, 24]). Moreover, the theory for a single map can easily be extended to maps of bounded variation [7]. Also, one would like to consider more general couplings A, like the standard diffusive coupling.

However, such extensions seem rather difficult, because even if the uncoupled map happens to have a Markov partition, the couplings tend to destroy these partitions. This is basically the reason for considering circle maps instead of expanding maps of the interval. We did not use explicitely the existence of a Markov partition, but we used it implicitely because no characteristic functions appeared in the formula (3.9) for the Perron-Frobenius operator (compare with the formula for P in [7]). The reader should not be misled by the fact that, in the statistical mechanics part of the argument (Section 2), we could handle a general transitive matrix $\mathcal{A}$, defining a subshift. Indeed this is a short-range hard-core interaction, in the statistical mechanics language, while the appearance of characteristic functions in the Perron-Frobenius operator may give rise to an infinite range hard core, and this is much more difficult to control.

Note, however, that existence results on SRB measures in this more general context were obtained in [21]. But there are no results on the exponential decay of correlation functions. Also, Blank has constructed examples of "pathological" behaviour for coupled non-smooth maps with arbitrarily weak coupling [1].

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