INTRODUCTION TO STATISTICS FALL 2010

Exercise 7

1. The probability distribution of a discrete random variable X is given by

   $P(x = -1) = \frac{1}{5} \quad P(x = 0) = \frac{2}{5} \quad P(x = 1) = \frac{2}{5}$

   (a) Compute $E(X)$.

   $\sum x_i P(X = x_i) = 0.2$

   (b) Give the probability distribution of $Y = X^2$ and compute $E(Y)$ using the distribution of Y.

   $P(y = x^2 = 0) = \frac{2}{5}$
   $P(y = x^2 = 1) = \frac{3}{5}$
   $E(Y) = \sum y_i P(Y = y_i) = \frac{3}{5}$

   (c) Determine $Var(X)$ and $Var(Y)$.

   $Var(X) = E(X^2) - [E(X)]^2 = E(Y) - [E(X)]^2 = \frac{14}{25}$ or 0.56
   $Var(Y) = E(Y^2) - [E(Y)]^2 = E(Y) - [E(Y)]^2 = \frac{6}{25}$ or 0.24

2. Find $a)E(X), b)E(X^2), c)E(X - \mu)^2$ for the following probability distribution
   $P(x = 8) = \frac{1}{8}$, $P(x = 12) = \frac{1}{6}$, $P(x = 16) = \frac{3}{8}$, $P(x = 20) = \frac{1}{4}$, $P(x = 24) = \frac{1}{2}$

   a) E(X) = 26
   b) E(X^2) $= \sum x_i^2 P(X = x_i) = 516$
   c) Because $\sum P(X = x) > 1$
   Var(X) $= E(X^2) - [E(X)]^2 = -160$

3. For a certain random variable with $E(X) = 2$, $Var(X) = 4$. Compute the expectation and variance of **3 - 2X**.

   Using properties of expectation and variance:
   $E(3 - 2X) = 3 - 2 \times E(X) = 3 - 2 \times 2 = -1$
   $Var(3 - 2X) = 0 + (-2)^2 \times Var(X)$ *The variance of a constant is 0!*
   $= 4 \times Var(X) = 4 \times 4 = 16$

4. If a man purchases a raffle ticket, he can win a first prize of 5000€or a second prize of 2000€with probabilities 0.001 and 0.003. What should be a fair price to pay for the ticket?.

   Let $P(win = 0) = P(not\, winning\, either\, prize)$.
   Fair price **(F)** is defined as the case where **expected gain = 0**.

   Then,
   $E(gain) = E(win) - E(loss) = 0$

$0 = 5000 \times P(win = 5000) + 2000 \times P(win = 2000) - F \times P(win = 0)$

Substituting in probability values and moving E(loss) to the left side of the equation gives us:
$F \times (1 - 0.001 - 0.003) = 5000 \times 0.001 + 2000 \times 0.001$
$F \times (0.996) = 11$
$F = 11/0.996 \approx 11.04418$

Fair price of the raffle ticket is approximately 11,05€.