## Malliteoria

Harjoitus 11

1.

- (i) Show that if  $\mathcal{A}$  and  $\mathcal{B}$  are countable and  $\mathcal{A} \equiv_{\infty,\omega} \mathcal{B}$  then  $\mathcal{A} \cong \mathcal{B}$ .
- (ii) Show that if  $\mathcal{A}, \mathcal{B} \models T_{dlo}$  then  $\mathcal{A} \equiv_{\infty, \omega} \mathcal{B}$ .
- (iii) Show that  $(\mathbf{R}, <) \not\cong (\mathbf{R} \{0\}, <)$ .

2. Suppose  $\kappa > \omega$  is such that  $L_{\kappa\omega}$  is  $\kappa$ -compact i.e. if T is a collection of  $L_{\kappa\omega}$ -sentences of size  $\kappa$  and every  $T' \subseteq T$  of size  $< \kappa$  has a model, then T has a model. Show that  $\kappa$  is regular i.e. every  $A \subseteq \kappa$  of size  $< \kappa$  is bounded in  $\kappa$ .

3. Suppose  $\kappa > \omega$  is such that  $L_{\kappa\omega}$  is  $\kappa$ -compact. Show that  $\kappa$  is not a successor cardinal.