1. $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 - 1 < xyz < \sin(1+y)\}$ avoin :
$f : \mathbb{R}^3 \to \mathbb{R}$, $g : \mathbb{R}^3 \to \mathbb{R}$
$f(x,y,z) = xyz - x^2 + 1$, $g(x,y,z) = \sin(1+y) - xyz$

$\Rightarrow A = f^{-1}\,]0,\infty[\, \cap\, g^{-1}\,]0,\infty[$
merk. $u := pr_1$, $v := pr_2$, $w := pr_3$
$\Rightarrow f$ jatk. sillä $f = uvw - u^2 + 1$
g jatk. sillä $g = \sin \circ (1+v) - uvw$

2. $f, g : \mathbb{R} \to \mathbb{R}$ jatk.
$h : \mathbb{R}^2 \to \mathbb{R}^2$, $h(x,y) = (f(x), g(y))$ jatk. :

$h_1(x,y) = f(x)$, $h_2(x,y) = g(y)$

$h_1 = f \circ u$, $h_2 = g \circ v$, missä $u := pr_1$, $v := pr_2$

h_1, h_2 jatk. $\Rightarrow h$ jatk.

3. $f(x) = x \cdot a$, $f : E \to \mathbb{R}$ Lipschitz :

$|f(x) - f(y)| = |x \cdot a - y \cdot a| = |(x-y) \cdot a| \overset{\text{Schwarz}}{\leq} \|x-y\| \, \|a\| \quad \forall x,y \in E$

$\Rightarrow f$ $\|a\|$-Lipschitz

4. E normiav., $I = [0,1]$, $f, g : I \to E$ jatk.
$\Rightarrow h : I^2 \to E$, $h(s,t) = (1-t) f(s) + t g(s)$ jatk.

todl.
merk. $u(s,t) = s$, $v(s,t) = t$, $(s,t) \in I^2$
$\Rightarrow u, v : I^2 \to \mathbb{R}$ jatk.
nyt
$h = (1-v)(f \circ u) + v(g \circ u)$ jatk., sillä
$1-v, v : I^2 \to \mathbb{R}$ jatk. ja $f \circ u, g \circ u : I^2 \to E$ jatk. $\square$

5. $f, g : X \to \mathbb{R}$ jatk. $\Rightarrow f \vee g,\ f \wedge g : X \to \mathbb{R}$ jatk. :

$H : X \to \mathbb{R}^2$, $H(x) = (f(x), g(x))$ jatk.

$f \vee g = \max \circ H$, $f \wedge g = \min \circ H$

os. $\max, \min : \mathbb{R}^2 \to \mathbb{R}$ jatk. :

$\min(s,t) = \frac{s+t}{2} - \frac{|s-t|}{2}$, $\max(s,t) = \frac{s+t}{2} + \frac{|s-t|}{2}$

$\Rightarrow \min = \frac{u+v}{2} - \frac{|u-v|}{2}$, $\max = \frac{u+v}{2} + \frac{|u-v|}{2}$ jatk.

huom. u jatk. $\Rightarrow \|u\|$ jatk.

6. $x_0 \in X$

$a \in X$, $f_a : X \to \mathbb{R}$, $f_a(x) = d(x,a) - d(x, x_0)$

(a) $|f_a(x)| = |d(x,a) - d(x,x_0)| \le d(a, x_0) \quad \forall x \in X$

(b) $\varphi : X \to \mathrm{raj}(X, \mathbb{R})$, $\varphi(a) = f_a$

$$\|\varphi(a) - \varphi(b)\| = \|f_a - f_b\| = \sup_{x \in X} |f_a(x) - f_b(x)|$$

$$= \sup_{x\in X} | d(x,a) - d(x,x_0) - d(x,b) + d(x,x_0)|$$

$$= \sup_{x \in X} \underbrace{|d(x,a) - d(x,b)|}_{\le d(a,b)} \le d(a,b)$$

(c) $\|(\varphi(a) - \varphi(b))(a)\| = |f_a(a) - f_b(a)| = |d(a,a) - d(a,x_0) - d(a,b) + d(a,x_0)|$

$= d(a,b)$