Time-frequency analysis — Winter 2012

1. Exercise set

1.1. Sketch the graph of the function w_7 (the seventh Walsh function) on the interval $[0,1)$.

1.2. Prove that $\displaystyle\int_0^1 w_n w_m = \delta_{mn} := \begin{cases} 1 & m = n, \\ 0 & m \neq n. \end{cases}$

1.3. A partial order for tiles $P = I_P \times \omega_P$ is defined as follows: we say that $P \leq P'$ if both $I_P \subseteq I_{P'}$ and $\omega_P \supseteq \omega_{P'}$. (No typo: the inclusions of time and frequency intervals are in different directions on purpose.) We say that two tiles are comparable if $P \leq P'$ or $P' \leq P$.

Prove that two tiles P, P' are comparable if and only if $P \cap P' \neq \varnothing$.

1.4. Let $\mathbb{T}$ be a collection of bitiles. We say that $\mathbb{T}$ is an *up-tree* if there exists a bitile T such that $P_u \leq T_u$ for all $P \in \mathbb{T}$.

Let $\mathbb{T}$ be an up-tree of bitiles. If $P, P' \in \mathbb{T}$ are two different bitiles, show that $P_d \cap P'_d = \varnothing$.

1.5. Let $P = I \times \omega$, and $P_i = I_i \times \omega^{(1)}$ for $i = 1, 2$, where I_0 and I_1 are the children (the left and right halves) of I, and $\omega^{(1)}$ is the parent of ω (the next larger dyadic interval containing ω). Prove that $w_P \in \operatorname{span}\{w_{P_0}, w_{P_1}\}$.

1.6. For two tiles P, P', prove that $\int_{\mathbb{R}_+} w_P w_{P'} = 0$ if and only if $P \cap P' = \varnothing$.