Malliteoria

Harjoitus 4

1. Where in the proof of Theorem 5.13 the atomic formula $\top$ was needed?

2. Let $L = \{E\}$, $\#E = 2$, and T_g consist of the following two sentences:

$$\forall v_0 \neg E(v_0, v_0)$$

$$\forall v_0 \forall v_1 (E(v_0, v_1) \rightarrow E(v_1, v_0)).$$

Show that T has AP, JEP and is closed under unions.

3. Let L and T_g be as above. Find a theory T such that the models of T are exactly the existentially closed models of T_g.

4. Let $L = \{+, 0\} \cup \{f_q \mid q \in \mathbf{Q}\}$, where $+$ is a 2-ary function symbol and f_q are 1-ary function symbols and 0 is a constant (instead of $+(t, u)$ we write $t + u$ and insted of $f_q(t)$ we write qt). Let T_{qv} consist of the following sentences:

$$\forall v_0 \forall v_1 \forall v_2 ((v_0 + v_1) + v_2 = v_0 + (v_1 + v_2))$$

$$\forall v_0 \forall v_1 (v_0 + v_1 = v_1 + v_0)$$

$$\forall v_0 \exists v_1 (v_0 + v_1 = 0)$$

$$\forall v_0 (v_0 + 0 = v_0)$$

$$\exists v_0 \neg (v_0 = 0)$$

for all $q, r \in \mathbf{Q}$,

$$\forall v_0 \forall v_1 (q(v_0 + v_1) = qv_0 + qv_1)$$

$$\forall v_0 ((q + r)v_0 = qv_0 + rv_0)$$

$$\forall v_0 ((qr)v_0 = q(rv_0))$$

$$\forall v_0 (1v_0 = v_0).$$

(I.e. the models of T_{qv} are the (non-trivial) vector spaces over $\mathbf{Q}$.) Show that T_{qv} is complete and has elimination of quantifiers.