Exercise 4: Introduction to Statistics, Fall 2010

1. We roll two dice, each with six faces numbered 1,2,3,4,5 and 6. Then the sample space related to results is
S = {(x, y)|x = 1, 2, 3, 4, 5, 6 ja y = 1, 2, 3, 4, 5, 6}

A = {With the first die we get 4 or more}
B = {The sum of results is 10 or more}
C = {We get the same number with both dice}

There are 6^6 = 36 possible combinations to roll!

S(x+y)	1	2	3	4	5	6
1	*2*	*3*	*4*	*5*	*6*	*7*
2	*3*	*4*	*5*	*6*	*7*	*8*
3	*4*	*5*	*6*	*7*	*8*	*9*
4	*5*	*6*	*7*	*8*	*9*	*10*
5	*6*	*7*	*8*	*9*	*10*	*11*
6	*7*	*8*	*9*	*10*	*11*	*12*

a) A U B

A completely overlaps B, so A U B = A = 18/36

b) A ∩ C

P(4-4, 5-5, 6-6) = 3/36 = 1/12

c) C^c

P(Inverse(C)) = 1 - C = 30/36 = 5/6

d) B \ C = B ∩ C^c

P(6-4, 6-5, 4-6,5-6) = 4/36 = 1/9

2. We roll two dice, each of which has six faces with numbers 1,2,3,4,5 and 6. Consider the sum of dice rolls as an event.

S(x+y)	1	2	3	4	5	6
1	*2*	*3*	*4*	*5*	*6*	*7*
2	*3*	*4*	*5*	*6*	*7*	*8*
3	*4*	*5*	*6*	*7*	*8*	*9*
4	*5*	*6*	*7*	*8*	*9*	*10*
5	*6*	*7*	*8*	*9*	*10*	*11*
6	*7*	*8*	*9*	*10*	*11*	*12*

(a) What is the sample space of the sum?

all possible outcomes S={2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

(b) What is the probability of the event A={Sum is equal to 1}?

P(A ∩ S) = 0

(c) What is the probability of the event B={Sum is equal to 8}?

P(B ∩ S) = subset of S equal to B = 5/36

(d) Let C be the event {With the first die we get 1}. What is the conditional probability of B with the condition C, i.e. Pr(B|C)?

P(sum is equal to 8 | first die rolls 1) = P(B ∩ C) / P(C) = 0 / (1/6) = 0

(e) Let D be the event {With the first die we get 5}. What is the conditional probability of B with the condition D, i.e. Pr(B|D)?

P(sum is equal to 8 | first die rolls 5) = P(B ∩ D) / P(D) = (1/36) / (6/36) = 1/6

3. We toss a coin. If the result is "heads" then we toss the coin twice more.

(a) What is the sample space?

S = { T, HTT, HTH, HHT, HHH }

(b) What is the probability of getting "tails" with the last tossing, if we assume that results are independent and the probability of getting "tails" is 0.5?

With S = { T, HTT, HTH, HHT, HHH } draw a probability tree. Then:

*T + HTT + HHT = 0.5 + 0.5*0.5*0.5 + 0.5*0.5*0.5 = 0.75*

Answer is never 3/5 because the probability of one toss does not occur with the same probability of three tosses!

4. Let A and B be two events with probabilities Pr(A) = 0.5 and Pr(B) = 0.6 respectively. Calculate the probability of the event A U B when

P(A U B) = P(A) + P(B) - P(A ∩B) = 0.5 + 0.6 - P(A∩B)

(a) Pr(A ∩ B) = 0.1 *P(A U B) = P(A) + P(B) - P(A ∩B) = 0.5 + 0.6 - 0.1 = 1*

(b) A and B are disjoint (i.e. mutually exclusive)

P(A∩B) = 0. Then P(A U B) = 0.5 + 0.6 -0 = 1.1

(c) A and B are independent

*P(A∩B) = P(A)P(B). Then P(A U B) = 0.5 + 0.6 - 0.5*0.6 = 0.8*

(d) Pr(A|B) = 0.1 P(A∩B) = P(A|B)*P(B) = 0.1 * 0.6 = 0.06

Then P(A U B) = 0.5 + 0.6 - 0.06 = 1.04

5. There are 8 bulbs in a box and 3 of them are broken

(a) We pick, randomly, 3 bulbs. Everytime we have picked up one of them we put it back into the box and then pick the next one. What is the probability that the three of them are broken?.

*P(all 3 chosen are broken) = 3/8 * 3/8 * 3/8 = 0.052734*

(b) We pick, randomly, 3 bulbs. Everytime we have picked up one of them we do not put it back into the box and then pick the next one. What is the probability that the three of them are broken?.

*P(all 3 chosen are broken) = 3/8 * 2/7 * 1/6 = 0.17857*