INTRODUCTION TO STATISTICS FALL 2010

Exercise 6

1. If n people are present in a room, what is the probability that no two of them celebrate their birthday on the same day of the year? How large need to be n so that this probability is less than $1/2$?

   Assume there are only 365 days in a year, no leap years, with each day having equal probabilityof being someone's birthday.
   That is, $P(the\, day\, is\, someone's\, birthday) = \frac{1}{365}$

   Then for person 1:
   $P(has\, a\, unique\, birthday) = 365\, days\, to\, choose\, from \times \frac{1}{365} = 1$

   Of course, every person is guaranteed to have a birthday. But because we want the probability of n people NOT having a birthday on the same day:

   Person 2 has $P(has\, a\, unique\, birthday) = \mathbf{364}\, days\, to\, choose\, from \times \frac{1}{365}$

   Person 3 has $P(has\, a\, unique\, birthday) = \mathbf{363}\, days\, to\, choose\, from \times \frac{1}{365}$

   etc..
   Such that probability that no two of them celebrate their birthday on the same day:

   $$\frac{365!}{(365-n+1)!} \times (\frac{1}{365})^n$$

   The above equation decreases as n increases and falls under 0.5 when n= 23 (0.492702766).

2. Delegates from 10 countries, including France, England, Spain, and the United States, are to be seated in a row. How many different seating arragements are possible if the French and Spanish delegates are to be seated next to each other, and the English and U.S delegates are not to be next to each other?

   There is more than one way to solve this problem. Choose the one that is easier for you to grasp. Here are two:

   Solution 1. Consider France and Spain as one block of reserved seats. There are **9** possible ways 2 consecutive chairs out of 10 can be reserved, with **2** ways the delegates can sit in the block (France then Spain or Spain then France). That leaves **8!** ways the other delegates can seat themselves. We then subtract the seating arrangements in which we consider the block of France & Spain and England & U.S. as single available "blocks". The arrangement then consists of 8 spots: 6 single seats and 2 block seats.

   $$2 \times 9 \times 8! - 2 \times 2 \times 8! = \mathbf{564{,}480}$$

Solution 2. Same as solution 1 but with a different subtraction approach. For each of the 9 possible ways 2 seats for France and Spain can be reserved, **7** allow for **6** ways another block of seats can be reserved with **6!** ways the remaining delegates with no preference can be seated. The remaining **2** possible ways, with France and Spain sitting either in the first 2 or last 2 seats, allows for **7** ways another block of seats can be reserved with **6!** ways the remaining delegates with no preference can be seated. Taking into account **2** ways Franch & Spain and England & U.S. can be arranged in their blocks:

$$2 \times 9 \times 8! - 2 \times 2 \times (7 \times 6 \times 6! + 2 \times 7 \times 6!) = \mathbf{564{,}480}$$