

The Lotka - Volterra asymmetric competition model

Full population dynamics

$$\frac{d}{dt} n_i = r[x_i] n_i \left(1 - \frac{\sum_{j=1}^l a[x_i, x_j] n_j}{k[x_i]} \right) \quad (i = 1, \dots, l)$$

DIMORPHIC RESIDENT POPULATION

Dimorphic resident population equilibrium:

```
Solve [ { 0 == 1 - (n1 + a[x1, x2] n2) / k[x1], 0 == 1 - (a[x2, x1] n1 + n2) / k[x2] }, {n1, n2} ];
Simplify[%]
```

```
{ {n1 -> (k[x1] - a[x1, x2] k[x2]) / (1 - a[x1, x2] a[x2, x1]), n2 -> (a[x2, x1] k[x1] - k[x2]) / (-1 + a[x1, x2] a[x2, x1]) } }
```

```
n1[x1_, x2_] := (k[x1] - a[x1, x2] k[x2]) / (1 - a[x1, x2] a[x2, x1]);
n2[x1_, x2_] := (k[x2] - a[x2, x1] k[x1]) / (1 - a[x1, x2] a[x2, x1]);
```

Dimorphic invasion fitness and derivatives

```
s_di[x1_, x2_, y_] := r[y] (1 - (a[y, x1] n1[x1, x2] + a[y, x2] n2[x1, x2]) / k[y]);
x1ds_di[x1_, x2_] := D[s_di[x1, x2, y], x1] /. {y -> x1};
x1dds_di[x1_, x2_] := D[s_di[x1, x2, y], {y, x1}] /. {y -> x1};
x2ds_di[x1_, x2_] := D[s_di[x1, x2, y], x2] /. {y -> x2};
x2dds_di[x1_, x2_] := D[s_di[x1, x2, y], {y, x2}] /. {y -> x2};
```

Default parameter values and functions :

```

In[10]:=
r[x_] := 1;
k[x_] := Exp[-(x - δ)^4] + Exp[-(x + δ)^2];
δ = 1;
a[x_, y_] := Exp[-α (x - y)^2 - β (x - y)];
α = 2;
β = -0.4;

```

Coexistence plot:

```

In[16]:=
coexBnd = ContourPlot[If[n1[x1, x2] > 0 && n2[x1, x2] > 0, 1, -1],
  {x1, -2, 2}, {x2, -2, 2}, Contours -> {0}, ContourStyle -> {Black, Thick},
  ContourShading -> False, PlotPoints -> 50];

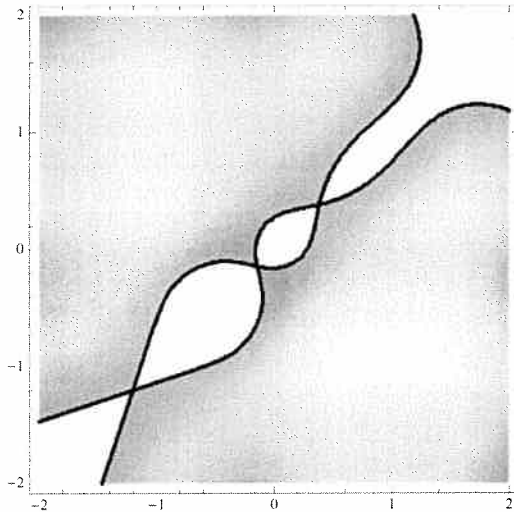
coexInt = DensityPlot[If[n1[x1, x2] > 0 && n2[x1, x2] > 0,
  n1[x1, x2] + n2[x1, x2]], {x1, -2, 2}, {x2, -2, 2}, PlotPoints -> 50];

```

```

In[18]:=
Show[coexInt, coexBnd]

```



Isocline plot

solid = x1-isocline; dashed = x2-isocline; black = evolutionarily stable; red = not evolutionarily stable

```

x1isoES =
  ContourPlot[If[n1[x1, x2] > 0 && n2[x1, x2] > 0 && x1ddsdi[x1, x2] ≤ 0,
    x1dsdi[x1, x2]], {x1, -2, 2}, {x2, -2, 2}, Contours → {0},
    ContourShading → False, ContourStyle → {Black, Thick}];

x1isoNES =
  ContourPlot[If[n1[x1, x2] > 0 && n2[x1, x2] > 0 && x1ddsdi[x1, x2] > 0,
    x1dsdi[x1, x2]], {x1, -2, 2}, {x2, -2, 2}, Contours → {0},
    ContourShading → False, ContourStyle → {Red, Thick}];

x2isoES =
  ContourPlot[If[n1[x1, x2] > 0 && n2[x1, x2] > 0 && x2ddsdi[x1, x2] ≤ 0,
    x2dsdi[x1, x2]], {x1, -2, 2}, {x2, -2, 2}, Contours → {0},
    ContourShading → False, ContourStyle → {Black, Thick, Dashed}];

x2isoNES =
  ContourPlot[If[n1[x1, x2] > 0 && n2[x1, x2] > 0 && x2ddsdi[x1, x2] > 0,
    x2dsdi[x1, x2]], {x1, -2, 2}, {x2, -2, 2}, Contours → {0},
    ContourShading → False, ContourStyle → {Red, Thick, Dashed}];

```

```
Show[coexInt, coexBnd, x1isoES, x1isoNES, x2isoES, x2isoNES]
```

