```
> ndot:=n*((x*n)/(1+n)-1-n);  #resident dynamics (monomorphic)
```

$$ndot := n\left(\frac{x\,n}{1+n} - 1 - n\right)$$

```
> solve(ndot=0,n);
```

$$0, \frac{x}{2} - 1 + \frac{\sqrt{x^2 - 4\,x}}{2}, \frac{x}{2} - 1 - \frac{\sqrt{x^2 - 4\,x}}{2}$$

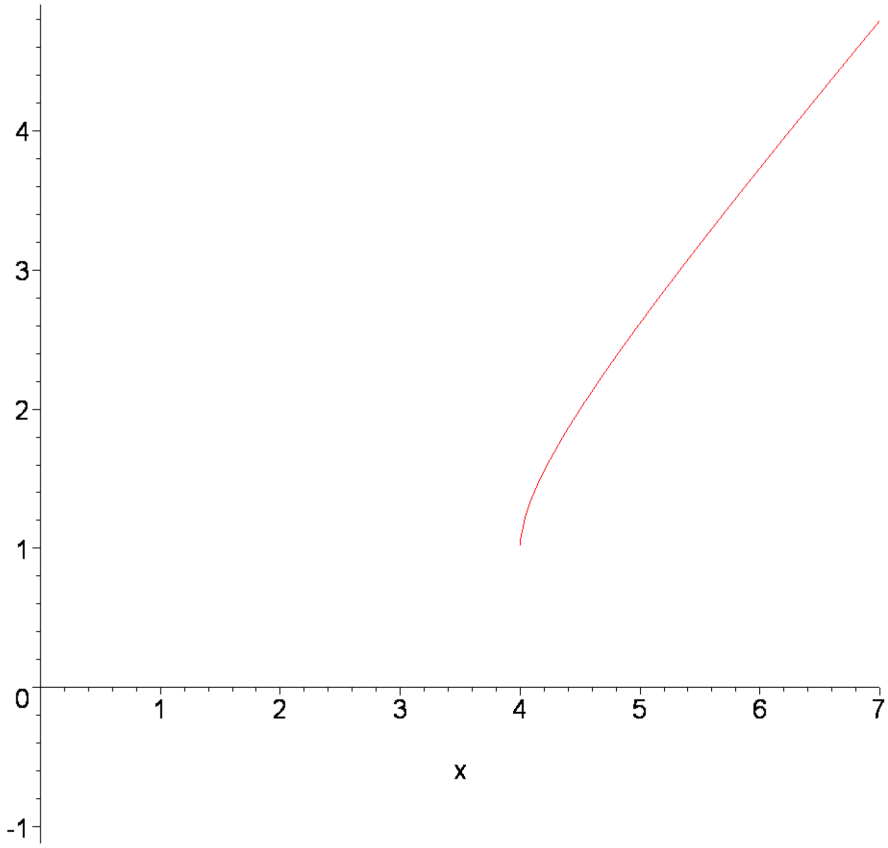
```
> a:=x->1/2*x-1+1/2*(x^2-4*x)^(1/2);
```

$$a := x \rightarrow \frac{1}{2}\,x - 1 + \frac{1}{2}\sqrt{x^2 - 4\,x}$$

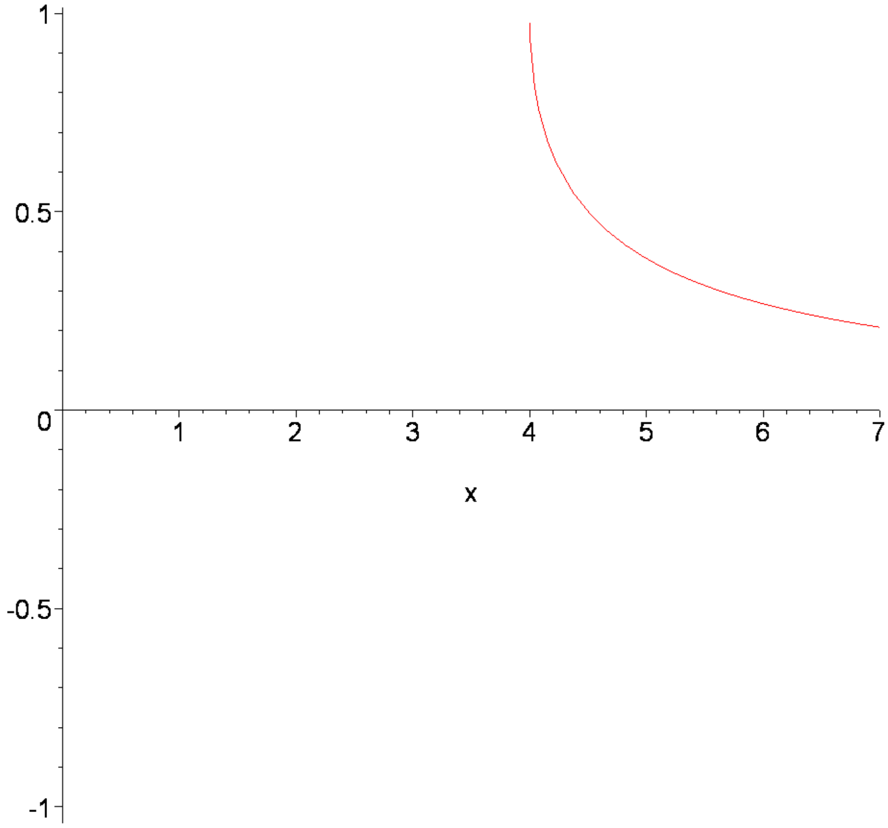
```
> b:=x->1/2*x-1-1/2*(x^2-4*x)^(1/2);
```

$$b := x \rightarrow \frac{1}{2}\,x - 1 - \frac{1}{2}\sqrt{x^2 - 4\,x}$$

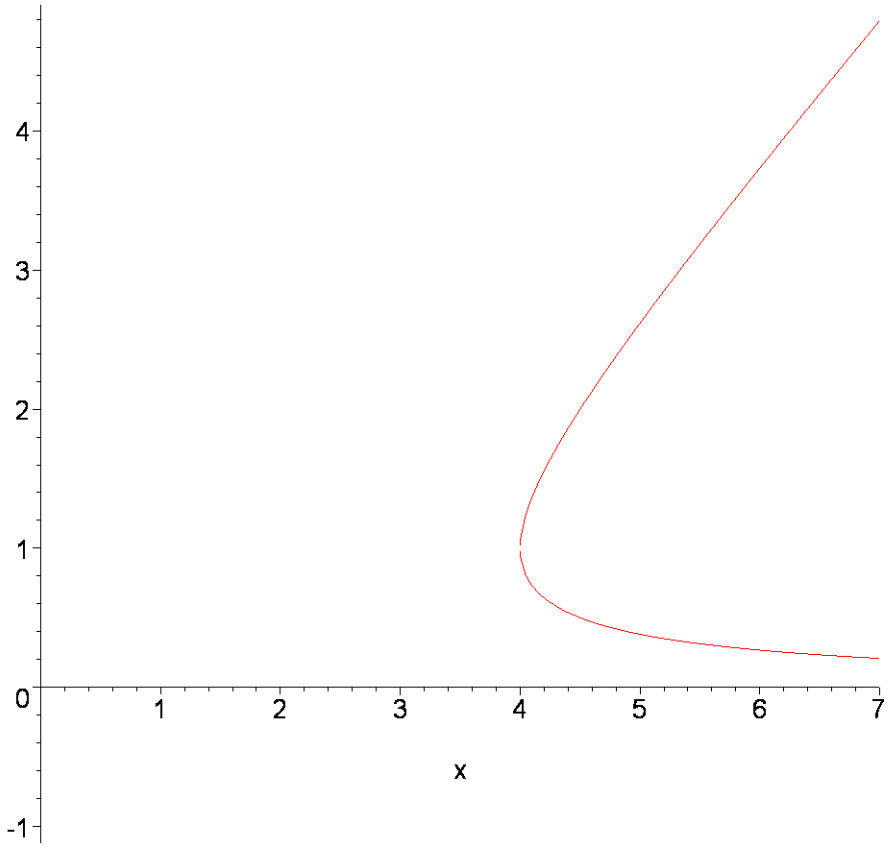
```
> plot(a(x),x=0..7);  #note that there are no real valued
  solutions of a(x), if 0<x<4
```



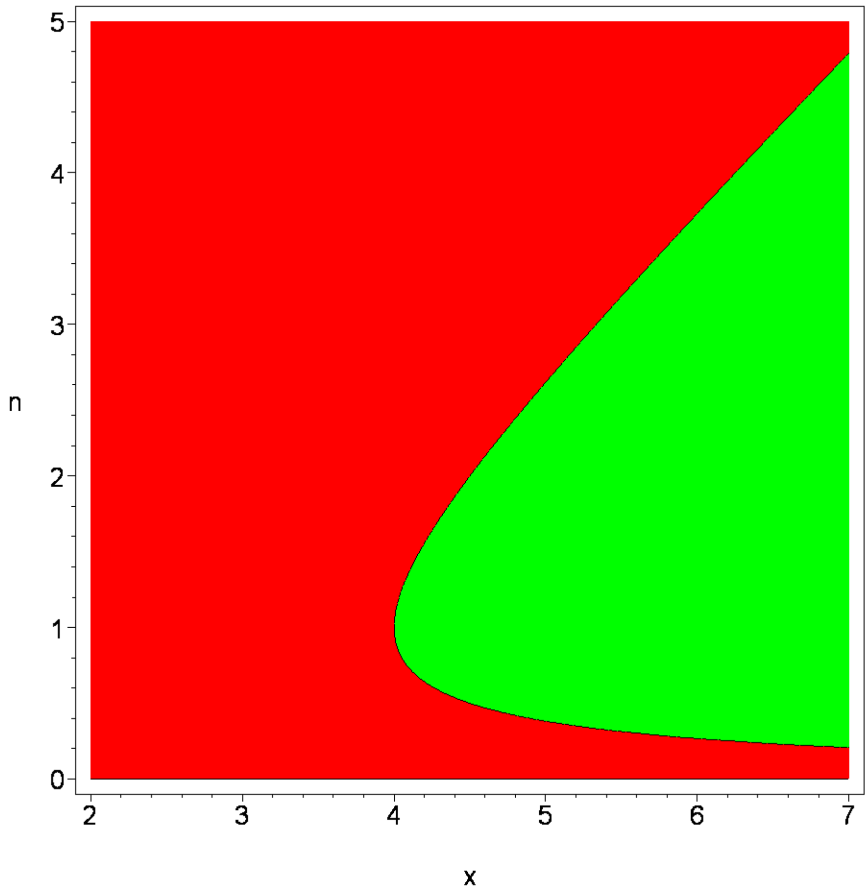

```
> plot(b(x),x=0..7);
```




```
> c:=plot(a(x),x=0..7):
> d:=plot(b(x),x=0..7):
> with(plots):
> display({c,d});  #combines plots of a(x) and b(x)
```




```
> with(plots):
> contourplot(ndot,x=2..7,n=0..5,contours=[0],filled=true,coloring
  =[red,green],grid=[100,100],axes=boxed);  #note how we don't
  need any of the previous calculations to make this plot, only
  the resident dynamics). In the green area we have ndot>0 and in
  the red ndot<0. Hence the upper curve is attracting and the
  lower repelling. So, if we take a fixed value of x, say x=6, we
  have convergence to a positive equilibrium density, if we start
  above the Allee threshold given by the lower curve.
```



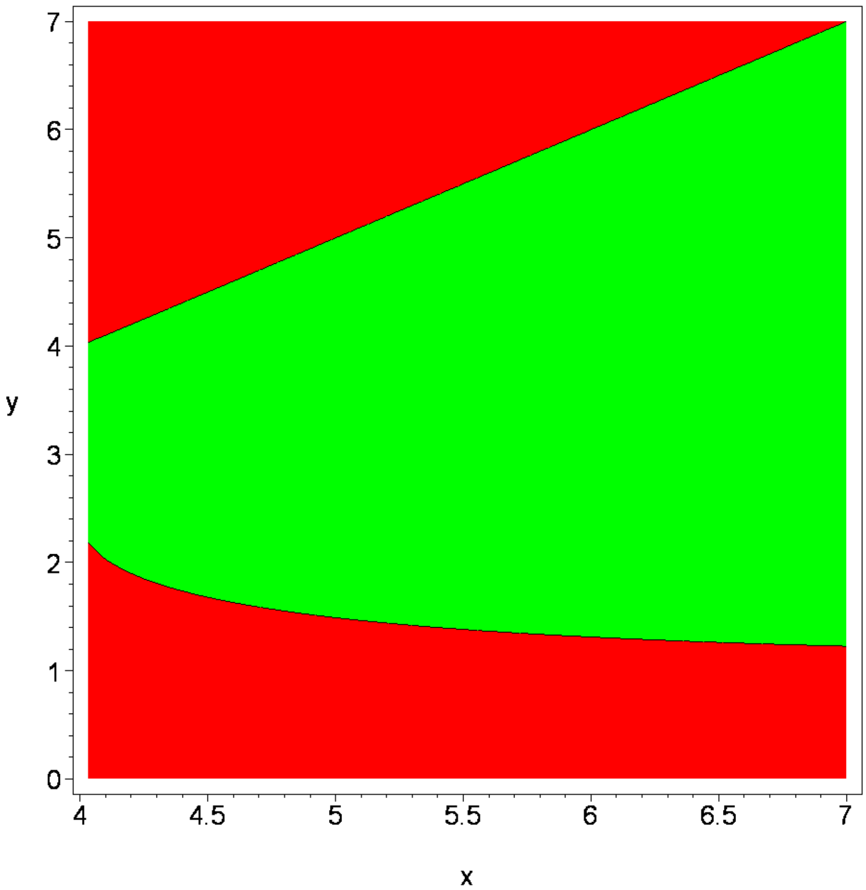

```
> subs(m=0,n=a(x),(y*(n+m)/(1+n+m)-1-(m+exp(y-x)*n))); #invader
dynamics, set (substitute) mutant density to zero and resident
density to the equilibrium density. Here it is reasonable to
pick the stable equilibrium density.
```

$$\frac{y\left(\frac{x}{2}-1+\frac{\sqrt{x^2-4x}}{2}\right)}{\frac{x}{2}+\frac{\sqrt{x^2-4x}}{2}}-1-e^{(y-x)}\left(\frac{x}{2}-1+\frac{\sqrt{x^2-4x}}{2}\right)$$

```
> s:=(x,y)->y*(1/2*x-1+1/2*(x^2-4*x)^(1/2))/(1/2*x+1/2*(x^2-4*x)^(
1/2))-1-exp(y-x)*(1/2*x-1+1/2*(x^2-4*x)^(1/2));  #invasion
fitness of a rare mutant with the res.pop. at the stable
equilibrium
```

$$s := (x, y) \to \frac{y\left(\frac{1}{2}x-1+\frac{1}{2}\sqrt{x^2-4x}\right)}{\frac{1}{2}x+\frac{1}{2}\sqrt{x^2-4x}}-1-e^{(y-x)}\left(\frac{1}{2}x-1+\frac{1}{2}\sqrt{x^2-4x}\right)$$

```
> contourplot(s(x,y),x=0..7,y=0..7,contours=[0],filled=true,colori
ng=[red,green],grid=[100,100],axes=boxed); #PIP
```


7
6
5
4
y
3
2
1
0
4
4.5
5
5.5
6
6.5
7
x

>

>

>