

1. An expert witness in a paternity suit testifies that the length (in days) of pregnancy (that is, the time from impregnation to the delivery of the child) is approximately normally distributed with parameter $\mu = 270$ and $\sigma^2 = 100$. The defendant in the suit is able to prove that he was out of the country during a period that began 290 days before the birth of the child and ended 240 days before the birth. If the defendant was, in fact, the father of the child, what is the probability that the mother could have had a very long or very short pregnancy indicated by the testimony?.
2. Early in the morning, a group of m people decides to use the elevator in an otherwise deserted building of 21 floors. Each of these persons chooses his or her floor independently of the others, and –from our point of view–completely random, so that each person selects a floor with probability $1/21$. Let S_m be the number of times the elevator stops. In order to study S_m , we introduce for $i = 1, 2, \dots, 21$ random variables R_i , given by

$$R_i = \begin{cases} 1 & \text{if the elevator stops at the } i\text{th floor} \\ 0 & \text{if the elevator does not stop at the } i\text{th floor} \end{cases}$$

- (a) Each R_i has $Ber(p)$ distribution. Calculate the parameter p . A discrete random variable X has *Bernoulli distribution* with parameter p , where $0 \leq p \leq 1$, if its probability mass function is given by $\mathbb{P}(X = 1) = p$ and $\mathbb{P}(X = 0) = 1 - p$. We denote it by $Ber(p)$
- (b) From the way we defined S_m , it follows that

$$S_m = R_1 + R_2 + \dots + R_{21}.$$

Can we conclude that S_m has $Bin(21, p)$ distribution, with p as in part a?. Why or why not?.

- (c) Clearly, if $m = 1$ one has that $\mathbb{P}(S_1 = 1) = 1$. Show that for $m = 2$

$$\mathbb{P}(S_2 = 1) = \frac{1}{21} = 1 - \mathbb{P}(S_2 = 2),$$

and that S_3 has the following distribution.

a	1	2	3
$\mathbb{P}(S_3 = a)$	1/441	60/441	380/441

3. Based on past experience, a bank believes that 7% of the people who receive loans will not make payments on time. The bank has recently approved 200 loans.
 - (a) What is the mean and standard deviation of the proportion of clients in this group who may not make timely payments?.
 - (b) What assumptions underlie the model? Are the conditions met? Explain.
 - (c) What is the probability that over 10% of these clients will not make timely payments?.