Malliteoria

Harjoitus 3

1. Using Theorem 10.3 (Ramsay's theorem) and compactness, show that for all $m \in \mathbb{N}$ there is $n \in \mathbb{N}$ such that $n \to (m)^2_2$ (see Definition 10.2).

2. Suppose $A \subseteq \mathcal{A}$ and $B = \{t^{\mathcal{A}}(a) |\ t(x_1, ..., x_n)$ is a term and $a \in \mathcal{A}^n\}$. Show that for all $f \in L$ and $b \in B^{\#f}$, $f^{\mathcal{A}}(b) \in B$.

3. Suppose $dom(\mathcal{B}) \subseteq dom(\mathcal{A})$, for all $R \in L$, $R^{\mathcal{B}} = R^{\mathcal{A}} \cap dom(\mathcal{B})^{\#R}$, for all $f \in L$, $f^{\mathcal{B}} = f^{\mathcal{A}} \restriction dom(\mathcal{B})^{\#f}$ and for all $c \in L$, $c^{\mathcal{B}} = c^{\mathcal{A}}$. Show that for all terms $t(x_1, ..., x_n)$ and $a \in dom(\mathcal{B})^n$, $t^{\mathcal{B}}(a) = t^{\mathcal{A}}(a)$.

4. Suppose $f : \mathcal{B} \to \mathcal{A}$ is a partial isomorphism, $\mathcal{C}$ is the submodel generated by $dom(f)$ (i.e. $dom(\mathcal{C}) = \{t^{\mathcal{B}}(a) |\ t(x_1, ..., x_n)$ is a term and $a \in dom(f)^n\})$ and define $g : \mathcal{C} \to \mathcal{A}$ so that for all terms $t(x_1, ..., x_n)$ and $a \in dom(f)^n$, $g(t^{\mathcal{B}}(a)) = t^{\mathcal{A}}(f(a))$. Show that g is well-defined and an embedding.

5. Prove Lemma 5.4.