

Malliteoria

Harjoitus 1

1. Show that the set of integers is definable without parameters in the structure $(\mathbf{C}, +, \times, \exp, 0, 1)$, where $+$ and \times are the addition and multiplication of complex numbers and $\exp(x) = e^x$.
2. Let $\mathcal{A} = (\mathbb{N} - \{0\}, R^{\mathcal{A}})$, where $(a, b) \in R^{\mathcal{A}}$ iff a divides b (i.e. for some $n \in \mathbb{N}$, $na = b$). Let X be the set of those $x \in \mathbb{N} - \{0\}$ for which there are a prime p and $n \in \mathbb{N}$ such that $x = p^n$. Show that X is definable without parameters in \mathcal{A} .
3. Let \mathcal{A} be an $L \cup \{R\}$ -structure, $\#R = n$, and $X \subseteq \mathcal{A}^m$ be definable in \mathcal{A} . Show that if $R^{\mathcal{A}}$ is definable in $\mathcal{A} \upharpoonright L$, then X is definable in $\mathcal{A} \upharpoonright L$ ($\mathcal{A} \upharpoonright L$ is what we get from \mathcal{A} by dropping $R^{\mathcal{A}}$ out).
4. Assume α is an ordinal and $\beta \in \alpha$. Show that $\alpha \cup \{\alpha\}$ and β are ordinals.