

Lukualueet, 1. harjoitus.

LUKUALUEET, HARJOITUS 1.

$$1. \quad \frac{3x+7}{x^2-2} = 2 \Leftrightarrow 3x+7 = 2x^2-4$$

$$\Leftrightarrow 2x^2 - 3x - 5 = 0 \Leftrightarrow x = \frac{3 \pm \sqrt{49}}{4}$$

$$\Leftrightarrow \underline{x = 2\frac{1}{2} \vee x = -1}$$

(Tässä jäsäällä olella koka ajan $x \neq \pm \sqrt{2}$)

$$2. \quad |x+7| = |2x-7| \Leftrightarrow |x+7|^2 = |2x-7|^2$$

$$\Leftrightarrow x^2 + 2x + 7 = 4x^2 - 4x + 7$$

$$\Leftrightarrow 3x^2 - 6x = 0 \Leftrightarrow 3x(x-2) = 0$$

$$\Leftrightarrow \underline{x = 0 \vee x = 2}$$

$$3. \quad |x+7| = |2x-7| \Leftrightarrow \begin{cases} x+7 = 2x-7, & x \geq -7 \\ -x-1 = 2x-7, & x < -7 \end{cases}$$

$$\Leftrightarrow \begin{cases} x-2 = 0, & x \geq -7 \\ 3x = 0, & x < -7 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 2, & x \geq -7 \\ x = 0, & x < -7 \end{cases}$$

$$\Leftrightarrow \underline{x = 2}$$

$$4. \quad |x+2| > |2x-7| \Leftrightarrow |x+2|^2 > |2x-7|^2$$

$$\Leftrightarrow x^2 + 4x + 4 > 4x^2 - 4x + 7 \Leftrightarrow 3x^2 - 8x - 3 < 0$$

$$\Leftrightarrow 9x^2 - 24x - 9 < 0 \Leftrightarrow (3x-4)^2 - 25 < 0$$

$$\Leftrightarrow (3x-4)^2 < 25 \Leftrightarrow |3x-4| < 5$$

$$\Leftrightarrow -5 < 3x-4 < 5 \Leftrightarrow -\frac{1}{3} < x < 3$$

$$(a) \quad \underline{x = 7 \vee x = 2}$$

$$(b) \quad \underline{x = 0 \vee x = 7 \vee x = 2}$$

$$(c) \quad \underline{-\frac{1}{3} < x < 3}$$

$$5. \quad x^4 - 2x^3 - 16x^2 + 2x + 15 = 0$$

$$\Leftrightarrow x^4 - 7 - 2x^3 + 2x - 16x^2 + 16 = 0$$

$$\Leftrightarrow (x^2-7)(x^2+7) - 2x(x^2-7) - 16(x^2-7) = 0$$

$$\Leftrightarrow (x^2-7)(x^2+7-2x-16) = 0$$

$$\Leftrightarrow (x^2-7)(x^2-2x-9) = 0$$

$$\Leftrightarrow (x^2-7)(x^2-9-2x-6) = 0$$

$$\Leftrightarrow (x^2-7)((x-3)(x+3) - 2(x+3)) = 0$$

$$\Leftrightarrow (x^2-7)(x+3)(x-5) = 0$$

$$\Leftrightarrow \underline{x = -7 \vee x = 7 \vee x = -3 \vee x = 5}$$

6. Yhtälölli: $x^2 + 2ax + 7 = 0$ on reaalista
ratkaisuja jos ratkaisuasteen diskriminantti

$$4a^2 - 4 \geq 0 \Leftrightarrow a^2 \geq 1 \Leftrightarrow a \leq -1 \vee a \geq 1$$

Tällöin $x^2 + 2ax + 7 = 0 \Leftrightarrow x = \frac{-2a \pm \sqrt{4a^2 - 4}}{2}$

$$\Leftrightarrow x = -a \pm \sqrt{a^2 - 1}$$