The Lotka - Volterra asymmetric competition model

Full population dynamics

$$\frac{d}{dt} n_i = r[x_i]\, n_i \left(1 - \frac{\sum_{j=1}^{l} a[x_i, x_j]\, n_j}{k[x_i]}\right) \qquad (i = 1, \ldots, l)$$

MONOMORPHIC RESIDENT POPULATION

- **Monomorphic resident population equilibrium:**

```
n[x_] := k[x];
```

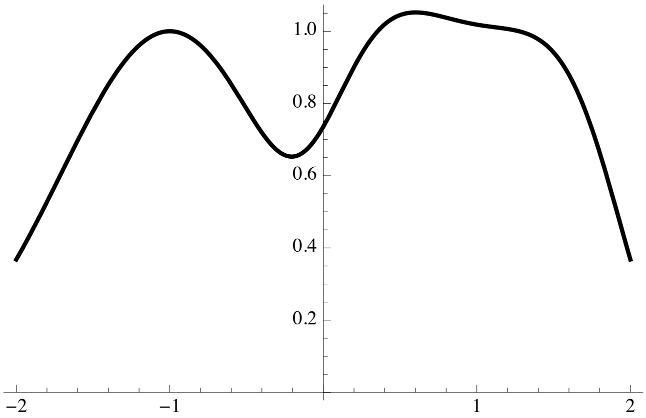
- **Invasion fitness and its derivatives:**

```
smo[x_, y_] := r[y] (1 - a[y, x] n[x] / k[y]);
dsmo[x_] := (∂y smo[x, y]) /. {y → x};
ddsmo[x_] := (∂y,y smo[x, y]) /. {y → x};
```

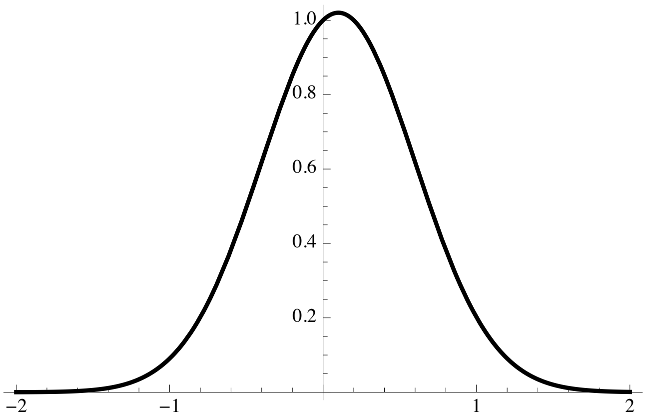
- **Default parameter values and functions :**

```
r[x_] := 1;
k[x_] := Exp[- (x - δ)^4] + Exp[- (x + δ)^2];
δ = 1;
a[x_, y_] := Exp[-α (x - y)^2 - β (x - y)];
α = 2;
β = -0.4;
```

```
Plot[k[x], {x, -2, 2},
 PlotStyle -> {Black, Thick}, AxesOrigin → {0, 0}]
```




```
Plot[a[x, 0], {x, -2, 2},
 PlotStyle -> {Black, Thick}, AxesOrigin → {0, 0}]
```

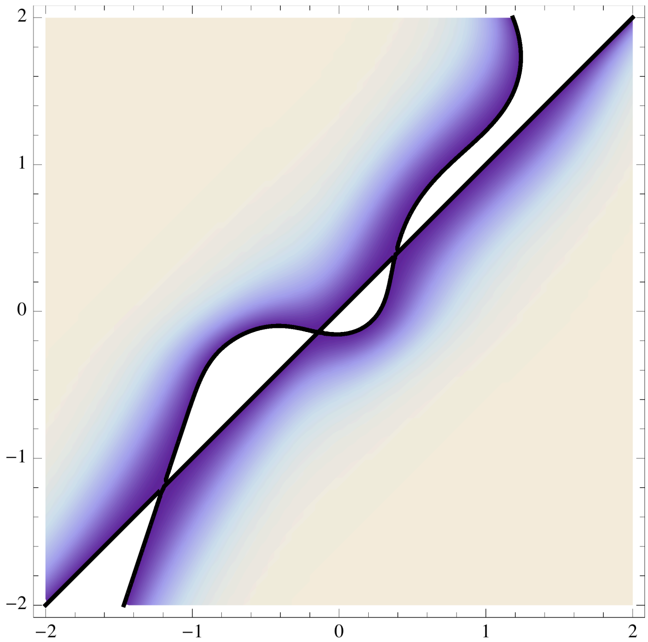


- ## Pairwise invadability plot (PIP):

```
PIPbnd = ContourPlot[s_mo[x, y], {x, -2, 2}, {y, -2, 2},
   Contours → {0}, ContourStyle → {Black, Thick},
   ContourShading → False, PlotPoints → 100];

PIPint = DensityPlot[If[s_mo[x, y] > 0, s_mo[x, y]],
   {x, -2, 2}, {y, -2, 2}, PlotPoints → 100];
```

```
Show[PIPint, PIPbnd]
```



- **Bifurcation plot singular strategy x versus parameter β:**

```
nDens = DensityPlot[n[x], {β, -1.5, 1.5}, {x, -2, 2}];

grad = StreamPlot[{0, ds_mo[x]}, {β, -1.5, 1.5},
    {x, -2, 2}, StreamStyle → Arrowheads[0.04]];

xES = ContourPlot[If[dds_mo[x] ≤ 0, ds_mo[x]],
    {β, -1.5, 1.5}, {x, -2, 2}, Contours → {0},
    ContourStyle → {Black, Thick},
    ContourShading → False, PlotPoints → 30];

xNES = ContourPlot[If[dds_mo[x] > 0, ds_mo[x]], {β, -1.5, 1.5},
    {x, -2, 2}, Contours → {0}, ContourStyle → {Red, Thick},
    ContourShading → False, PlotPoints → 30];
```

- **Singular strategy x versus asymmetry parameter β:**

```
Show[nDens, grad, xES, xNES]
```