(2)

Jetzt $\underline{z = i\frac{7}{3}}$.

Tai (zweitens)

$$3z + \overline{2+i} = 2 \Leftrightarrow 3z = 2 - \overline{2+i}$$

$$\Leftrightarrow 3z = 2 - (2-i) = i$$

$$\Leftrightarrow \underline{z = i\frac{7}{3}}$$

(3.)

a) $$\overline{(7+8i)(7-i)} = \overline{(7 + 7\cdot(-i) + 8i\cdot 7 + (8i)\cdot(-i))}$$

$$= \overline{(7 - i + 8i + 8\cdot(-7)\cdot(i^2))}$$

$$= \overline{(7 + 7i - 8\cdot(-7))} = \overline{(7 + 7i + 8)}$$

$$= \overline{(9+7i)} = \underline{9 - 7i}$$

b) $$(7+i)\,\overline{(7-3i)} = (7+i)(7+3i)$$

$$= 7\cdot 7 + 7\cdot(3i) + i\cdot 7 + i\cdot(3i)$$

$$= 7 + 3i + i + 3\cdot(i^2) = 7 + 4i + 3\cdot(-7)$$

$$= 7 + 4i - 3 = \underline{-2 + 4i}$$

Lukualueet Harjoitus 3

(7)

(7.)

a)

$$(7+i)^2 = (7+i)(7+i) = 7+7\cdot i + i\cdot 7 + i^2$$
$$= 7+2i+(-7) = 2i$$
$$(2-i)^2 = (2-i)(2-i) = 2\cdot 2 + 2\cdot(-i) + (-i)\cdot 2 + (-i)^2$$
$$= 4 - 4i + i^2 = 4-4i-7 = 3-4i$$

Joten

$$(7+i)^2(2-i)^2 = (2i)(3-4i) = 3\cdot(2i) + (2i)\cdot(-4i)$$
$$= 6i + (2\cdot(-4))(i\cdot i) = 6i + (-8)\cdot(-7)$$
$$= 6i + 8 = 8+6i$$

b)

$$(7+i)^3 = (7+i)^2\cdot(7+i) \overset{a)}{=} (2i)(7+i)$$
$$= 7\cdot(2i) + (2i)\cdot i = 2i + 2\cdot(i^2)$$
$$= 2i + 2\cdot(-7) = -2+2i$$

(2.)

Merkitään $z = x+iy$.

Nyt

$$3z + \overline{2+i} = 2$$
$$\Rightarrow 3(x+iy) + \overline{2+i} = 2$$
$$\Rightarrow 3x + i3y + 2 - i = 2$$
$$\Rightarrow \underline{3x + i3y} = 2 - (2-i) \underline{= i}$$
$$\Rightarrow \begin{cases} 3x = 0 \\ 3y = 7 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 7/3 \end{cases}$$

(4.)

$$\overline{6z + (3-i)} = 7-2i$$

$$\Leftrightarrow \overline{6z} + \overline{(3-i)} = 7-2i$$

$$\Leftrightarrow 6\bar{z} + (3+i) = 7-2i$$

$$\Leftrightarrow 6\bar{z} = 7-2i-(3+i) = -2-3i$$

$$\Leftrightarrow \bar{z} = \frac{1}{6}(-2-3i) = -\frac{1}{3} - \frac{1}{2}i$$

$$\Leftrightarrow \underline{z = -\frac{1}{3} + i\frac{1}{2}}$$

(5.)

$$(2-i)z = (7+i) + iz$$

$$\Leftrightarrow (2-i)z - iz = (7+i)$$

$$\Leftrightarrow (2-i-i)z = (7+i)$$

$$\Leftrightarrow (2-2i)z = (7+i)$$

Tässä on helpompaa merkitä $z = x+iy$ jolloin saamme

$$(2-2i)(x+iy) = 7+i$$

$$\Leftrightarrow 2x + 2\cdot(iy) + (-2i)\cdot x + (-2i)\cdot(iy) = 7+i$$

$$\Leftrightarrow 2x + i2y - i2x - 2\cdot\underbrace{(i^2)}_{=-1}\cdot y = 7+i$$

$$\Leftrightarrow 2x + i(2y-2x) + 2y = 7+i$$

$$\Leftrightarrow (2x+2y) + i(2y-2x) = 7+i$$

$$\Leftrightarrow \begin{cases} 2x+2y = 7 \\ 2y-2x = 1 \end{cases} \overset{+}{\Leftrightarrow} \begin{cases} 2x+2y = 7 \\ 4y = 8 \end{cases}$$

$$\Rightarrow \begin{cases} 2x + 2\cdot\frac{7}{2} = 7 \\ y = 7/2 \end{cases} \Rightarrow \begin{cases} 2x = 0 \\ y = 7/2 \end{cases} \Rightarrow \begin{cases} x = 0 \\ y = 7/2 \end{cases}.$$

Siis $\underline{z = i\frac{7}{2}}$.

(6.)

$(z :=)\ (1 - i\lambda)(\lambda - i)^{-1}$

Q.73 laventaminen

$$= (1 - i\lambda)\frac{1}{\lambda - i} \overset{\lambda+i}{=} (1 - i\lambda)\frac{(\lambda + i)}{\lambda^2 + 1}$$

$$= \frac{(1 - i\lambda)(\lambda + i)}{\lambda^2 + 1} = \frac{(1 \cdot \lambda + 1 \cdot i + (-i\lambda)\cdot\lambda + (-i\lambda)\cdot i)}{\lambda^2 + 1}$$

$$= \frac{(\lambda + i - i\lambda^2 + \lambda)}{\lambda^2 + 1} = \frac{(2\lambda + i(1 - \lambda^2))}{\lambda^2 + 1}$$

Siis $\operatorname{Re} z = \frac{2\lambda}{\lambda^2 + 1}$ ja $\operatorname{Im} z = \frac{1 - \lambda^2}{\lambda^2 + 1}$

Reaalinen, kun $\operatorname{Im} z = 0 \iff 1 - \lambda^2 = 0$

$\iff \lambda^2 = 1 \iff \underline{\lambda = \pm 1}$.

Puhtaasti imaginaarinen, kun $\operatorname{Re} z = 0$

$\iff 2\lambda = 0 \iff \underline{\lambda = 0}$.