```
N = 50000; dt = 1; mu = 1; sigma = 0.01;
alpha = 2; beta = -0.4; delta = 1; %parameters
k = 4; %number of different runs

dW = zeros(k,N); % preallocate arrays ...
W = zeros(k,N);
n = zeros(k,N);
Ds = zeros(k,N);

f = @(x,delta) exp(-(x-delta)^4)+exp(-(x+delta)^2); %monomorphic population↙
equilibrium (=carrying capacity)
g = @(x,beta,delta) -(-beta*exp(-(x-delta)^4)-beta*exp(-(x+delta)^2)+4*exp(-(x-delta)↙
^4)*x^3-12*exp(-(x-delta)^4)*x^2*delta+12*exp(-(x-delta)^4)*x*delta^2-4*exp(-(x-↙
delta)^4)*delta^3+2*exp(-(x+delta)^2)*x+2*exp(-(x+delta)^2)*delta)/(exp(-(x-delta)^4)↙
+exp(-(x+delta)^2)); %selection gradient, calculated with Maple

x = [-1.5 -0.5 0 2]; %starting points of different runs

for i = 1:k
    n(i,1) = f(x(i),delta); %monomorphic population equilibrium
    Ds(i,1) = g(x(i),beta,delta); %selection gradient
    dW(i,1) = mu*sigma^2*n(i,1)*Ds(i,1)*dt; % first evaluation outside the following↙
for-loop ...
    W(i,1) = x(i)+dW(i,1); %starting point + small increment by the drift
        for j = 2:N
            n(i,j) = f((W(i,j-1)),delta);
            Ds(i,j) = g((W(i,j-1)),beta,delta);
            dW(i,j) = mu*sigma^2*n(i,j)*Ds(i,j)*dt; % general increment
            W(i,j) = W(i,j-1) + dW(i,j); %previous location + small increment, gives↙
the new location
        end
end

plot(W(1:k,1:N),1:N,'r-') % plot W against t
axis([-2 2 1 N])
xlabel('W(t)','FontSize',16)
ylabel('t','FontSize',16,'Rotation',0)
```

```
randn('state',100) % set the state of randn
N = 50000; dt = 1; mu = 1; sigma = 0.01;
alpha = 2; beta = -0.4; delta = 1; %parameters
k = 5; %number of different runs

dW = zeros(k,N); % preallocate arrays ...
W = zeros(k,N);
n = zeros(k,N);
Ds = zeros(k,N);
D = zeros(k,N);

f = @(x,delta) exp(-(x-delta)^4)+exp(-(x+delta)^2); %monomorphic population↙
equilibrium (=carrying capacity)
g = @(x,beta,delta) -(-beta*exp(-(x-delta)^4)-beta*exp(-(x+delta)^2)+4*exp(-(x-delta)↙
^4)*x^3-12*exp(-(x-delta)^4)*x^2*delta+12*exp(-(x-delta)^4)*x*delta^2-4*exp(-(x-↙
delta)^4)*delta^3+2*exp(-(x+delta)^2)*x+2*exp(-(x+delta)^2)*delta)/(exp(-(x-delta)^4)↙
+exp(-(x+delta)^2)); %selection gradient, calculated with Maple

%x = [-1.5 -0.5 0 1 2]; %different starting points for different runs
x = -1.5*ones(1,k); %same starting point (-1.5), k different runs

for i = 1:k
    n(i,1) = f(x(i),delta); %monomorphic population equilibrium
    Ds(i,1) = g(x(i),beta,delta); %selection gradient
    D(i,1) = mu*sigma^3*sqrt(2/pi)*n(i,1)*abs(Ds(i,1)); %diffusion coefficient
    dW(i,1) = mu*sigma^2*n(i,1)*Ds(i,1)*dt+sqrt(D(i,1)*dt)*randn; % first evaluation↙
outside the following for-loop ...
    W(i,1) = x(i)+dW(i,1); %starting point + small increment by the drift
        for j = 2:N
            n(i,j) = f((W(i,j-1)),delta);
            Ds(i,j) = g((W(i,j-1)),beta,delta);
            D(i,j) = mu*sigma^3*sqrt(2/pi)*n(i,j)*abs(Ds(i,j));
            dW(i,j) = mu*sigma^2*n(i,j)*Ds(i,j)*dt+sqrt(D(i,j)*dt)*randn; % general↙
increment
            W(i,j) = W(i,j-1) + dW(i,j);
        end
end

plot(W(1:k,1:N),1:N,'r-') % plot W against t
%axis([-2 2 1 N]) forces the range of the axis, if not specified Matlab will give↙
suitable ones
xlabel('W(t)','FontSize',16)
ylabel('t','FontSize',16,'Rotation',0)
```

```
N = 50000; dt = 10; mu = 1; sigma = 0.01; %change dt from 1 to 10 to speed up
convergence of the solution path
alpha = 2; beta = -0.4; delta = 1;

dW = zeros(2,N); % preallocate arrays ...
W = zeros(2,N);
n = zeros(2,N);
Ds = zeros(2,N);

f1 = @(x,y,alpha,beta,delta) (-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)
+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-
y))*exp(-alpha*(y-x)^2-beta*(y-x))); %dimorphic population equilibrium of n[1]

f2 = @(x,y,alpha,beta,delta) -1/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)
^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-
x))+exp(-(y-delta)^4)+exp(-(y+delta)^2));  %dimorphic population equilibrium of n[2]

g1 = @(x,y,alpha,beta,delta) -(-beta*exp(0)*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+
(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*
(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))-(-2*alpha*(x-y)-beta)*exp(-alpha*
(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-
x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-
delta)^4)+exp(-(y+delta)^2)))/(exp(-(x-delta)^4)+exp(-(x+delta)^2))+(exp(0)*(-exp(-
(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)
^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))-
exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-
beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))
+exp(-(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(x-delta)^4)+exp(-(x+delta)^2))^2*(-4*
(x-delta)^3*exp(-(x-delta)^4)+(-2*x-2*delta)*exp(-(x+delta)^2)); %selection gradient
for n[1]

g2 = @(x,y,alpha,beta,delta) -((-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x))*
(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-
alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-
beta*(y-x)))+beta*exp(0)/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*
(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-
(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(y-delta)^4)+exp(-(y+delta)^2))+(exp(-alpha*
(y-x)^2-beta*(y-x))*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-
(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp
(-alpha*(y-x)^2-beta*(y-x)))-exp(0)/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-
x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-
x))+exp(-(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(y-delta)^4)+exp(-(y+delta)^2))^2*(-4
*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*delta)*exp(-(y+delta)^2)); %selection gradient
for n[2]

x = 0.38;  %starting point x-co-ordinate
y = 0.41; %starting point y-co-ordinate

n(1,1) = f1(x,y,alpha,beta,delta);
n(2,1) = f2(x,y,alpha,beta,delta);
Ds(1,1) = g1(x,y,alpha,beta,delta);
Ds(2,1) = g2(x,y,alpha,beta,delta);
dW(1,1) = mu*sigma^2*n(1,1)*Ds(1,1)*dt;
dW(2,1) = mu*sigma^2*n(2,1)*Ds(2,1)*dt;
W(1,1) = x+dW(1,1);
```

```
W(2,1) = y+dW(2,1);

for j = 2:N
    n(1,j) = f1((W(1,j-1)),W(2,j-1),alpha,beta,delta);
    n(2,j) = f2((W(1,j-1)),W(2,j-1),alpha,beta,delta);
    Ds(1,j) = g1((W(1,j-1)),W(2,j-1),alpha,beta,delta);
    Ds(2,j) = g2((W(1,j-1)),W(2,j-1),alpha,beta,delta);
    dW(1,j) = mu*sigma^2*n(1,j)*Ds(1,j)*dt; % general increment in x
    dW(2,j) = mu*sigma^2*n(2,j)*Ds(2,j)*dt; % general increment in y
    W(1,j) = W(1,j-1) + dW(1,j);
    W(2,j) = W(2,j-1) + dW(2,j);
end

s = @(x,y,alpha,beta,delta) 1-exp(-alpha*(y-x)^2-beta*(y-x))*(exp(-(x-delta)^4)+exp(-↙
(x+delta)^2))/(exp(-(y-delta)^4)+exp(-(y+delta)^2)); %invasion fitness in a↙
monomorphic population
s1 = @(x,y) s(x,y,2,-0.4,1); %s_x(y)-> x is the resident strategy, y mutant
s2 = @(x,y) s(y,x,2,-0.4,1); %s_y(x)-> y is the resident strategy, x mutant
xiso = @(x,y) g1(x,y,2,-0.4,1);
yiso = @(x,y) g2(x,y,2,-0.4,1);

ezplot(s1,[-2 2 -2 2]); %gives a contourplot of s1=0
hold on; %tells Matlab not to plot immediately and therefore we can have several↙
plots on top of each other in the same Figure
ezplot(s2,[-2 2 -2 2]);
hold on;
ezplot(xiso,[-2 2 -2 2]);
hold on;
ezplot(yiso,[-2 2 -2 2]);
hold on;
plot(W(1,1:N),W(2,1:N))
axis([0 1 0 1.5])
```

```
randn('state',300) % set the state of randn
N = 50000; dt = 10; mu = 1; sigma = 0.01;
alpha = 2; beta = -0.4; delta = 1; %parameters
k = 10; %number of different runs

dW = zeros(2*k,N); % preallocate arrays ...
W = zeros(2*k,N);
n = zeros(2*k,N);
Ds = zeros(2*k,N);
D = zeros(2*k,N);

f1 = @(x,y,alpha,beta,delta) (-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)
+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-
y))*exp(-alpha*(y-x)^2-beta*(y-x))); %dimorphic population equilibrium of n[1]

f2 = @(x,y,alpha,beta,delta) -1/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)
^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-
x))+exp(-(y-delta)^4)+exp(-(y+delta)^2)); %dimorphic population equilibrium of n[2]

g1 = @(x,y,alpha,beta,delta) -(-beta*exp(0)*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+
(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*
(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))-(-2*alpha*(x-y)-beta)*exp(-alpha*
(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-
x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-
delta)^4)+exp(-(y+delta)^2)))/(exp(-(x-delta)^4)+exp(-(x+delta)^2))+(exp(0)*(-exp(-
(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)
^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))-
exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-
beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))
+exp(-(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(x-delta)^4)+exp(-(x+delta)^2))^2*(-4*
(x-delta)^3*exp(-(x-delta)^4)+(-2*x-2*delta)*exp(-(x+delta)^2)); %selection gradient
for n[1]

g2 = @(x,y,alpha,beta,delta) -((-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x))*
(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-
alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-
beta*(y-x)))+beta*exp(0)/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*
(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-
(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(y-delta)^4)+exp(-(y+delta)^2))+(exp(-alpha*
(y-x)^2-beta*(y-x))*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-
(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp
(-alpha*(y-x)^2-beta*(y-x)))-exp(0)/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-
x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-
x))+exp(-(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(y-delta)^4)+exp(-(y+delta)^2))^2*(-4
*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*delta)*exp(-(y+delta)^2)); %selection gradient
for n[2]

x = 0.38; %same starting point x=0.38, k different runs
y = 0.41;

for i = 1:k
    n(i,1) = f1(x,y,alpha,beta,delta); %the values related to x strategy are stored
in rows from 1 to k
    n(k+i,1) = f2(x,y,alpha,beta,delta); %the values related to y strategy are stored
in rows from k+1 to 2*k
```

```
    Ds(i,1) = g1(x,y,alpha,beta,delta);
    Ds(k+i,1) = g2(x,y,alpha,beta,delta);
    D(i,1) = mu*sigma^3*sqrt(2/pi)*n(i,1)*abs(Ds(i,1)); %diffusion coefficient
    D(k+i,1) = mu*sigma^3*sqrt(2/pi)*n(k+i,1)*abs(Ds(k+i,1));
    dW(i,1) = mu*sigma^2*n(i,1)*Ds(i,1)*dt+sqrt(D(i,1)*dt)*randn;
    dW(k+i,1) = mu*sigma^2*n(k+i,1)*Ds(k+i,1)*dt+sqrt(D(k+i,1)*dt)*randn;
    W(i,1) = x+dW(i,1);
    W(k+i,1) = y+dW(k+i,1);
        for j = 2:N
            n(i,j) = f1((W(i,j-1)),W(k+i,j-1),alpha,beta,delta);
            n(k+i,j) = f2((W(i,j-1)),(W(k+i,j-1)),alpha,beta,delta);
            Ds(i,j) = g1((W(i,j-1)),W(k+i,j-1),alpha,beta,delta);
            Ds(k+i,j) = g2((W(i,j-1)),W(k+i,j-1),alpha,beta,delta);
            D(i,j) = mu*sigma^3*sqrt(2/pi)*n(i,j)*abs(Ds(i,j));
            D(k+i,j) = mu*sigma^3*sqrt(2/pi)*n(k+i,j)*abs(Ds(k+i,j));
            dW(i,j) = mu*sigma^2*n(i,j)*Ds(i,j)*dt+sqrt(D(i,j)*dt)*randn; % general↙
increment
            dW(k+i,j) = mu*sigma^2*n(k+i,j)*Ds(k+i,j)*dt+sqrt(D(k+i,j)*dt)*randn;
            W(i,j) = W(i,j-1) + dW(i,j);
            W(k+i,j) = W(k+i,j-1) + dW(k+i,j);
        end
end

s = @(x,y,alpha,beta,delta) 1-exp(-alpha*(y-x)^2-beta*(y-x))*(exp(-(x-delta)^4)+exp(-↙
(x+delta)^2))/(exp(-(y-delta)^4)+exp(-(y+delta)^2)); %invasion fitness
s1 = @(x,y) s(x,y,2,-0.4,1);
s2 = @(x,y) s(y,x,2,-0.4,1);
xiso = @(x,y) g1(x,y,2,-0.4,1);
yiso = @(x,y) g2(x,y,2,-0.4,1);

ezplot(s1,[-1.5 1 0 1.5]);
hold on;
ezplot(s2,[-1.5 1 0 1.5]);
hold on;
ezplot(xiso,[-1.5 1 0 1.5]);
hold on;
ezplot(yiso,[-1.5 1 0 1.5]);
hold on;
for i = 1:k-1
    plot(W(i,1:N),W(k+i,1:N));
    hold on;
end
plot(W(k,1:N),W(2*k,1:N))
axis([-1.5 1 0 1.5]) %forces the range of the axis, if not specified Matlab will give↙
suitable ones
```

```
> s:=(x,y,z)->r(z)*( 1-((a(z,x)*n[1](x,y)+a(z,y)*n[2](x,y))/K(z))
  );
```

$$s := (x, y, z) \to \mathrm{r}(z)\left(1 - \frac{\mathrm{a}(z, x)\, n_1(x, y) + \mathrm{a}(z, y)\, n_2(x, y)}{\mathrm{K}(z)}\right)$$

```
> K:=x->exp(-(x-delta)^4)+exp(-(x+delta)^2);
```

$$K := x \to \mathrm{e}^{(-(x-\delta)^4)} + \mathrm{e}^{(-(x+\delta)^2)}$$

```
> a:=(x,y)->exp(-alpha*(x-y)^2-beta*(x-y));
```

$$a := (x, y) \to \mathrm{e}^{(-\alpha\,(x-y)^2 - \beta\,(x-y))}$$

```
> r:=x->1;
```

$$r := x \to 1$$

```
> n[1]:=(x,y)->(-a(y,y)*K(x)+K(y)*a(x,y))/(-a(x,x)*a(y,y)+a(x,y)*a
  (y,x));
```

$$n_1 := (x, y) \to \frac{-\mathrm{a}(y, y)\,\mathrm{K}(x) + \mathrm{K}(y)\,\mathrm{a}(x, y)}{-\mathrm{a}(x, x)\,\mathrm{a}(y, y) + \mathrm{a}(x, y)\,\mathrm{a}(y, x)}$$

```
> n[2]:=(x,y)->-1/(-a(x,x)*a(y,y)+a(x,y)*a(y,x))*(-K(x)*a(y,x)+a(x
  ,x)*K(y));
```

$$n_2 := (x, y) \to -\frac{-\mathrm{K}(x)\,\mathrm{a}(y, x) + \mathrm{a}(x, x)\,\mathrm{K}(y)}{-\mathrm{a}(x, x)\,\mathrm{a}(y, y) + \mathrm{a}(x, y)\,\mathrm{a}(y, x)}$$

```
> subs(z=x,diff(s(x,y,z),z));
```

$$-\left(\frac{0.4\,\mathrm{e}^{0.}\,(-1.\,\mathrm{e}^{(-(x-1)^4)} - 1.\,\mathrm{e}^{(-(x+1)^2)} + (\mathrm{e}^{(-(y-1)^4)} + \mathrm{e}^{(-(y+1)^2)})\,\mathrm{e}^{(-2\,(x-y)^2 + 0.4\,x - 0.4\,y)})}{-1. + \mathrm{e}^{(-2\,(x-y)^2 + 0.4\,x - 0.4\,y)}\,\mathrm{e}^{(-2\,(y-x)^2 + 0.4\,y - 0.4\,x)}} - \right.$$

$$(-4\,x + 4\,y + 0.4)\,\mathrm{e}^{(-2\,(x-y)^2 + 0.4\,x - 0.4\,y)}$$

$$(-(\mathrm{e}^{(-(x-1)^4)} + \mathrm{e}^{(-(x+1)^2)})\,\mathrm{e}^{(-2\,(y-x)^2 + 0.4\,y - 0.4\,x)} + 1.\,\mathrm{e}^{(-(y-1)^4)} + 1.\,\mathrm{e}^{(-(y+1)^2)}) \Big/ ($$

$$\left.-1. + \mathrm{e}^{(-2\,(x-y)^2 + 0.4\,x - 0.4\,y)}\,\mathrm{e}^{(-2\,(y-x)^2 + 0.4\,y - 0.4\,x)}\right) \Big/ (\mathrm{e}^{(-(x-1)^4)} + \mathrm{e}^{(-(x+1)^2)}) +$$

$$\left(\frac{\mathrm{e}^{0.}\,(-1.\,\mathrm{e}^{(-(x-1)^4)} - 1.\,\mathrm{e}^{(-(x+1)^2)} + (\mathrm{e}^{(-(y-1)^4)} + \mathrm{e}^{(-(y+1)^2)})\,\mathrm{e}^{(-2\,(x-y)^2 + 0.4\,x - 0.4\,y)})}{-1. + \mathrm{e}^{(-2\,(x-y)^2 + 0.4\,x - 0.4\,y)}\,\mathrm{e}^{(-2\,(y-x)^2 + 0.4\,y - 0.4\,x)}} - \right.$$

$$\mathrm{e}^{(-2\,(x-y)^2 + 0.4\,x - 0.4\,y)}$$

$$(-(\mathrm{e}^{(-(x-1)^4)} + \mathrm{e}^{(-(x+1)^2)})\,\mathrm{e}^{(-2\,(y-x)^2 + 0.4\,y - 0.4\,x)} + 1.\,\mathrm{e}^{(-(y-1)^4)} + 1.\,\mathrm{e}^{(-(y+1)^2)}) \Big/ ($$

$$\left.\left.-1. + \mathrm{e}^{(-2\,(x-y)^2 + 0.4\,x - 0.4\,y)}\,\mathrm{e}^{(-2\,(y-x)^2 + 0.4\,y - 0.4\,x)}\right)\right)$$

$$(-4\,(x-1)^3\,\mathrm{e}^{(-(x-1)^4)} + (-2\,x - 2)\,\mathrm{e}^{(-(x+1)^2)}) \Big/ (\mathrm{e}^{(-(x-1)^4)} + \mathrm{e}^{(-(x+1)^2)})^2$$

```
> Dsx:=(x,y)->-(-beta*exp(0)*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)
  +(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(
  x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-bet
  a*(y-x)))-(-2*alpha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y))/(
  -1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))
  )*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-bet
  a*(y-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(x-delta)^4
  )+exp(-(x+delta)^2))+(exp(0)*(-exp(-(x-delta)^4)-exp(-(x+delta)^
  2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta
  *(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-b
  eta*(y-x)))-exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^
  2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^
  4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-del
  ta)^4)+exp(-(y+delta)^2)))/(exp(-(x-delta)^4)+exp(-(x+delta)^2))
  ^2*(-4*(x-delta)^3*exp(-(x-delta)^4)+(-2*x-2*delta)*exp(-(x+delt
  a)^2)):
> subs(z=y,diff(s(x,y,z),z));
```

$$
\begin{aligned}
&-\Bigg(\Big(-4\,y+4\,x+0.4\Big)\,\mathrm{e}^{\left(-2\,(y-x)^2+0.4\,y-0.4\,x\right)}\\
&\Big(-1.\,\mathrm{e}^{\left(-(x-1)^4\right)}-1.\,\mathrm{e}^{\left(-(x+1)^2\right)}+\left(\mathrm{e}^{\left(-(y-1)^4\right)}+\mathrm{e}^{\left(-(y+1)^2\right)}\right)\mathrm{e}^{\left(-2\,(x-y)^2+0.4\,x-0.4\,y\right)}\Big)\Big/\Big(\\
&-1.+\mathrm{e}^{\left(-2\,(x-y)^2+0.4\,x-0.4\,y\right)}\,\mathrm{e}^{\left(-2\,(y-x)^2+0.4\,y-0.4\,x\right)}\Big)\\
&-\frac{0.4\,\mathrm{e}^{0.}\left(-\left(\mathrm{e}^{\left(-(x-1)^4\right)}+\mathrm{e}^{\left(-(x+1)^2\right)}\right)\mathrm{e}^{\left(-2\,(y-x)^2+0.4\,y-0.4\,x\right)}+1.\,\mathrm{e}^{\left(-(y-1)^4\right)}+1.\,\mathrm{e}^{\left(-(y+1)^2\right)}\right)}{-1.+\mathrm{e}^{\left(-2\,(x-y)^2+0.4\,x-0.4\,y\right)}\,\mathrm{e}^{\left(-2\,(y-x)^2+0.4\,y-0.4\,x\right)}}\Bigg)\Big/\Big(\\
&\mathrm{e}^{\left(-(y-1)^4\right)}+\mathrm{e}^{\left(-(y+1)^2\right)}\Big)+\Bigg(\mathrm{e}^{\left(-2\,(y-x)^2+0.4\,y-0.4\,x\right)}\\
&\Big(-1.\,\mathrm{e}^{\left(-(x-1)^4\right)}-1.\,\mathrm{e}^{\left(-(x+1)^2\right)}+\left(\mathrm{e}^{\left(-(y-1)^4\right)}+\mathrm{e}^{\left(-(y+1)^2\right)}\right)\mathrm{e}^{\left(-2\,(x-y)^2+0.4\,x-0.4\,y\right)}\Big)\Big/\Big(\\
&-1.+\mathrm{e}^{\left(-2\,(x-y)^2+0.4\,x-0.4\,y\right)}\,\mathrm{e}^{\left(-2\,(y-x)^2+0.4\,y-0.4\,x\right)}\Big)\\
&-\frac{\mathrm{e}^{0.}\left(-\left(\mathrm{e}^{\left(-(x-1)^4\right)}+\mathrm{e}^{\left(-(x+1)^2\right)}\right)\mathrm{e}^{\left(-2\,(y-x)^2+0.4\,y-0.4\,x\right)}+1.\,\mathrm{e}^{\left(-(y-1)^4\right)}+1.\,\mathrm{e}^{\left(-(y+1)^2\right)}\right)}{-1.+\mathrm{e}^{\left(-2\,(x-y)^2+0.4\,x-0.4\,y\right)}\,\mathrm{e}^{\left(-2\,(y-x)^2+0.4\,y-0.4\,x\right)}}\Bigg)\\
&\Big(-4\,(y-1)^3\,\mathrm{e}^{\left(-(y-1)^4\right)}+\left(-2\,y-2\right)\mathrm{e}^{\left(-(y+1)^2\right)}\Big)\Big/\Big(\mathrm{e}^{\left(-(y-1)^4\right)}+\mathrm{e}^{\left(-(y+1)^2\right)}\Big)^2
\end{aligned}
$$

```
> Dsy:=(x,y)->-((-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x
  ))*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(
  -(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x
  -y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))+beta*exp(0)/(-
  1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))
```

```
*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta
*(y-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(y-delta)^4)
+exp(-(y+delta)^2))+(exp(-alpha*(y-x)^2-beta*(y-x))*(-exp(-(x-de
lta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*
exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y
))*exp(-alpha*(y-x)^2-beta*(y-x)))-exp(0)/(-1+exp(-alpha*(x-y)^2
-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4
)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-delt
a)^4)+exp(-(y+delta)^2)))/(exp(-(y-delta)^4)+exp(-(y+delta)^2))^
2*(-4*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*delta)*exp(-(y+delta
)^2)):
> diff(Dsx(x,y),x);
```

$$
\begin{aligned}
&-\Bigg(-\beta\,(4\,(x-\delta)^3\,e^{(-(x-\delta)^4)}-(-2x-2\delta)\,e^{(-(x+\delta)^2)}\\
&+(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})\,(-2\alpha\,(x-y)-\beta)\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))})\Big/(\\
&-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))})+\beta\\
&(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))})\,(\\
&(-2\alpha\,(x-y)-\beta)\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\\
&+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,(2\alpha\,(y-x)+\beta)\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))})\Big/\\
&(-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))})^2+\\
&\frac{2\alpha\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,(-(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)})\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})}{-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}}\\
&-(-2\alpha\,(x-y)-\beta)^2\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\\
&(-(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)})\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})\Big/(\\
&-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))})+(-2\alpha\,(x-y)-\beta)\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\\
&(-(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)})\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})\,(\\
&(-2\alpha\,(x-y)-\beta)\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\\
&+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,(2\alpha\,(y-x)+\beta)\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))})\Big/\\
&(-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))})^2-(-2\alpha\,(x-y)-\beta)\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,(\\
&-(-4\,(x-\delta)^3\,e^{(-(x-\delta)^4)}+(-2x-2\delta)\,e^{(-(x+\delta)^2)})\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}
\end{aligned}
$$

$$-\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)\left(2\,\alpha\,(y-x)+\beta\right)e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\Big)\Big/\Big($$

$$-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\Big)\Bigg)\Big/\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)+\Bigg($$

$$-\frac{\beta\left(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\right)}{-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}}-$$

$$\left(-2\,\alpha\,(x-y)-\beta\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}$$

$$\left(-\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)\Big/\Big($$

$$-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\Big)\Bigg(-4\,(x-\delta)^3\,e^{(-(x-\delta)^4)}+(-2\,x-2\,\delta)\,e^{(-(x+\delta)^2)}\Bigg)$$

$$\Big/\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)^2+\Big(\left(4\,(x-\delta)^3\,e^{(-(x-\delta)^4)}-(-2\,x-2\,\delta)\,e^{(-(x+\delta)^2)}\right.$$

$$+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)\left(-2\,\alpha\,(x-y)-\beta\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\Big)\Big/\Big($$

$$-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\Big)-$$

$$\left(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\right)\Big($$

$$\left(-2\,\alpha\,(x-y)-\beta\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}$$

$$+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\left(2\,\alpha\,(y-x)+\beta\right)e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\Big)\Big/$$

$$\left(-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\right)^2-\left(-2\,\alpha\,(x-y)-\beta\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}$$

$$\left(-\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)\Big/\Big($$

$$-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\Big)+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}$$

$$\left(-\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)\Big($$

$$\left(-2\,\alpha\,(x-y)-\beta\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}$$

$$+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\left(2\,\alpha\,(y-x)+\beta\right)e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\Big)\Big/$$

$$\left(-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\right)^2-e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\Big($$

$$-\left(-4\,(x-\delta)^3\,e^{(-(x-\delta)^4)}+(-2\,x-2\,\delta)\,e^{(-(x+\delta)^2)}\right)e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}$$

$$-\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)\left(2\,\alpha\,(y-x)+\beta\right)e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\Big)\Big/\Big($$

$$-1+e^{(-\alpha(x-y)^2-\beta(x-y))}e^{(-\alpha(y-x)^2-\beta(y-x))}))\left(-4(x-\delta)^3e^{(-(x-\delta)^4)}+(-2x-2\delta)e^{(-(x+\delta)^2)}\right)$$

$$\Big/\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)^2-2\left(\frac{-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)e^{(-\alpha(x-y)^2-\beta(x-y))}}{-1+e^{(-\alpha(x-y)^2-\beta(x-y))}e^{(-\alpha(y-x)^2-\beta(y-x))}}\right.$$

$$\left.-\frac{e^{(-\alpha(x-y)^2-\beta(x-y))}\left(-\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)e^{(-\alpha(y-x)^2-\beta(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)}{-1+e^{(-\alpha(x-y)^2-\beta(x-y))}e^{(-\alpha(y-x)^2-\beta(y-x))}}\right)$$

$$\left(-4(x-\delta)^3e^{(-(x-\delta)^4)}+(-2x-2\delta)e^{(-(x+\delta)^2)}\right)^2\Big/\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)^3+\left(\frac{-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)e^{(-\alpha(x-y)^2-\beta(x-y))}}{-1+e^{(-\alpha(x-y)^2-\beta(x-y))}e^{(-\alpha(y-x)^2-\beta(y-x))}}\right.$$

$$\left.-\frac{e^{(-\alpha(x-y)^2-\beta(x-y))}\left(-\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)e^{(-\alpha(y-x)^2-\beta(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)}{-1+e^{(-\alpha(x-y)^2-\beta(x-y))}e^{(-\alpha(y-x)^2-\beta(y-x))}}\right)$$

$$\left(-12(x-\delta)^2e^{(-(x-\delta)^4)}+16(x-\delta)^6e^{(-(x-\delta)^4)}-2e^{(-(x+\delta)^2)}+(-2x-2\delta)^2e^{(-(x+\delta)^2)}\right)\Big/$$

$$\left(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)}\right)^2$$

```
> a11:=(x,y)->-(-beta*(4*(x-delta)^3*exp(-(x-delta)^4)-(-2*x-2*del
ta)*exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*(-2*
alpha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha
*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))+beta*(-exp(
-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta
)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-bet
a*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))^2*((-2*alpha*(x-y)-beta
)*exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))+
exp(-alpha*(x-y)^2-beta*(x-y))*(2*alpha*(y-x)+beta)*exp(-alpha*(
y-x)^2-beta*(y-x)))+2*alpha*exp(-alpha*(x-y)^2-beta*(x-y))/(-1+e
xp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-
(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y
-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2))-(-2*alpha*(x-y)-beta)^
2*exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-
y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x
+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-delta)^4)+exp
(-(y+delta)^2))+(-2*alpha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x
-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*
(y-x)))^2*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-
```

```
x)^2-beta*(y-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2))*((-2*alpha
*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-b
eta*(y-x))+exp(-alpha*(x-y)^2-beta*(x-y))*(2*alpha*(y-x)+beta)*e
xp(-alpha*(y-x)^2-beta*(y-x)))-(-2*alpha*(x-y)-beta)*exp(-alpha*
(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alph
a*(y-x)^2-beta*(y-x)))*(-(-4*(x-delta)^3*exp(-(x-delta)^4)+(-2*x
-2*delta)*exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))-(exp
(-(x-delta)^4)+exp(-(x+delta)^2))*(2*alpha*(y-x)+beta)*exp(-alph
a*(y-x)^2-beta*(y-x))))/(exp(-(x-delta)^4)+exp(-(x+delta)^2))+(-
beta*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+ex
p(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*
(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))-(-2*alpha*(x
-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-
beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)
+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-delta
)^4)+exp(-(y+delta)^2)))/(exp(-(x-delta)^4)+exp(-(x+delta)^2))^2
*(-4*(x-delta)^3*exp(-(x-delta)^4)+(-2*x-2*delta)*exp(-(x+delta)
^2))+((4*(x-delta)^3*exp(-(x-delta)^4)-(-2*x-2*delta)*exp(-(x+de
lta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*(-2*alpha*(x-y)-be
ta)*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*
(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))-(-exp(-(x-delta)^4)-exp(-
(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x
-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha
*(y-x)^2-beta*(y-x)))^2*((-2*alpha*(x-y)-beta)*exp(-alpha*(x-y)^
2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-alpha*(x-y)^2-
beta*(x-y))*(2*alpha*(y-x)+beta)*exp(-alpha*(y-x)^2-beta*(y-x)))
-(-2*alpha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-a
lpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(
-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+
exp(-(y-delta)^4)+exp(-(y+delta)^2))+exp(-alpha*(x-y)^2-beta*(x-
y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(
y-x)))^2*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x
)^2-beta*(y-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2))*((-2*alpha*
(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-be
ta*(y-x))+exp(-alpha*(x-y)^2-beta*(x-y))*(2*alpha*(y-x)+beta)*ex
p(-alpha*(y-x)^2-beta*(y-x)))-exp(-alpha*(x-y)^2-beta*(x-y))/(-1
+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*
(-(-4*(x-delta)^3*exp(-(x-delta)^4)+(-2*x-2*delta)*exp(-(x+delta
)^2))*exp(-alpha*(y-x)^2-beta*(y-x))-(exp(-(x-delta)^4)+exp(-(x+
delta)^2))*(2*alpha*(y-x)+beta)*exp(-alpha*(y-x)^2-beta*(y-x))))
/(exp(-(x-delta)^4)+exp(-(x+delta)^2))^2*(-4*(x-delta)^3*exp(-(x
-delta)^4)+(-2*x-2*delta)*exp(-(x+delta)^2))-2*((-exp(-(x-delta)
^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(
-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*e
xp(-alpha*(y-x)^2-beta*(y-x)))-exp(-alpha*(x-y)^2-beta*(x-y))/(-
1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))
*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta
*(y-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(x-delta)^4)
```

```
+exp(-(x+delta)^2))^3*(-4*(x-delta)^3*exp(-(x-delta)^4)+(-2*x-2*
delta)*exp(-(x+delta)^2))^2+((-exp(-(x-delta)^4)-exp(-(x+delta)^
2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta
*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-b
eta*(y-x)))-exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^
2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^
4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-del
ta)^4)+exp(-(y+delta)^2)))/(exp(-(x-delta)^4)+exp(-(x+delta)^2))
^2*(-12*(x-delta)^2*exp(-(x-delta)^4)+16*(x-delta)^6*exp(-(x-del
ta)^4)-2*exp(-(x+delta)^2)+(-2*x-2*delta)^2*exp(-(x+delta)^2)):
> diff(Dsx(x,y),y);
```

$$
\begin{aligned}
&-\left(0.4\left(\left(-4\,(y-1)^3\,e^{(-(y-1)^4)}+(-2\,y-2)\,e^{(-(y+1)^2)}\right)e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\right.\right.\\
&\quad\left.+\left(e^{(-(y-1)^4)}+e^{(-(y+1)^2)}\right)(4\,x-4\,y-0.4)\,e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\right)\Big/\Big(\\
&\quad -1+e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\Big)-0.4\\
&\quad\left(-e^{(-(x-1)^4)}-e^{(-(x+1)^2)}+\left(e^{(-(y-1)^4)}+e^{(-(y+1)^2)}\right)e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\right)\Big(\\
&\quad (4\,x-4\,y-0.4)\,e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\\
&\quad +e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,(-4\,y+4\,x+0.4)\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\Big)\Big/\\
&\quad \left(-1+e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\right)^2-4\,e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\\
&\quad\left(-\left(e^{(-(x-1)^4)}+e^{(-(x+1)^2)}\right)e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}+e^{(-(y-1)^4)}+e^{(-(y+1)^2)}\right)\Big/\Big(\\
&\quad -1+e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\Big)-(-4\,x+4\,y+0.4)\,(4\,x-4\,y-0.4)\\
&\quad e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\left(-\left(e^{(-(x-1)^4)}+e^{(-(x+1)^2)}\right)e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}+e^{(-(y-1)^4)}+e^{(-(y+1)^2)}\right)\\
&\quad \Big/\left(-1+e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\right)+(-4\,x+4\,y+0.4)\\
&\quad e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\left(-\left(e^{(-(x-1)^4)}+e^{(-(x+1)^2)}\right)e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}+e^{(-(y-1)^4)}+e^{(-(y+1)^2)}\right)\Big(\\
&\quad (4\,x-4\,y-0.4)\,e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\\
&\quad +e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,(-4\,y+4\,x+0.4)\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\Big)\Big/\\
&\quad \left(-1+e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\right)^2-(-4\,x+4\,y+0.4)\,e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\Big(\\
&\quad -\left(e^{(-(x-1)^4)}+e^{(-(x+1)^2)}\right)(-4\,y+4\,x+0.4)\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}-4\,(y-1)^3\,e^{(-(y-1)^4)}\\
&\quad +(-2\,y-2)\,e^{(-(y+1)^2)}\Big)\Big/\left(-1+e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\,e^{(-2\,(y-x)^2+0.4\,y-0.4\,x)}\right)\Big)\Big/\Big(\\
&\quad e^{(-(x-1)^4)}+e^{(-(x+1)^2)}\Big)+\left(\left(\left(-4\,(y-1)^3\,e^{(-(y-1)^4)}+(-2\,y-2)\,e^{(-(y+1)^2)}\right)e^{(-2\,(x-y)^2+0.4\,x-0.4\,y)}\right.\right.
\end{aligned}
$$

$$+\left(e^{(-(y-1)^4)}+e^{(-(y+1)^2)}\right)(4x-4y-0.4)\,e^{(-2(x-y)^2+0.4x-0.4y)}\Big)\Big/\Big($$

$$-1+e^{(-2(x-y)^2+0.4x-0.4y)}\,e^{(-2(y-x)^2+0.4y-0.4x)}\Big)-$$

$$\left(-e^{(-(x-1)^4)}-e^{(-(x+1)^2)}+\left(e^{(-(y-1)^4)}+e^{(-(y+1)^2)}\right)e^{(-2(x-y)^2+0.4x-0.4y)}\right)\Big($$

$$(4x-4y-0.4)\,e^{(-2(x-y)^2+0.4x-0.4y)}\,e^{(-2(y-x)^2+0.4y-0.4x)}$$

$$+e^{(-2(x-y)^2+0.4x-0.4y)}(-4y+4x+0.4)\,e^{(-2(y-x)^2+0.4y-0.4x)}\Big)\Big/$$

$$\left(-1+e^{(-2(x-y)^2+0.4x-0.4y)}\,e^{(-2(y-x)^2+0.4y-0.4x)}\right)^2-(4x-4y-0.4)\,e^{(-2(x-y)^2+0.4x-0.4y)}$$

$$\left(-\left(e^{(-(x-1)^4)}+e^{(-(x+1)^2)}\right)e^{(-2(y-x)^2+0.4y-0.4x)}+e^{(-(y-1)^4)}+e^{(-(y+1)^2)}\right)\Big/\Big($$

$$-1+e^{(-2(x-y)^2+0.4x-0.4y)}\,e^{(-2(y-x)^2+0.4y-0.4x)}\Big)+e^{(-2(x-y)^2+0.4x-0.4y)}$$

$$\left(-\left(e^{(-(x-1)^4)}+e^{(-(x+1)^2)}\right)e^{(-2(y-x)^2+0.4y-0.4x)}+e^{(-(y-1)^4)}+e^{(-(y+1)^2)}\right)\Big($$

$$(4x-4y-0.4)\,e^{(-2(x-y)^2+0.4x-0.4y)}\,e^{(-2(y-x)^2+0.4y-0.4x)}$$

$$+e^{(-2(x-y)^2+0.4x-0.4y)}(-4y+4x+0.4)\,e^{(-2(y-x)^2+0.4y-0.4x)}\Big)\Big/$$

$$\left(-1+e^{(-2(x-y)^2+0.4x-0.4y)}\,e^{(-2(y-x)^2+0.4y-0.4x)}\right)^2-e^{(-2(x-y)^2+0.4x-0.4y)}\Big($$

$$-\left(e^{(-(x-1)^4)}+e^{(-(x+1)^2)}\right)(-4y+4x+0.4)\,e^{(-2(y-x)^2+0.4y-0.4x)}-4(y-1)^3\,e^{(-(y-1)^4)}$$

$$+(-2y-2)\,e^{(-(y+1)^2)}\Big)\Big/\left(-1+e^{(-2(x-y)^2+0.4x-0.4y)}\,e^{(-2(y-x)^2+0.4y-0.4x)}\right)\Big)$$

$$\left(-4(x-1)^3\,e^{(-(x-1)^4)}+(-2x-2)\,e^{(-(x+1)^2)}\right)\Big/\left(e^{(-(x-1)^4)}+e^{(-(x+1)^2)}\right)^2$$

```
> a12:=(x,y)->-(-beta*((-4*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*d
elta)*exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y))+(exp(-(y
-delta)^4)+exp(-(y+delta)^2))*(2*alpha*(x-y)+beta)*exp(-alpha*(x
-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha
*(y-x)^2-beta*(y-x)))+beta*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)
+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(
x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-bet
a*(y-x)))^2*((2*alpha*(x-y)+beta)*exp(-alpha*(x-y)^2-beta*(x-y))
*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-alpha*(x-y)^2-beta*(x-y))*(
-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x)))-2*alpha*exp
(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*e
xp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delt
a)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-delta)^4)+exp(-(y+
delta)^2))-(-2*alpha*(x-y)-beta)*(2*alpha*(x-y)+beta)*exp(-alpha
*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alp
ha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*
exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-delta)^4)+exp(-(y+delta)^
```

```
2))+(-2*alpha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp
(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))^2*(-
(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y
-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2))*((2*alpha*(x-y)+beta)*
exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))+ex
p(-alpha*(x-y)^2-beta*(x-y))*(-2*alpha*(y-x)-beta)*exp(-alpha*(y
-x)^2-beta*(y-x)))-(-2*alpha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta
*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-be
ta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*(-2*alpha*(y-
x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x))-4*(y-delta)^3*exp(-(y-de
lta)^4)+(-2*y-2*delta)*exp(-(y+delta)^2)))/(exp(-(x-delta)^4)+ex
p(-(x+delta)^2))+(((-4*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*del
ta)*exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y))+(exp(-(y-d
elta)^4)+exp(-(y+delta)^2))*(2*alpha*(x-y)+beta)*exp(-alpha*(x-y
)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(
y-x)^2-beta*(y-x)))-(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-
(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/
(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)
))^2*((2*alpha*(x-y)+beta)*exp(-alpha*(x-y)^2-beta*(x-y))*exp(-a
lpha*(y-x)^2-beta*(y-x))+exp(-alpha*(x-y)^2-beta*(x-y))*(-2*alph
a*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x)))-(2*alpha*(x-y)+bet
a)*exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-beta*(x
-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(
x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-delta)^4)+ex
p(-(y+delta)^2))+exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(
x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))^2*(-(exp(-(x-
delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(
-(y-delta)^4)+exp(-(y+delta)^2))*((2*alpha*(x-y)+beta)*exp(-alph
a*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-alpha*
(x-y)^2-beta*(x-y))*(-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-bet
a*(y-x)))-exp(-alpha*(x-y)^2-beta*(x-y))/(-1+exp(-alpha*(x-y)^2-
beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)
+exp(-(x+delta)^2))*(-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-bet
a*(y-x))-4*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*delta)*exp(-(y+
delta)^2)))/(exp(-(x-delta)^4)+exp(-(x+delta)^2))^2*(-4*(x-delta
)^3*exp(-(x-delta)^4)+(-2*x-2*delta)*exp(-(x+delta)^2)):
> diff(Dsy(x,y),x);
```

$$-\left(\frac{2\,\alpha\,e^{(-\alpha(y-x)^2-\beta(y-x))}\left(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)e^{(-\alpha(x-y)^2-\beta(x-y))}\right)}{-1+e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))}}\right.$$

$$\left.+\left(-2\,\alpha\,(y-x)-\beta\right)\left(2\,\alpha\,(y-x)+\beta\right)e^{(-\alpha(y-x)^2-\beta(y-x))}\right)$$

$$
\left(-e^{\left(-(x-\delta)^4\right)}-e^{\left(-(x+\delta)^2\right)}+\left(e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\right)\Big/\left(
$$

$$
-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)+\left(-2\alpha(y-x)-\beta\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\left(
$$

$$
4(x-\delta)^3e^{\left(-(x-\delta)^4\right)}-(-2x-2\delta)e^{\left(-(x+\delta)^2\right)}
$$

$$
+\left(e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)\left(-2\alpha(x-y)-\beta\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\Big)\Big/\left(
$$

$$
-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)-\left(-2\alpha(y-x)-\beta\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}
$$

$$
\left(-e^{\left(-(x-\delta)^4\right)}-e^{\left(-(x+\delta)^2\right)}+\left(e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\right)\left(
$$

$$
\left(-2\alpha(x-y)-\beta\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}
$$

$$
+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\left(2\alpha(y-x)+\beta\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\Big)\Big/
$$

$$
\left(-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)^2-\beta
$$

$$
\left(-\left(e^{\left(-(x-\delta)^4\right)}+e^{\left(-(x+\delta)^2\right)}\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}+e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)\left(
$$

$$
\left(-2\alpha(x-y)-\beta\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}
$$

$$
+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\left(2\alpha(y-x)+\beta\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\Big)\Big/
$$

$$
\left(-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)^2+\beta\left(
$$

$$
-\left(-4(x-\delta)^3e^{\left(-(x-\delta)^4\right)}+(-2x-2\delta)e^{\left(-(x+\delta)^2\right)}\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}
$$

$$
-\left(e^{\left(-(x-\delta)^4\right)}+e^{\left(-(x+\delta)^2\right)}\right)\left(2\alpha(y-x)+\beta\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\Big)\Big/\left(
$$

$$
-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)\Bigg)\Big/\left(e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)+\left(\left(2\alpha(y-x)+\beta\right)\right.
$$

$$
e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\left(-e^{\left(-(x-\delta)^4\right)}-e^{\left(-(x+\delta)^2\right)}+\left(e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\right)\Big/\left(
$$

$$
-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)+e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\left(4(x-\delta)^3e^{\left(-(x-\delta)^4\right)}\right.
$$

$$
\left.-(-2x-2\delta)e^{\left(-(x+\delta)^2\right)}+\left(e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)\left(-2\alpha(x-y)-\beta\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\right)
$$

$$
\Big/\left(-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)-e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}
$$

$$
\left(-e^{\left(-(x-\delta)^4\right)}-e^{\left(-(x+\delta)^2\right)}+\left(e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\right)\left(
$$

$$
\left(-2\alpha(x-y)-\beta\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}
$$

$$
+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\left(2\alpha(y-x)+\beta\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\Big)\Big/
$$

$$\left(-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)^2+$$
$$\left(-\left(e^{\left(-(x-\delta)^4\right)}+e^{\left(-(x+\delta)^2\right)}\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}+e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)\left(\right.$$
$$\left(-2\alpha(x-y)-\beta\right)e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}$$
$$\left.+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}\left(2\alpha(y-x)+\beta\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)\Big/$$
$$\left(-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)^2-\left(\right.$$
$$-\left(-4(x-\delta)^3e^{\left(-(x-\delta)^4\right)}+(-2x-2\delta)e^{\left(-(x+\delta)^2\right)}\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}$$
$$\left.-\left(e^{\left(-(x-\delta)^4\right)}+e^{\left(-(x+\delta)^2\right)}\right)\left(2\alpha(y-x)+\beta\right)e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)\Big/\left(\right.$$
$$\left.-1+e^{\left(-\alpha(x-y)^2-\beta(x-y)\right)}e^{\left(-\alpha(y-x)^2-\beta(y-x)\right)}\right)\left(-4(y-\delta)^3e^{\left(-(y-\delta)^4\right)}+(-2y-2\delta)e^{\left(-(y+\delta)^2\right)}\right)$$
$$\Big/\left(e^{\left(-(y-\delta)^4\right)}+e^{\left(-(y+\delta)^2\right)}\right)^2$$

```
> a21:=(x,y)->-(2*alpha*exp(-alpha*(y-x)^2-beta*(y-x))*(-exp(-(x-d
elta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))
*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-
y))*exp(-alpha*(y-x)^2-beta*(y-x)))+(-2*alpha*(y-x)-beta)*(2*alp
ha*(y-x)+beta)*exp(-alpha*(y-x)^2-beta*(y-x))*(-exp(-(x-delta)^4
)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-a
lpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp
(-alpha*(y-x)^2-beta*(y-x)))+(-2*alpha*(y-x)-beta)*exp(-alpha*(y
-x)^2-beta*(y-x))*(4*(x-delta)^3*exp(-(x-delta)^4)-(-2*x-2*delta
)*exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*(-2*al
pha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(
x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))-(-2*alpha*(y-
x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x))*(-exp(-(x-delta)^4)-exp(
-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(
x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alph
a*(y-x)^2-beta*(y-x)))^2*((-2*alpha*(x-y)-beta)*exp(-alpha*(x-y)
^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-alpha*(x-y)^2
-beta*(x-y))*(2*alpha*(y-x)+beta)*exp(-alpha*(y-x)^2-beta*(y-x))
)-beta/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-bet
a*(y-x)))^2*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(
y-x)^2-beta*(y-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2))*((-2*alp
ha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2
-beta*(y-x))+exp(-alpha*(x-y)^2-beta*(x-y))*(2*alpha*(y-x)+beta)
*exp(-alpha*(y-x)^2-beta*(y-x)))+beta/(-1+exp(-alpha*(x-y)^2-bet
a*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(-4*(x-delta)^3*exp(-
(x-delta)^4)+(-2*x-2*delta)*exp(-(x+delta)^2))*exp(-alpha*(y-x)^
2-beta*(y-x))-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*(2*alpha*(y-
x)+beta)*exp(-alpha*(y-x)^2-beta*(y-x))))/(exp(-(y-delta)^4)+exp
(-(y+delta)^2))+((2*alpha*(y-x)+beta)*exp(-alpha*(y-x)^2-beta*(y
```

```
-x))*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+ex
p(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*
(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))+exp(-alpha*(
y-x)^2-beta*(y-x))*(4*(x-delta)^3*exp(-(x-delta)^4)-(-2*x-2*delt
a)*exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*(-2*a
lpha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*
(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))-exp(-alpha*(
y-x)^2-beta*(y-x))*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(
y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(
-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))
)^2*((-2*alpha*(x-y)-beta)*exp(-alpha*(x-y)^2-beta*(x-y))*exp(-a
lpha*(y-x)^2-beta*(y-x))+exp(-alpha*(x-y)^2-beta*(x-y))*(2*alpha
*(y-x)+beta)*exp(-alpha*(y-x)^2-beta*(y-x)))+1/(-1+exp(-alpha*(x
-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))^2*(-(exp(-(x-d
elta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-
(y-delta)^4)+exp(-(y+delta)^2))*((-2*alpha*(x-y)-beta)*exp(-alph
a*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-alpha*
(x-y)^2-beta*(x-y))*(2*alpha*(y-x)+beta)*exp(-alpha*(y-x)^2-beta
*(y-x)))-1/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2
-beta*(y-x)))*(-(-4*(x-delta)^3*exp(-(x-delta)^4)+(-2*x-2*delta)
*exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))-(exp(-(x-delt
a)^4)+exp(-(x+delta)^2))*(2*alpha*(y-x)+beta)*exp(-alpha*(y-x)^2
-beta*(y-x))))/(exp(-(y-delta)^4)+exp(-(y+delta)^2))^2*(-4*(y-de
lta)^3*exp(-(y-delta)^4)+(-2*y-2*delta)*exp(-(y+delta)^2)):
> diff(Dsy(x,y),y);
```

$$
-\left(
-\frac{2\,\alpha\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\left(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\right)}{-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}}
+(-2\,\alpha\,(y-x)-\beta)^2\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}
\left(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\right)\Big/\left(
-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\right)+(-2\,\alpha\,(y-x)-\beta)\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\left(
(-4\,(y-\delta)^3\,e^{(-(y-\delta)^4)}+(-2\,y-2\,\delta)\,e^{(-(y+\delta)^2)})\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}
+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)(2\,\alpha\,(x-y)+\beta)\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\right)\Big/\left(
-1+e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}\right)-(-2\,\alpha\,(y-x)-\beta)\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}
\left(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+\left(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)}\right)e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\right)\left(
(2\,\alpha\,(x-y)+\beta)\,e^{(-\alpha\,(x-y)^2-\beta\,(x-y))}\,e^{(-\alpha\,(y-x)^2-\beta\,(y-x))}
\right.
$$

$$+e^{(-\alpha(x-y)^2-\beta(x-y))}(-2\alpha(y-x)-\beta)\,e^{(-\alpha(y-x)^2-\beta(y-x))})\Big/$$

$$(-1+e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))})^2-\beta$$

$$(-(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)})\,e^{(-\alpha(y-x)^2-\beta(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})\,($$

$$(2\alpha(x-y)+\beta)\,e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))}$$

$$+e^{(-\alpha(x-y)^2-\beta(x-y))}(-2\alpha(y-x)-\beta)\,e^{(-\alpha(y-x)^2-\beta(y-x))})\Big/$$

$$(-1+e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))})^2+\beta\,($$

$$-(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)})(-2\alpha(y-x)-\beta)\,e^{(-\alpha(y-x)^2-\beta(y-x))}-4(y-\delta)^3\,e^{(-(y-\delta)^4)}$$

$$+(-2y-2\delta)\,e^{(-(y+\delta)^2)})\Big/(-1+e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))})\Big)\Big/($$

$$e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})+\Bigg((-2\alpha(y-x)-\beta)\,e^{(-\alpha(y-x)^2-\beta(y-x))}$$

$$(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})\,e^{(-\alpha(x-y)^2-\beta(x-y))})\Big/($$

$$-1+e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))})$$

$$+\frac{\beta(-(e^{(-(x-\delta)^4)}+e^{(-(x+\delta)^2)})\,e^{(-\alpha(y-x)^2-\beta(y-x))}+e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})}{-1+e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))}}\Bigg)$$

$$(-4(y-\delta)^3\,e^{(-(y-\delta)^4)}+(-2y-2\delta)\,e^{(-(y+\delta)^2)})\Big/(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})^2+($$

$$(-2\alpha(y-x)-\beta)\,e^{(-\alpha(y-x)^2-\beta(y-x))}$$

$$(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})\,e^{(-\alpha(x-y)^2-\beta(x-y))})\Big/($$

$$-1+e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))})+e^{(-\alpha(y-x)^2-\beta(y-x))}($$

$$(-4(y-\delta)^3\,e^{(-(y-\delta)^4)}+(-2y-2\delta)\,e^{(-(y+\delta)^2)})\,e^{(-\alpha(x-y)^2-\beta(x-y))}$$

$$+(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})(2\alpha(x-y)+\beta)\,e^{(-\alpha(x-y)^2-\beta(x-y))})\Big/($$

$$-1+e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))})-e^{(-\alpha(y-x)^2-\beta(y-x))}$$

$$(-e^{(-(x-\delta)^4)}-e^{(-(x+\delta)^2)}+(e^{(-(y-\delta)^4)}+e^{(-(y+\delta)^2)})\,e^{(-\alpha(x-y)^2-\beta(x-y))})\,($$

$$(2\alpha(x-y)+\beta)\,e^{(-\alpha(x-y)^2-\beta(x-y))}\,e^{(-\alpha(y-x)^2-\beta(y-x))}$$

$$+ e^{(-\alpha (x-y)^2 - \beta (x-y))} (-2\,\alpha (y-x) - \beta)\, e^{(-\alpha (y-x)^2 - \beta (y-x))}) \Big/$$

$$(-1 + e^{(-\alpha (x-y)^2 - \beta (x-y))}\, e^{(-\alpha (y-x)^2 - \beta (y-x))})^2 +$$

$$(-(e^{(-(x-\delta)^4)} + e^{(-(x+\delta)^2)})\, e^{(-\alpha (y-x)^2 - \beta (y-x))} + e^{(-(y-\delta)^4)} + e^{(-(y+\delta)^2)})\,($$

$$(2\,\alpha (x-y) + \beta)\, e^{(-\alpha (x-y)^2 - \beta (x-y))}\, e^{(-\alpha (y-x)^2 - \beta (y-x))}$$

$$+ e^{(-\alpha (x-y)^2 - \beta (x-y))} (-2\,\alpha (y-x) - \beta)\, e^{(-\alpha (y-x)^2 - \beta (y-x))}) \Big/$$

$$(-1 + e^{(-\alpha (x-y)^2 - \beta (x-y))}\, e^{(-\alpha (y-x)^2 - \beta (y-x))})^2 - ($$

$$-(e^{(-(x-\delta)^4)} + e^{(-(x+\delta)^2)})\,(-2\,\alpha (y-x) - \beta)\, e^{(-\alpha (y-x)^2 - \beta (y-x))} - 4\,(y-\delta)^3\, e^{(-(y-\delta)^4)}$$

$$+ (-2\,y - 2\,\delta)\, e^{(-(y+\delta)^2)}) \Big/ (-1 + e^{(-\alpha (x-y)^2 - \beta (x-y))}\, e^{(-\alpha (y-x)^2 - \beta (y-x))}))$$

$$(-4\,(y-\delta)^3\, e^{(-(y-\delta)^4)} + (-2\,y - 2\,\delta)\, e^{(-(y+\delta)^2)}) \Big/ (e^{(-(y-\delta)^4)} + e^{(-(y+\delta)^2)})^2 - 2 \left(\right.$$

$$\frac{e^{(-\alpha (y-x)^2 - \beta (y-x))}\,(-e^{(-(x-\delta)^4)} - e^{(-(x+\delta)^2)} + (e^{(-(y-\delta)^4)} + e^{(-(y+\delta)^2)})\, e^{(-\alpha (x-y)^2 - \beta (x-y))})}{-1 + e^{(-\alpha (x-y)^2 - \beta (x-y))}\, e^{(-\alpha (y-x)^2 - \beta (y-x))}}$$

$$\left. - \frac{-(e^{(-(x-\delta)^4)} + e^{(-(x+\delta)^2)})\, e^{(-\alpha (y-x)^2 - \beta (y-x))} + e^{(-(y-\delta)^4)} + e^{(-(y+\delta)^2)}}{-1 + e^{(-\alpha (x-y)^2 - \beta (x-y))}\, e^{(-\alpha (y-x)^2 - \beta (y-x))}} \right)$$

$$(-4\,(y-\delta)^3\, e^{(-(y-\delta)^4)} + (-2\,y - 2\,\delta)\, e^{(-(y+\delta)^2)})^2 \Big/ (e^{(-(y-\delta)^4)} + e^{(-(y+\delta)^2)})^3 + \left(\right.$$

$$\frac{e^{(-\alpha (y-x)^2 - \beta (y-x))}\,(-e^{(-(x-\delta)^4)} - e^{(-(x+\delta)^2)} + (e^{(-(y-\delta)^4)} + e^{(-(y+\delta)^2)})\, e^{(-\alpha (x-y)^2 - \beta (x-y))})}{-1 + e^{(-\alpha (x-y)^2 - \beta (x-y))}\, e^{(-\alpha (y-x)^2 - \beta (y-x))}}$$

$$\left. - \frac{-(e^{(-(x-\delta)^4)} + e^{(-(x+\delta)^2)})\, e^{(-\alpha (y-x)^2 - \beta (y-x))} + e^{(-(y-\delta)^4)} + e^{(-(y+\delta)^2)}}{-1 + e^{(-\alpha (x-y)^2 - \beta (x-y))}\, e^{(-\alpha (y-x)^2 - \beta (y-x))}} \right)$$

$$(-12\,(y-\delta)^2\, e^{(-(y-\delta)^4)} + 16\,(y-\delta)^6\, e^{(-(y-\delta)^4)} - 2\, e^{(-(y+\delta)^2)} + (-2\,y - 2\,\delta)^2\, e^{(-(y+\delta)^2)}) \Big/$$

$$(e^{(-(y-\delta)^4)} + e^{(-(y+\delta)^2)})^2$$

```
> a22:=(x,y)->-(-2*alpha*exp(-alpha*(y-x)^2-beta*(y-x))*(-exp(-(x-
delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2)
)*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x
-y))*exp(-alpha*(y-x)^2-beta*(y-x)))+(-2*alpha*(y-x)-beta)^2*exp
(-alpha*(y-x)^2-beta*(y-x))*(-exp(-(x-delta)^4)-exp(-(x+delta)^2
```

```
)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*
(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-be
ta*(y-x)))+(-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x))*
((-4*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*delta)*exp(-(y+delta)
^2))*exp(-alpha*(x-y)^2-beta*(x-y))+(exp(-(y-delta)^4)+exp(-(y+d
elta)^2))*(2*alpha*(x-y)+beta)*exp(-alpha*(x-y)^2-beta*(x-y)))/(
-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))
)-(-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x))*(-exp(-(x
-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2
))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(
x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))^2*((2*alpha*(x-y)+beta)*ex
p(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(
-alpha*(x-y)^2-beta*(x-y))*(-2*alpha*(y-x)-beta)*exp(-alpha*(y-x
)^2-beta*(y-x)))-beta/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-al
pha*(y-x)^2-beta*(y-x)))^2*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2
))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-delta)^4)+exp(-(y+delt
a)^2))*((2*alpha*(x-y)+beta)*exp(-alpha*(x-y)^2-beta*(x-y))*exp(
-alpha*(y-x)^2-beta*(y-x))+exp(-alpha*(x-y)^2-beta*(x-y))*(-2*al
pha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x)))+beta/(-1+exp(-al
pha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-
(x-delta)^4)+exp(-(x+delta)^2))*(-2*alpha*(y-x)-beta)*exp(-alpha
*(y-x)^2-beta*(y-x))-4*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*del
ta)*exp(-(y+delta)^2)))/(exp(-(y-delta)^4)+exp(-(y+delta)^2))+((
-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x))*(-exp(-(x-de
lta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*
exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y
))*exp(-alpha*(y-x)^2-beta*(y-x)))+beta/(-1+exp(-alpha*(x-y)^2-b
eta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+
exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-(y-delta)
^4)+exp(-(y+delta)^2)))/(exp(-(y-delta)^4)+exp(-(y+delta)^2))^2*
(-4*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*delta)*exp(-(y+delta)^
2))+((-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x))*(-exp(
-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta
)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-bet
a*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))+exp(-alpha*(y-x)^2-beta
*(y-x))*((-4*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*delta)*exp(-(
y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y))+(exp(-(y-delta)^4)+e
xp(-(y+delta)^2))*(2*alpha*(x-y)+beta)*exp(-alpha*(x-y)^2-beta*(
x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-bet
a*(y-x)))-exp(-alpha*(y-x)^2-beta*(y-x))*(-exp(-(x-delta)^4)-exp
(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*
(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alp
ha*(y-x)^2-beta*(y-x)))^2*((2*alpha*(x-y)+beta)*exp(-alpha*(x-y)
^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(-alpha*(x-y)^2
-beta*(x-y))*(-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x)
))+1/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*
(y-x)))^2*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-
x)^2-beta*(y-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2))*((2*alpha*
```

```
(x-y)+beta)*exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-be
ta*(y-x))+exp(-alpha*(x-y)^2-beta*(x-y))*(-2*alpha*(y-x)-beta)*e
xp(-alpha*(y-x)^2-beta*(y-x)))-1/(-1+exp(-alpha*(x-y)^2-beta*(x-
y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-delta)^4)+exp(-(x
+delta)^2))*(-2*alpha*(y-x)-beta)*exp(-alpha*(y-x)^2-beta*(y-x))
-4*(y-delta)^3*exp(-(y-delta)^4)+(-2*y-2*delta)*exp(-(y+delta)^2
)))/(exp(-(y-delta)^4)+exp(-(y+delta)^2))^2*(-4*(y-delta)^3*exp(
-(y-delta)^4)+(-2*y-2*delta)*exp(-(y+delta)^2))-2*(exp(-alpha*(y
-x)^2-beta*(y-x))*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y
-delta)^4)+exp(-(y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-
1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))
-1/(-1+exp(-alpha*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y
-x)))*(-(exp(-(x-delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2
-beta*(y-x))+exp(-(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(y-delt
a)^4)+exp(-(y+delta)^2))^3*(-4*(y-delta)^3*exp(-(y-delta)^4)+(-2
*y-2*delta)*exp(-(y+delta)^2))^2+(exp(-alpha*(y-x)^2-beta*(y-x))
*(-exp(-(x-delta)^4)-exp(-(x+delta)^2)+(exp(-(y-delta)^4)+exp(-(
y+delta)^2))*exp(-alpha*(x-y)^2-beta*(x-y)))/(-1+exp(-alpha*(x-y
)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))-1/(-1+exp(-alpha
*(x-y)^2-beta*(x-y))*exp(-alpha*(y-x)^2-beta*(y-x)))*(-(exp(-(x-
delta)^4)+exp(-(x+delta)^2))*exp(-alpha*(y-x)^2-beta*(y-x))+exp(
-(y-delta)^4)+exp(-(y+delta)^2)))/(exp(-(y-delta)^4)+exp(-(y+del
ta)^2))^2*(-12*(y-delta)^2*exp(-(y-delta)^4)+16*(y-delta)^6*exp(
-(y-delta)^4)-2*exp(-(y+delta)^2)+(-2*y-2*delta)^2*exp(-(y+delta
)^2)):
```

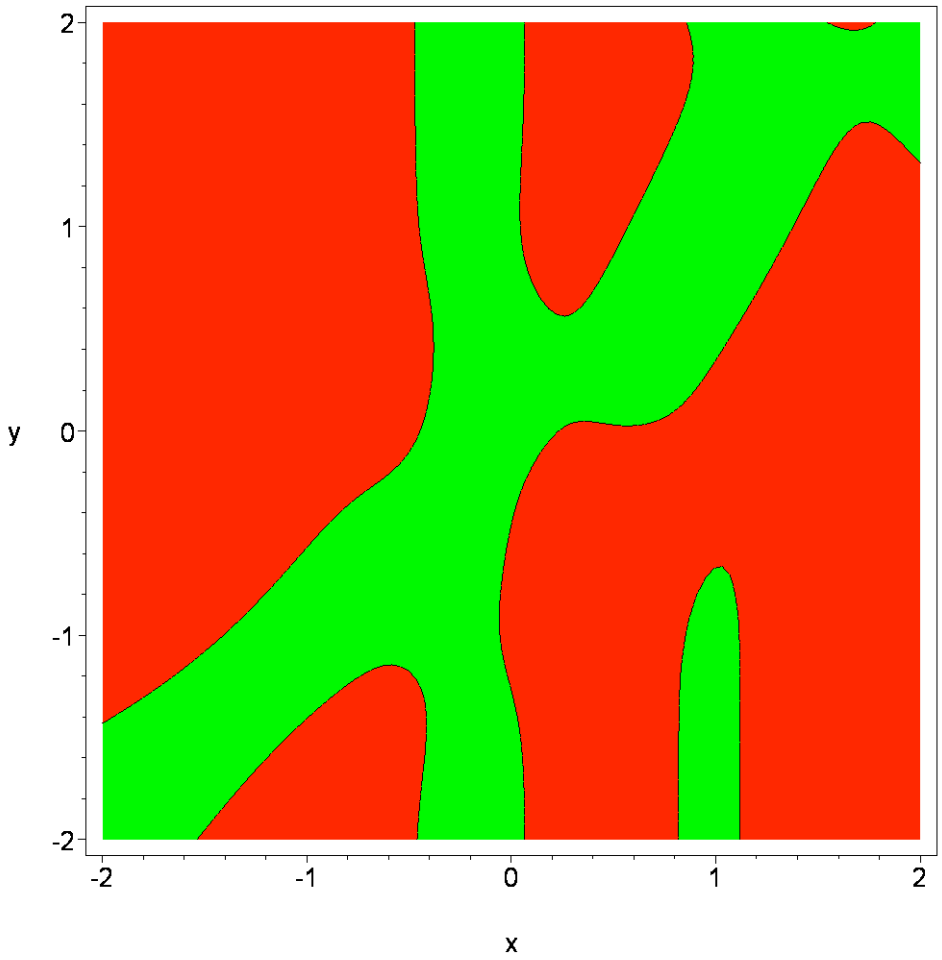
```
> alpha:=2;beta:=-0.4;delta:=1;
```

$$\alpha := 2$$

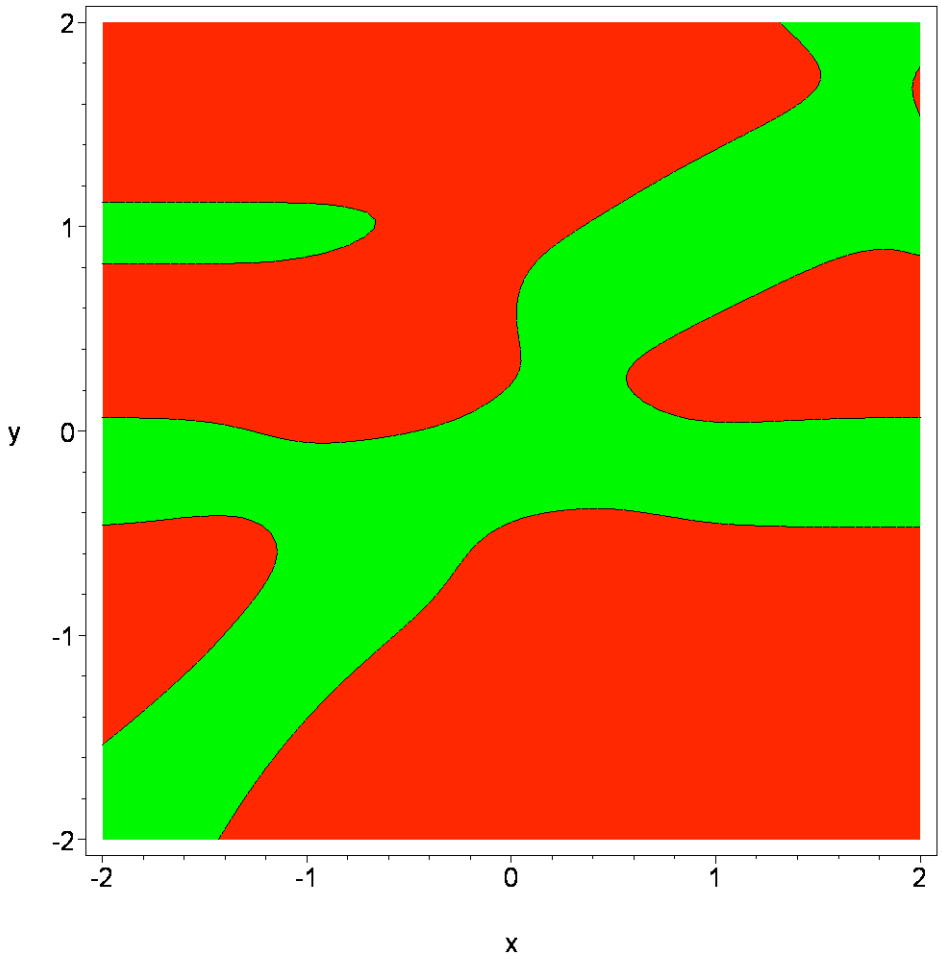
$$\beta := -0.4$$

$$\delta := 1$$

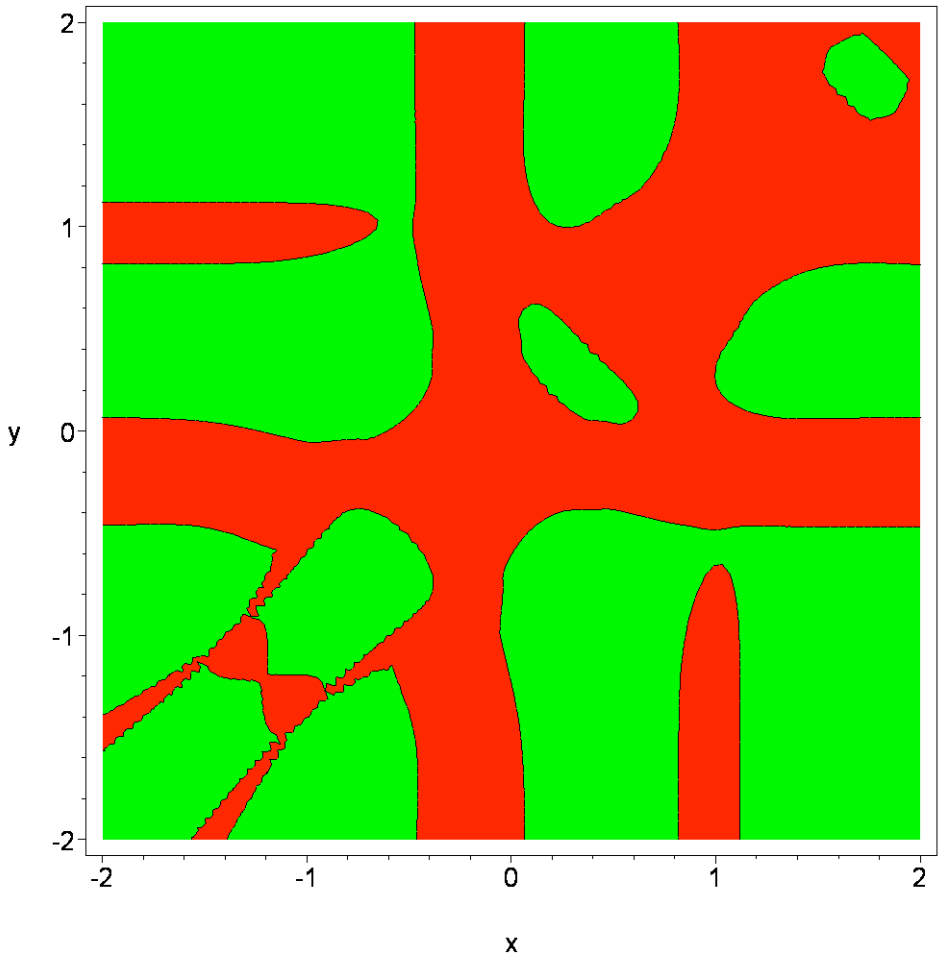
```
> with(plots):
> contourplot(piecewise(abs(x-y)<0.01,0,a11(x,y)),x=-2..2,y=-2..2,
  contours=[0],filled=true,coloring=[red,green],grid=[100,100],axe
  s=boxed);
```



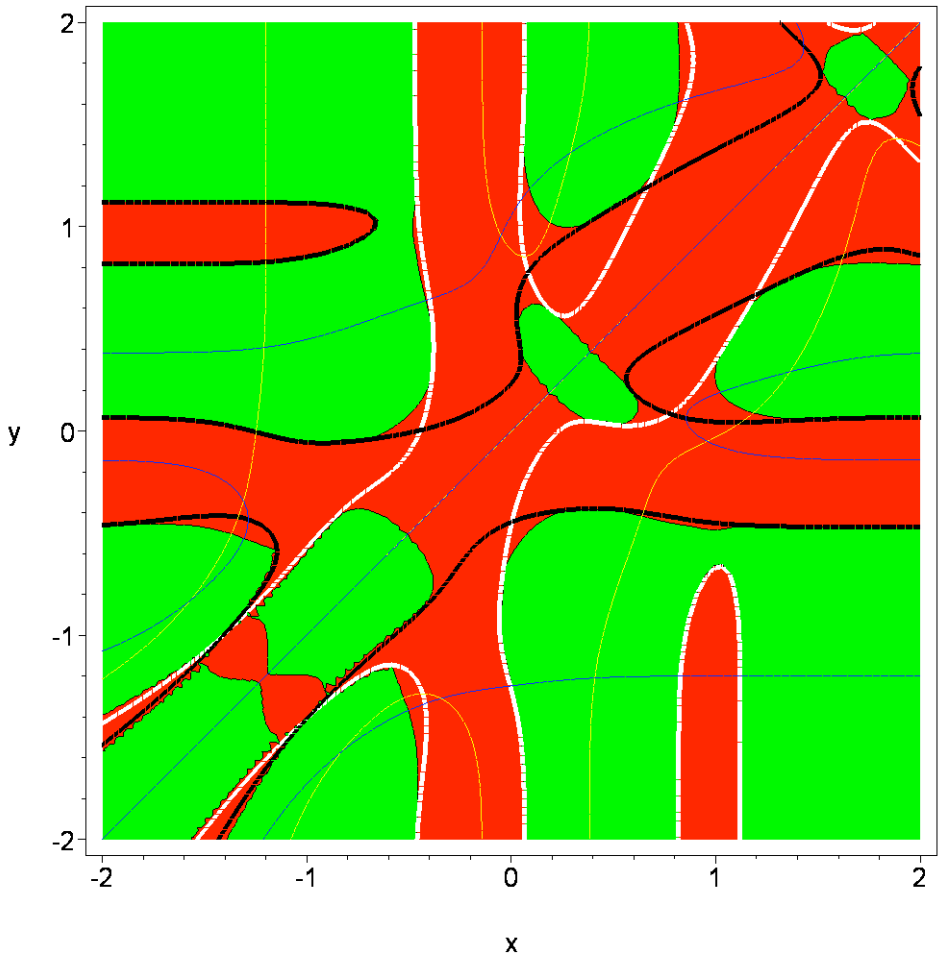

```
> contourplot(piecewise(abs(x-y)<0.01,0,a22(x,y)),x=-2..2,y=-2..2,
  contours=[0],filled=true,coloring=[red,green],grid=[100,100],axe
  s=boxed);
```



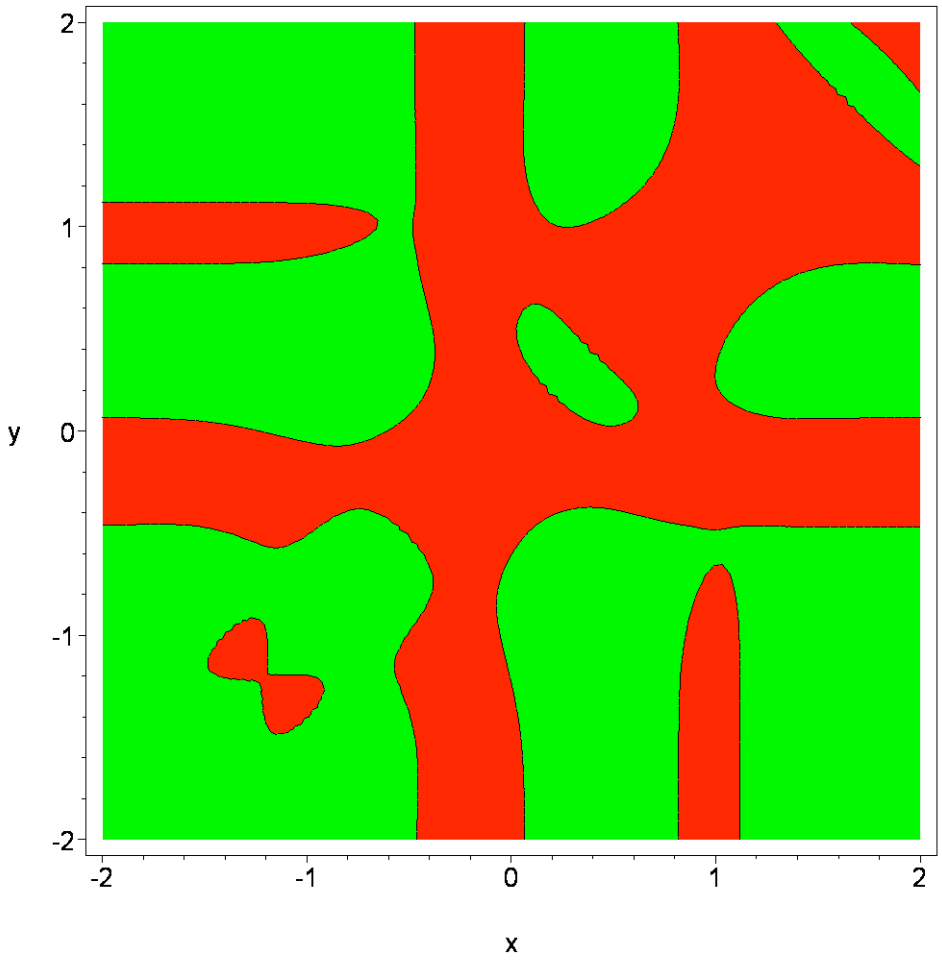

```
> contourplot(piecewise(abs(x-y)<0.01,0,a11(x,y)*a22(x,y)-abs(a12(
  x,y)*a21(x,y))),x=-2..2,y=-2..2,contours=[0],filled=true,colorin
  g=[red,green],grid=[100,100],axes=boxed);
```



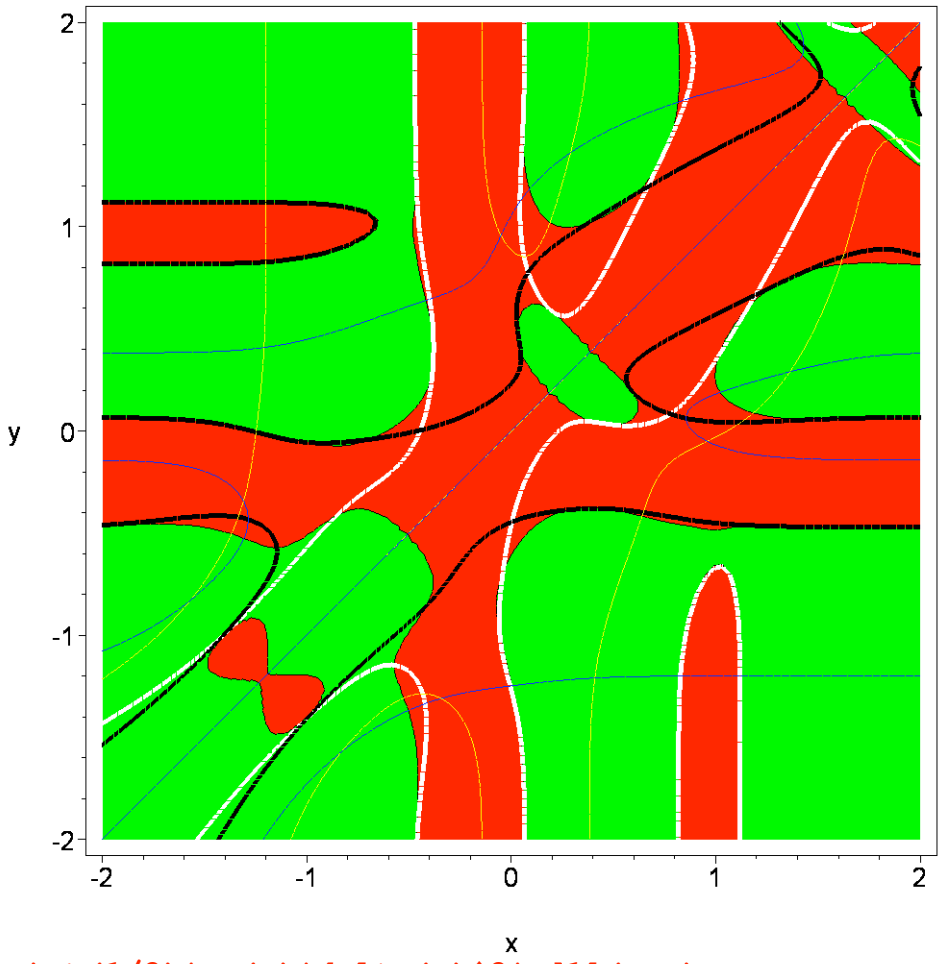

```
> A:=contourplot(piecewise(abs(x-y)<0.01,0,a11(x,y)),x=-2..2,y=-2.
  .2,contours=[0],color=white,grid=[100,100],axes=boxed,thickness=
  5):
> B:=contourplot(piecewise(abs(x-y)<0.01,0,a22(x,y)),x=-2..2,y=-2.
  .2,contours=[0],color=black,grid=[100,100],axes=boxed,thickness=
  5):
> C:=contourplot(piecewise(abs(x-y)<0.01,0,a11(x,y)*a22(x,y)-abs(a
  12(x,y)*a21(x,y))),x=-2..2,y=-2..2,contours=[0],filled=true,colo
  ring=[red,green],grid=[100,100],axes=boxed):
> E:=contourplot(piecewise(abs(x-y)<0.01,0,Dsx(x,y)),x=-2..2,y=-2.
  .2,contours=[0],color=yellow,grid=[100,100],axes=boxed):
> F:=contourplot(piecewise(abs(x-y)<0.01,0,Dsy(x,y)),x=-2..2,y=-2.
  .2,contours=[0],color=blue,grid=[100,100],axes=boxed):
> display({A,B,C,E,F}); #there are three singularities that have
  total stability
```




```
> contourplot(piecewise(abs(x-y)<0.01,0,a11(x,y)*a22(x,y)-a12(x,y)
  *a21(x,y)),x=-2..2,y=-2..2,contours=[0],filled=true,coloring=[re
  d,green],grid=[100,100],axes=boxed);
```




```
> G:=contourplot(piecewise(abs(x-y)<0.01,0,a11(x,y)*a22(x,y)-a12(x
  ,y)*a21(x,y)),x=-2..2,y=-2..2,contours=[0],filled=true,coloring=
  [red,green],grid=[100,100],axes=boxed):
> display({A,B,G,E,F}); #the three totally stable singularities
  are also strongly stable (naturally!)
```




```
> c1:=(x,y)->(1/2)*mu(x)*delta(x)^2*n[1](x,y);
```

$$c1 := (x, y) \to \frac{1}{2}\,\mu(x)\,\delta(x)^2\,n_1(x, y)$$

```
> c2:=(x,y)->(1/2)*mu(y)*delta(y)^2*n[2](x,y);
```

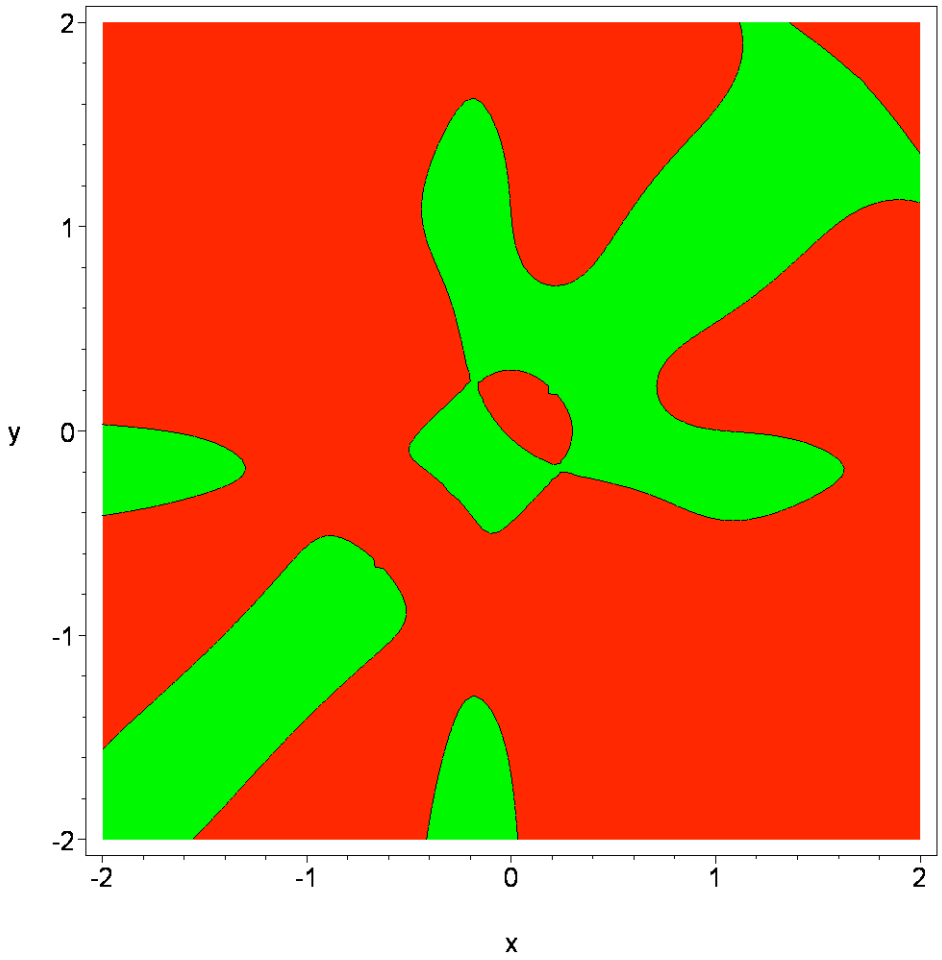
$$c2 := (x, y) \to \frac{1}{2}\,\mu(y)\,\delta(y)^2\,n_2(x, y)$$

```
> mu:=x->1; sigma:=x->0.01;
```

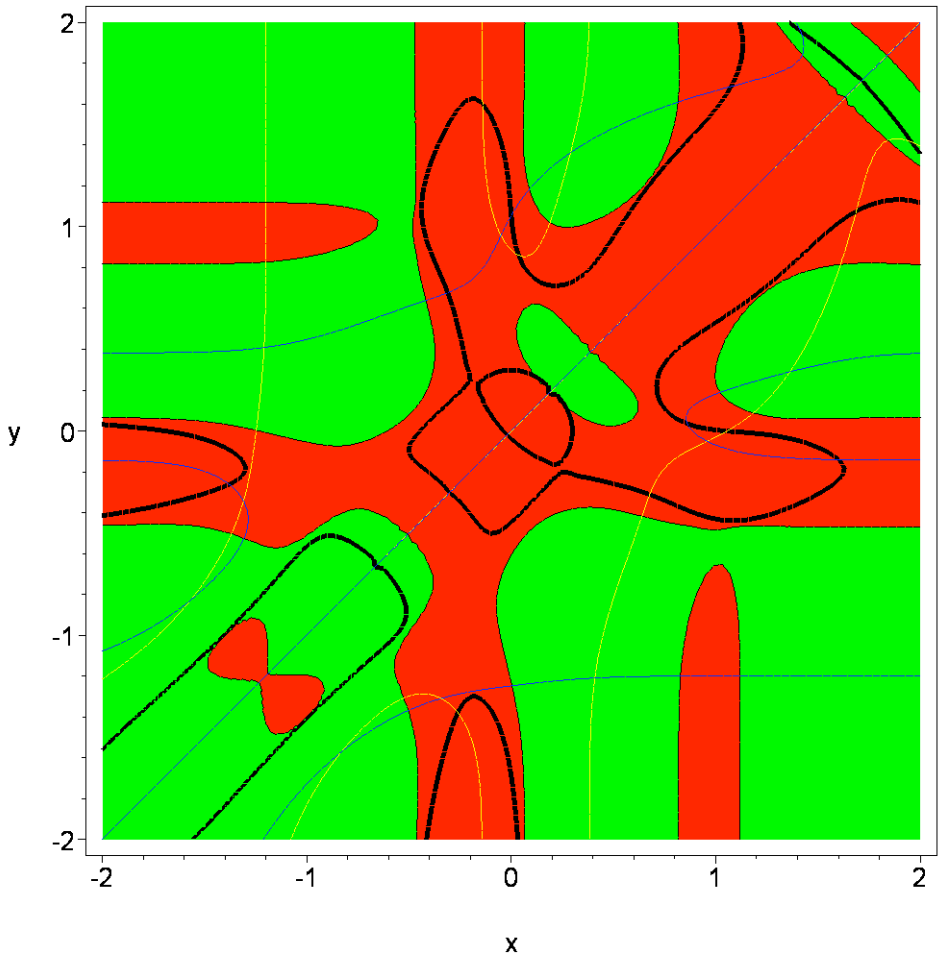
$$\mu := x \to 1$$

$$\sigma := x \to 0.01$$

```
> contourplot(piecewise(abs(x-y)<0.01,0,c1(x,y)*a11(x,y)+c2(x,y)*a
  22(x,y)),x=-2..2,y=-2..2,contours=[0],filled=true,coloring=[red,
  green],grid=[100,100],axes=boxed);
```




```
> H:=contourplot(piecewise(abs(x-y)<0.01,0,c1(x,y)*a11(x,y)+c2(x,y
  )*a22(x,y)),x=-2..2,y=-2..2,contours=[0],color=black,grid=[100,1
  00],axes=boxed,thickness=5):
> display({G,E,F,H}); #there are two singularities that lack any
  of the three types of stability
```


2
1
0
-1
-2
y
-2
-1
0
1
2
x

>